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Comparsion between the Twisting and the Quasi Continuous Controller for a Coupled Tanks System

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***Abstract*— In this paper, two Second order sliding mode controllers have been successfully used to regulate the liquid level in the second tank of a coupled tanks system. The effectiveness of the controllers is verified through computer simulations. Comparison between the controllers is based on the time domain performance measures such the rise time, settling time and the integral absolute error. Results showed that controllers are able to regulate the liquid level with small differences in their performance.**

***Keywords*— Coupled tank system, sliding mode, twisting controller, quasi-continues controller**

I. INTRODUCTION

In process control, the control of liquid level in multiple connected tanks by controlling the liquid flow is a typical nonlinear control problem that is present in many industrial processes. Many Researchers have attempted the design and implementation of controllers for the liquid level of a coupled tanks system such as PID type controllers [1], a parallel structure of fuzzy PID control systems [2], a nonlinear constrained predictive algorithms based on feedback linearization control [3] and, fractional PID controller [4] have also been used to control coupled tanks systems.

Sliding mode control is an efficient method for robust control of uncertain systems [5,6,7]. The basic idea of the first order sliding mode control (1-SMC) is to let the system converge towards a selected surface and then to stay there in spite of uncertainties and disturbances. The first order sliding mode control (1-SMC) method can be designed by performing two steps. The first step is to select a appropriate sliding surface to constrain the state trajectory on it. The second step is the design of a discontinuous control law to force the system state to reach the designed surface preferably in finite time. 1-SMC requires sliding variable relative degree (the relative degree is defined as the order of the derivative of the controlled variable, in which the control input appears explicitly) to be equal to one with respect to the control input which limits the choice of the sliding variable. The 1-SMC also used to regulate the liquid level. A input-dependent sliding surface has been used in [8] to regulate the liquid level in a coupled tanks system. A sliding mode controller which has a state varying sliding surface parameter has been designed in [9]. A neuro-fuzzy-sliding mode controller using nonlinear sliding surface has been proposed in [10].

In the addition to the restriction regarding the relative degree of 1-SMC has the drawback of the chattering due to the high switching frequency of the control. The drawbacks of 1-SMC can be successfully eliminated by the use of higher order sliding mode controllers (HOSMC). HOSMC force the sliding variable and its (r-1) successive derivatives to zero. There is no restriction on relative degrees. As the high frequency control switching is pushed in the higher derivative of the sliding variable, chattering is significantly reduced. Another feature of HOSMC is the detailed mathematical model of the plant is not required. The most widely used HOSMC are second order sliding mode controllers (2-SMC). Examples for 2-SMC controllers are the widely used twisting and its modified variant the super twisting controllers, the quasi-continuous controllers, the suboptimal control algorithm, and the control algorithm with prescribed convergence law. Khan and Spurgeon [10] applied a second order sliding mode control idea to control a coupled tank system.

This paper presents the use of two 2-SMC to regulate the water level of the second tank in the coupled tanks system. These controllers are, twisting (TA), and the quasi-continues controller (QCC). The 2-SMC requires only the error (the difference between the reference set point and the output of the system. To obtain the needed first derivative of the error a sliding mode differentiator is used. The comparison of the performance of the controllers is based on time domain control performance measures.

The remaining structure of this paper is as follows. In the next section the dynamical model of the coupled tank system will be revisited. Section three briefly provides the basics 2-SMC. In section four 2-SMC controllers will be briefly described. In section 5 the simulation results from the application of the controller will be presented and discussed. Finally section 6 concludes this paper.

II. MATHEMATICAL MODELING OF THE COUPLED TANKS SYSTEM

Figure 1 shows a schematic diagram of the two coupled tanks system. The tanks system consists of two connected tanks. A pump supplies the water into the first tank. The second tank is filled from the first tank via a connecting pipe. An outlet is located at the bottom of the second tank to change the output flow q_2. The Mathematical model of the coupled tanks system is nonlinear and can be derived as follows:

Applying the flow balance equation for tank 1 and 2:

$$\frac{dh_1}{dt} = \frac{1}{A}(q - q_1) \quad (1)$$

$$\frac{dh_2}{dt} = \frac{1}{A}(q_1 - q_2) \quad (2)$$

In (1) and (2) q1 and q2 are defined as:

$$q_1 = a_1\sqrt{2g(h_1 - h_2)} \quad \text{for} \quad h_1 > h_2 \quad (3)$$

$$q_2 = a_2\sqrt{2gh_2} \quad \text{for} \quad h_2 > 0 \quad (4)$$

Where, h_1 and h_2 are the water level in Tank 1 and Tank 2, respectively. q is the inlet flow rate, q_1 is the flow rate from Tank 1 to Tank 2. A is the cross-section area for both Tank, a_1 is the area of pipe connecting the two tanks, a2 is the area of the outlet, and g is the constant of gravity. If the inlet flow q is selected as input and the liquid level h_2 in the second tank as output, the system can be considered as a single input single output system (SISO). The two tanks system can be modeled by the following two differential equations:

$$\frac{dh_1}{dt} = -k_1\mathrm{sign}(h_1 - h_2)\sqrt{|h_1 - h_2|} + \frac{q}{A} \quad (5)$$

$$\frac{dh_2}{dt} = k_1\mathrm{sign}(h_1 - h_2)\sqrt{|h_1 - h_2|} - k_2\sqrt{h_2} \quad (6)$$

The parameters k1and k2 are defined by

$$k_1 = \frac{a_1\sqrt{2g}}{A} \quad (7)$$

$$k_2 = \frac{a_2\sqrt{2g}}{A} \quad (8)$$

Note q is always positive which means that the pump can pumps water into the tank ($q \geq 0$). At equilibrium, for constant water level set point, the derivatives w.r.t of the water levels in the two tanks must be zero so that the following condition can be written:

$$\frac{dh_1}{dt} = \frac{dh_2}{dt} = 0 \quad (9)$$

therefore, using (9) in (5) and (6), the following algebraic relationships holds:

$$-k_1\mathrm{sign}(h_1 - h_2)\sqrt{|h_1 - h_2|} + \frac{q}{A} \quad (10)$$

$$k_1\mathrm{sign}(h_1 - h_2)\sqrt{|h_1 - h_2|} - k_2\sqrt{h_2} \quad (11)$$

The equilibrium flow rate q can be calculated as:

$$q = -Ak_1\mathrm{sign}(h_1 - h_2)\sqrt{|h_1 - h_2|} \quad (12)$$

To satisfy the constraint on the input flow rate the term $k_1\mathrm{sign}(h_1 - h_2) \geq 0$, which implies $h_1 \geq h_2$ .Then the dynamics model can be written as:

$$\frac{dh_1}{dt} = -k_1\sqrt{h_1 - h_2} + k_2\sqrt{h_2} \quad (13)$$

$$\frac{dh_2}{dt} = -k_1\sqrt{h_1 - h_2} - 2k_2\sqrt{h_2} + \frac{1}{A}u \quad (14)$$

Using the transformation in (16) ,(13) and (14) can be written as in (15).

$$\begin{gathered} x_1 = h_1 \\ x_2 = -k_1\sqrt{h_1} + k_2\sqrt{h_1 - h_2} \end{gathered} \quad (15)$$

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = f(x,t) + g(x,t)u \\ y = x_1 \end{cases} \quad (16)$$

It can be easily checked that f(x,t), and g(x,t) in (15) have the following form:

$$f(x,t) = \frac{k_1k_2}{2}\left(\frac{\sqrt{h_2}}{\sqrt{h_1 - h_2}} - \frac{\sqrt{h_1 - h_2}}{\sqrt{h_2}}\right) + \frac{k_1{}^2}{2} - k_2^2 \quad (17)$$

$$g(x,t) = \frac{k_2}{2A}\frac{1}{\sqrt{h_1 - h_2}} \quad (18)$$

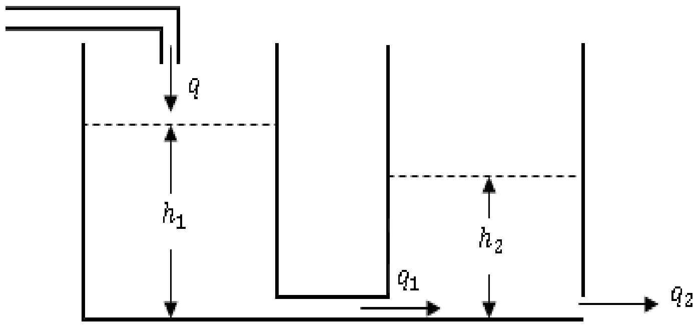

Fig. 1. A schematic diagram of the two coupled tanks system.

III. SECOND ORDER SLIDING MODE CONTROL

2-SMC control is a subset of HOSMC differs from the 1-SMC by including the first order derivative of the sliding variable while maintaining the same robustness and performance as that of the 1-SMC. $\sigma \in \mathbb{R}$ in an output of (19) to be exactly stabilized in finite time at $\sigma = 0, u \in \mathbb{R}$ is the control input and $x \in \mathbb{R}^n$ is the state. If the output σ have a fixed and known relative degree $r \in \mathbb{R}^n$. For the positive constants Km, KM, and C the following inequalities

$$0 < K_m \leq b \leq K_M \quad (19)$$

$$|f| \leq C$$

hold globally. 2-SM controllers may be considered as controllers for the following differential inclusion (levant, 2003)

$$\ddot{\sigma} \in [-C, C] + [K_m, K_M]u \tag{20}$$

2-SM controllers allows to solve the problem of finite-time stabilization of a black box system as shown in Figure 2. The only information needed from the system is the output. The required derivative can be obtained by using HOSM arbitrary order differentiator [12].The real time differentiator has the general form as in the following set of equations. In (28) f(t) represents signal to be differentiated, k-1 times.

$$\begin{aligned} \dot{z}_0 &= v_0 \\ \dot{z}_1 = v_1 v_0 &= -\lambda_k L^{\frac{1}{k+1}} |z_0 - f(t)|^{\frac{k}{k+1}} sign\big(z_0 - f(t)\big) \\ &\quad + z_1 \\ v_1 &= -\lambda_{k-1} L^{1/k} |z_1 - v_0|^{(k-1)/k} sign(z_1 - v_0) + z_2 \\ \dot{z}_{k-1} = v_{k-1} &= -\lambda_1 L^{1/2} |z_{k-1} - v_{k-2}|^{k/(k+1)} sign(z_{k-1} \\ &\quad - v_{k-2}) + z_k \\ \dot{z}_k &= \lambda_0 L sign(z_k - v_{k-1}) \end{aligned} \tag{21}$$

A first order differentiator can be obtained by setting k=1 and has the form:

$$\begin{aligned} \dot{z_0} &= -\lambda_1 |z_0 - f(t)|^{\frac{1}{2}} sign(z_0 - \mathrm{f(t)}) + z_1 \\ \dot{z_1} &= -\lambda_0 \mathrm{sign}(z_0 - \mathrm{f(t)}) \end{aligned} \tag{22}$$

Where z_0 and z_1 are the estimation of $f(t)$ and $\dot{f}(t)$ respectively. The parameters $\lambda_1 = 1.5L^{1/2}$, $\lambda_0 = 1.1L$, and L a positive constant to be selected via simulation as recommended in [12].

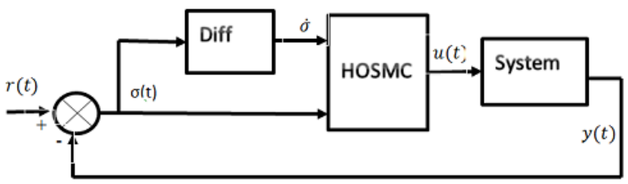

Fig. 2. Block diagram of a black box controller.

IV. Controllers Design

A. *Twisting Controller*

The twisting controller is historically the first 2-sliding mode controller which was proposed. It is defined by the formula

$$u(t) = -(r_1 sign(\sigma) + r_2 sign(\dot{\sigma})) \tag{23}$$

This controller guarantees the appearance of a 2-sliding mode $\sigma = \dot{\sigma} = 0$ attracting the trajectories of the sliding variable dynamics in finite time if the controller; parameters r_1 and r_2 satisfy the following conditions:

$$\begin{gathered} r_1 > r_2 > 0 \\ (r_1 + r_2)K_m - C > (r_1 - r_2)K_M \\ (r_1 - r_2)K_m > 0 \end{gathered} \tag{24}$$

B. *Quasi-Continues Controller (QCC)*

The homogeneous Quasi continuous (QCC) controller [14 requires the first order derivative of the sliding surface σ .With the use of differentiator in (22) the second order quasi continues- sliding mode controller can be written as:

$$\mathrm{u(t)} = -\alpha \frac{z_1 + \beta |\sigma|^{\frac{1}{2}} \mathrm{sign}(\sigma)}{|z_1| + \beta |\sigma|^{1/2}} \tag{25}$$

The parameters α and β in (25) are positive constant to be selected via computer simulation.

V. Simulation Results and Discussion

The characteristic dynamics model of the coupled tank system are presented in Table 1 [8].The control input is restricted to be between umin= 0 and umax= 50 [cm3/s].The computer simulations were performed using a time interval of [0:150] s. The parameters of the controller were optimized using a step input with a final value of 6 cm for the water level. For the others tested water levels no more adjustment of the controllers parameters were performed. Through extensive simulations the optimum controllers parameters for the two controllers tested in this study are reported in Table 2.

TABLE I. Characteristic of the coupled tanks system

Gravitational rate g	981 cm/s^2
Cross-sectional area of both tanks	208.2 cm^2
Area of the connecting pipe a_1	0.58 cm^2
Area of the outlet a_2	0.24 cm^2

TABLE II. *Controllers parameters*

Controller	STC		QCC	
parameters	α	λ	α	β
value	220	14	65	2

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Figure 3 shows the regulation performance for the tested controllers for a desired level of 6 cm. From the figure it can be concluded that the controllers regulate the water level successfully with approximately the same performance. Note that the controllers are able to regulate any desired water levels without new adjusting of any parameter.

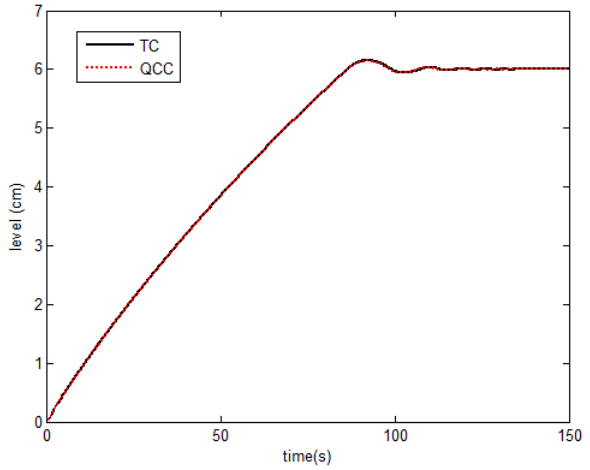

Fig. 3. level tracking test for the different controllers

The behaviour of sliding surfaces/error are represented in Fig. 4 for the desired level of 6 cm. As can be seen from the figure the two controllers show typical sliding mode behaviour that is the error reaches 0 in finite time, and stay 0 afterwards. The derivative of error w.r.t. is also shown for the two controllers. The error derivative converge also to zero.

Fig. 5 shows the control signal of the two controllers. The controllers have the same control signal until reaching the sliding surface.

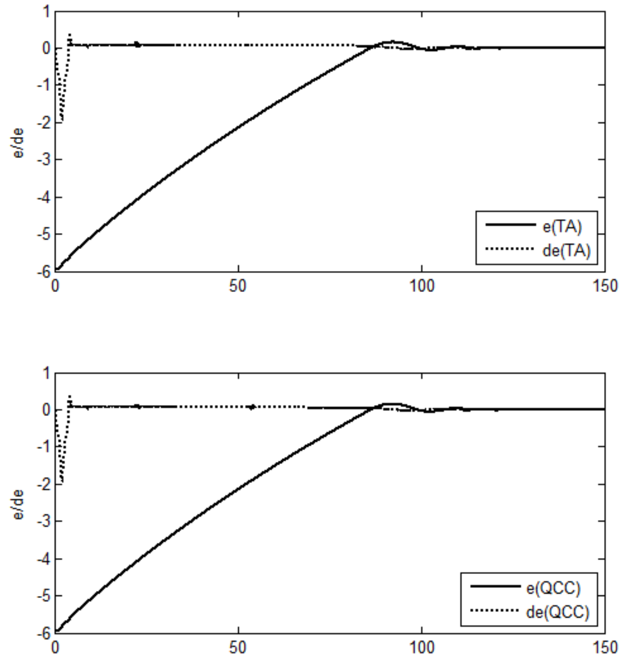

Fig. 4. Error and the its derivative for the two controllers

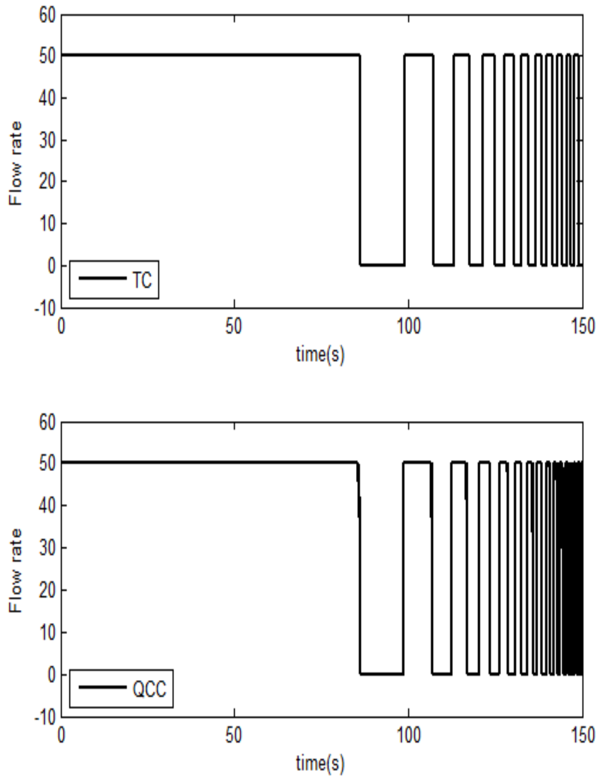

Fig. 5. The control signals of the designed controllers

Fig. 6.

To compare the performance of the controllers. The time domain performance measures such as the rise time, the settling time, the percentage overshoot, and the IAE are used. The settling time is defined as the time required for the response to settle within 1% of the steady state value. The rise time is defined as the time required for the output to change from 10% to 90% of its final value. The IAE is given by the following eq.

$$IAE = \int_0^t |\sigma| \, dt \qquad (26)$$

The performance measures have been computed for three different desired level of 3 ,6, 8 cm as listed in table 3. As can be seen from the table the two controllers have approximately the same rise time in all cases. The QCC controller has a slightly less settling, overshoot, and IAE.

The results of the tracking test of the five controllers using a square signal reference input are shown in Fig. 6. As excepted the tracking performance for the all the controller is good. In term of the IAE the values obtained are listed in table 4.

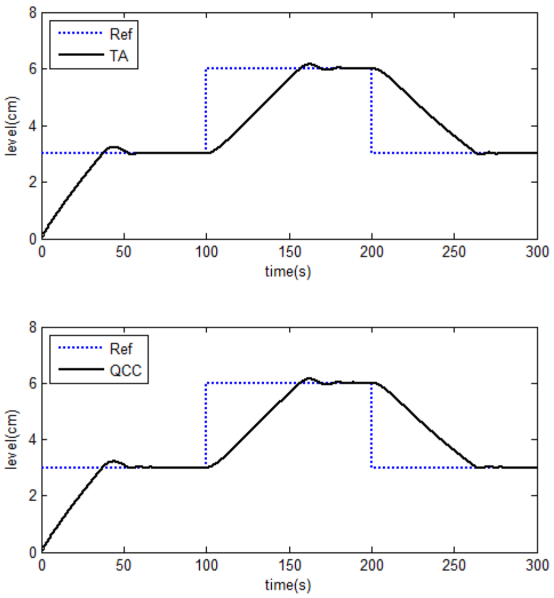

Fig. 7. Level tracking test for the two controllers

TABLE III. *TIME DOMAIN PERFORMANCE MEASURES FOR DIFFERENT H VALUES.*

H [cm]	Controller	T_r [s]	T_s [s]	Os[%]	IAE
3	TA	30.010	51.783	7.506	54.823
	QCC	30.010	51.250	7.085	54.580
6	TA	69.335	95.179	2.522	236.066
	QCC	69.335	94.630	2.393	235.909
8	TA	101.384	123.330	1.392	447.400
	QCC	101.384	123.330	1.327	447.329

VI. CONCLUSIONS

In this paper, two second order sliding mode controllers namely the twisting, and the quasi-continues controllers have been successfully designed to regulate the water level in the second tank of a coupled tanks system. The efficacy and usability of the proposed controllers are verified through computer simulation tests. Comparison between the controllers is based on the time domain performance measures. Results showed that both controllers are able to regulate the water level without major differences in their performance.

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