Completeness-Compatibility and Reviews for Finite Element Analysis Results

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Abstract

This paper presents the theory and solution implementation of Completeness-Compatibility and Reviews for finite element analysis results as applied to nonlinear problems like geometric, material and contact nonlinearities. In addition, guidelines check given for Review and interpret FEA results, since all the numerical methods including FEA are approximate and one should not believe the results blindly.

1. Introduction

Finite Element Analysis has become an integral part of design process in aerospace, automotive, Nuclear reactors and various consumer goods and industrial products. FE analysis helps in accurating design and development of products by minimising number of physical tests, thereby reducing cost of prototyping and testing. Here an attempt is made to throw a light on Stress-Strain measures and Convergence issues.

2. Linear and Non-Linear FEA

2.1. Features of Linear FEA Problems

When structure response (deformation, stress and strain) is linearly proportional to the magnitude of the load (force, pressure, moment, torque, temperature etc.) then analysis of such a structure is known as linear analysis.

Displacements vary linearly with applied loads. Thus stiffness is constant.

Changes in geometry due to displacement are assumed to be small and hence are ignored. Original or Undeformed state is always used as reference state.

Stress-Strain Relationship linear up to proportional/elastic limit.

2.2. Features of Non-Linear FEA Problems

When the load to response relationship is not linearly proportional, then analysis comes under Non-linear analysis.

$$[K(u)]\ \{D\} = \{F\}$$

This behavior leads to an FEA Formulation with Stiffness K response that depends on deformation. Since Direct Inversion of the Stiffness Matrix is impossible, other methods must be used to solve Non-linear Problems that includes Incremental or Stepwise Procedures, Iterative or Newton methods, Mixed type- Iterative Techniques.

The solution to Non-linear problems may be sensitive to Initial and/or Boundary Conditions.

In General Superposition and Scalability will not apply to Non-linear Problems.

2.3. Types of Non-Linear analysis

Many problems of engineering interest involve non-linear behavior. Such behavior commonly arises from the following three sources:

***2.3.1 Non-linear Material Behavior*:** This is one of the most common forms of non-linearity and would include non-linear elastic, plastic, and viscoelastic behavior. For thermal problems, a temperature dependent thermal conductivity will produce non-linear equations.

2.3.2. Large Deformation Theory (Geometric Non-linearity): If a continuum body under study undergoes large finite deformations, the strain-displacement relations will become non-linear.

Also for structural mechanics problems under large deformations, the stiffness will change with deformation thus making the problem non-linear. Buckling problems are also non-linear.

2.3.3. Non-linear Boundary or Initial Conditions (Contact): Problems involving contact mechanics normally include a boundary condition that depends on the deformation thereby producing a non-linear formulation. Thermal problems involving melting or freezing (phase change) also include such non-linear boundary conditions.

3. Convergence Issues

There is no doubt that finite-element analysis is getting a bigger role in development projects. One reason is that it helps slash expensive prototype testing. The technology is also seen as another way to improve product integrity.

Despite FEA's reputation for accurately pinpointing weak spots in designs, a few faulty assumptions and organizational flaws may render analysis work unusable. For instance, some companies treat FEA as an extension to CAD packages. In fact, it requires specialized training all its own.

In FEA Convergence means multiple meaning like Mesh convergence, Time Integration accuracy, Convergence of non-linear solution procedure and solution accuracy.

A sudden change in stiffness or sudden change in load can cause convergence difficulty. To settle this issue, it is always better to break sudden changes down into a many small incremental changes. i.e. we must use ramped loading small time steps.

Convergence requirements can be grouped into three:

3.1. Completeness

The elements must have enough approximation power to capture the analytical solution in the limit of a mesh refinement process.
The element shape functions must represent exactly all polynomial terms of order ≤ m in the Cartesian coordinates. A set of shape functions that satisfies this condition is called m-complete. Note that this requirement applies at the element level and involves all shape functions of the element. The completeness is satisfied if the sum of the shape functions is unity and the element is compatible.

3.2. Compatibility.

The shape functions should provide displacement continuity between elements. Physically these insure that no material gaps appear as the elements deform. As the mesh is refined, such gaps would multiply and may absorb or release spurious energy.

3.2. Stability.

The system of finite element equations must satisfy certain well posedness conditions that preclude non-physical zero-energy modes in elements, as well as the absence of excessive element distortion.

Stability may be informally characterised as ensuring that the finite element model enjoys the same solution uniqueness properties of the analytical solution of the mathematical model. For example, if the only motions that produce zero internal energy in the mathematical model are rigid body motions, the finite element model must inherit that property. Since FEM can handle arbitrary assemblies of elements, including individual elements, this property is required to hold at the element level.

Completeness and compatibility are two aspects of the so-called consistency condition between the discrete and mathematical models. A finite element model that passes both completeness and continuity requirements is called consistent. This is the FEM analog of the famous Lax-Wendroff theorem, which says that consistency and stability imply convergence.

4. Review FEA Results

Poorly defined FEA objectives lead to wasted effort. When the analyst receives a project requirement saying only "perform FEA on the control arm," then objectives need work. Goals should cover why the analysis is needed, the expectations, and how results will be used. The perform-FEA syndrome often stems from bureaucratic misunderstanding rather than engineering need for the results.
Lack of project monitoring leads to time-and-cost overruns. Standard checks should take place during an FEA project to monitor progress and provide guidance for analysts. This helps the manager follow the project and quickly spot problems. Failing that, mistakes may never be uncovered.
No lessons-learned database means mistakes are often repeated. Each project should be well documented so third parties can recreate results long after the analyst is gone. A sample of completed results should be confirmed through testing. Where there are discrepancies, an appendix to the project report should address the problem. Users need to verify results with experiments until they get confidence in the method.
An FEA report should be self-explanatory and contain enough information to duplicate analysis results. A good report together with backup, provides sufficient detail for rerunning the analysis without any additional instructions.
No real commitment to FEA is an attribute of managers with a short attention span who become disappointed quickly. Building confidence in the method, and accumulating and maintaining in-house expertise, takes years of considerable effort and commitment. Nobody should expect instant savings.
Once introduced, FEA is considered an omnipotent method to assure quality designs. But a few unsuccessful application attempts can make people give up not realizing that the discipline failed simply because of lack of a quality assurance system.

Here are a few check points where the FEA manager should provide guidance to the analyst and designer.

Are the loads, supports, and modeling approach acceptable?
Are the mesh and elements appropriate?
Is the error value within specified criteria?
Do results agree with an independent analysis method?

5. Conclusion

The finite element analysis (FEA) is a computer-based technique which can be used to analyse linear and nonlinear characteristic materials. FEA is also a popular numerical methodology that is widely used to solve engineering problems. Major applications for FEA include static, dynamic and thermal characterizations of mechanical components or parts. Advances in computer hardware have made FEA easier and very efficient into solving complex engineering problems on desktop computers.

A computer-integrated manufacturing system is use of computers to integrates CAD (computer-aided design), CAE (computer-aided engineering), CAM (computer-aided manufacturing) and business functions is not the same as a "lights-out" factory, which would run completely independent of human intervention, although it is a big step in that direction. In earlier days FEA was in localised area but now it is plant wise or globalised.

10. References

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