International Journal of Engineering Research & Technology (IJERT)
ISSN: 2278-0181
Vol. 3 Issue 11, November-2014

Comprehensive Analysis based on Various Communication Parameters for Spectrum Sensing in Cognitive Radio Systems

Meghana Joshi
P.E.S's Modern College of Engineering
Shivaji Nagar, Pune, India

S. D. Borde
P.E.S's Modern College of Engineering
Shivaji Nagar, Pune, India

***Abstract*—The radio spectrum is the most vital resource that needs to be utilized efficiently. The cognitive radio technology has been proposed to overcome the problem of spectrum under utilization. One of the key challenges for a cognitive radio system is to detect the presence of primary licensed users over the entire dynamic spectrum at a specific time and a particular geographic location. Thus for effective band utilization cognitive radio provides a unique solution in terms of spectrum sensing. In spectrum sensing, the aim of secondary user is to identify the occupancy or vacancy of a particular radio channel over the entire radio spectrum within a short detection time. This paper presents the performance analysis of energy detection and the match filter detection schemes of spectrum sensing. It also focuses on the comparative analysis of the effects of communication parameters like Signal to noise ratio, number of samples, noise uncertainty and dynamic threshold on the performance metric like probability of detection, probability of misdetection and probability of false alarm for both the methods. The numerical results show how noise uncertainty issue can be overcome by introduction of dynamic threshold for the schemes which are sensitive to noise uncertainty, especially in low SNR conditions. Another set of results show that energy detector works with minimal information about the primary transmitter signal whereas match filter detection algorithm outperforms well with less sensing time even in low SNR regimes. The simulations are plotted using MATLAB software.**

***Keywords—Cognitive radio, spectrum sensing, energy detection, match filter detection, noise uncertainty, dynamic threshold, probability of detection, probability of false alarm*.**

I. INTRODUCTION

The radio frequency Spectrum has become a sparse resource nowadays with the increasing demand of various wireless services and thus has made it vital to address spectrum scarcity problem. But fortunately, according to a recent survey made by Spectrum Policy Task Force (SPTF) within FCC the actual licensed spectrum is largely under-utilized in vast temporal and geographic dimensions [1]. If we examine a section of radio spectrum, we find that some frequency bands in the spectrum are heavily used while others are partially occupied or just vacant [2].This leads to an underutilization of radio spectrum. A solution to spectrum insufficiency can be initiated by allowing the unlicensed secondary users to access dynamically the under-utilized licensed bands wherever or whenever licensed users are not present. The main challenge with secondary user is that it should sense the PU signal without any interference.To accomplish this, the cognitive radio (CR) must constantly sense the spectrum in order to detect the recurrence of the primary user (PU). If the primary user is detected, the cognitive radio should at once vacate from that particular spectrum band which it was using, so that interference can be reduced to a minimal. This is enormously challenging task as the various primary users will be employing different data rates, modulation schemes and transmission powers in the presence of interference and uneven propagation environments generated by other secondary users. The Spectrum sensing is first and foremost step that needs to be performed for opportunities for secondary transmissions. But it is a very sensitive task since Cognitive Radio has to decide the best spectrum band while maintaining the Quality of service for the entire band of frequencies since interfering with other users is illegal. Spectrum holes are not constant, they migrate with frequency and time. The spectrum sensing algorithm should be quick enough to rapidly detect the changes in moving holes in real time for the entire spectrum [3]. Again in low SNR scenarios the noise power affects the hole detection process. Also the threshold used for PU detection depends on the noise statistics [4]. For spectrum sensing, three methods are usually used such as Match filter detection (MF), Energy Detection (ED) and Cyclostationary detection [1] [6] [7]. The main contribution of this paper is performance analysis of spectrum sensing algorithms that are based on primary transmitter detection like the energy detector and match filter detector and effects of Signal to noise ratio, number of samples and noise uncertainty and dynamic threshold on the performance parameters like probability of detection, probability of misdetection and probability of false alarm in cognitive radio systems in presence of Additive White Gaussian noise (AWGN). We also show how Energy detection algorithm is sensitive to noise uncertainty and leads to performance degradation, but introduction of dynamic threshold algorithm enhances the robustness of system by solving the noise uncertainty issue. The noise uncertainty issues, does not affect the performance of match filter detector, thus can perform well even in low SNR conditions. Match filter scheme requires less number of samples as compared to energy detection scheme to meet a given probability of detection constraint and thus sensing time required for match filter scheme is also less. But it needs complete information about the primary users signaling type. Energy Detector does not require prior information about the primary user. But it does not perform well in low SNR conditions. Lastly, simulations also revisit the fact, how usage of digital communication in a system can improve the overall performance of a system as compared to analog communication.

The rest of the paper is organized as follows: In Section II, Spectrum sensing model in AWGN is formulated. Section III focuses on various detection schemes like energy detector and match filter detector and effect of various communication parameters on the performance of these methods. Section IV show the simulation results of the various analysis performed using these two algorithms. Section V ultimately concludes the different findings observed using these algorithms.

II. SPECTRUM SENSING MODEL

The performance of spectrum sensing algorithm depends on different parameters like Signal to noise ratio, number of samples and noise uncertainty. The aim of spectrum sensing is to make a decision between the two hypotheses (choose H0 or H1) based on the received signal.

$$\begin{aligned} &H0: X(n) = W(n) \\ &H1: X(n) = S(n) + W(n) \end{aligned} \quad (1)$$

where S (n) is the primary user's transmitted signal, W (n) is the white noise and X (n) is the received signal at the CR node. H_0 and H_1 hypotheses denote that the primary user is present or not, respectively. The noise is assumed to be AWGN with zero mean and is a random process. The signal is assumed to be independent of noise. The signal to noise ratio is defined as the ratio of signal power to noise power [5]. The key metric in spectrum sensing are the probability of correct detection (Pd) and two types of errors in spectrum sensor. The first error occurs when the channel is vacant (H0) but the spectrum sensor decides that the channel is occupied, the probability of this event is the probability of false alarm (Pfa).The second error occurs when channel is occupied (H1), but the spectrum sensor decides that the channel is unoccupied, the probability of this event is probability of misdetection (Pm).

III. SENSING SCHEMES

A. ENERGY DETECTION ALGORITHM

It is a non coherent detection method that detects the primary signal based on the sensed energy. Due to its simplicity and no requirement of a priori knowledge of primary user signal, energy detection algorithm is the most popular sensing technique in spectrum sensing.

In scenarios where the signal X (n) is a not deterministic one and if only the average power of the signal is known, the energy detector is the most optimal choice. Thus energy detector involves estimation of energy of the received signal at receiver, and comparison with a set threshold to indicate whether primary signal is present or not. This detector can be expressed as [4].

$$D(Y) = \frac{1}{N}\sum_{n=0}^{N-1} X(n)X(n) \quad \begin{matrix} > \xi & H1 \\ < \xi & H0 \end{matrix} \quad (2)$$

where D(Y) is the decision variable, N is the number of samples and ξ is the decision threshold. Now if the noise variance is completely known, then from the central limit theorem the following approximations can be made [9].

$$D(Y|H0) \sim \mathrm{Normal}(\sigma_n^{\,2}, 2\sigma_n^{\,4}/N) \quad (3)$$

$$D(Y|H1) \sim \mathrm{Normal}(P + \sigma_n^{\,2}, 2(P + \sigma_n^{\,2})^2/N) \quad (4)$$

Where P is the average of signal power and σ_n^2 is the variance of noise. The expressions for the probabilities are

$$P_{fa} = Q\left(\frac{\xi - \sigma_n^{\,2}}{\sqrt{2\sigma_n^{\,4}/N}}\right) \quad (5)$$

$$P_d = Q\left(\frac{\xi - (P + \sigma_n^{\,2})}{\sqrt{2(P + \sigma_n^{\,2})^2/N}}\right) \quad (6)$$

where Q (·) is the standard Gaussian complementary cumulative distribution function (CDF) and Q^{-1} (·) is the inverse standard Gaussian complementary CDF. From (5) and (6) we can get relationship between N, SNR, P_d and P_{fa} after eliminating the threshold ξ

$$N = 2\,[Q^{-1}(P_{fa}) - Q^{-1}(P_d)]^2(SNR)^{-2} \quad (7)$$

1. *Energy Detector under noise uncertainty*

In realistic scenarios, we need to take into account the noise uncertainty factor that is caused due to noise power variations. So, we introduce a noise uncertainty factor ρ ($\rho > 1$) in the noise model. The distributional uncertainty of noise can be represented as in [7] $\sigma^2 \in [\sigma_n^{\,2}/\rho, \rho\sigma_n^{\,2}]$

Thus (5) and (6) are modified as

$$P_{fa} = Q\left(\frac{\xi - \rho\sigma_n^{\,2}}{\sqrt{2\rho^2\sigma_n^{\,4}/N}}\right) \quad (8)$$

$$P_d = Q\left(\frac{\xi - (P + \sigma_n^{\,2}/\rho)}{\sqrt{2(P + \sigma_n^{\,2}/\rho)^2/N}}\right) \quad (9)$$

Eliminating threshold ξ and equating (8) and (9), we get

$$N = 2\left[\rho Q^{-1}(P_{fa}) - (1/\rho + SNR)Q^{-1}(P_d)\right]^2 * (SNR - (\rho - 1/\rho))^{-2} \quad (10)$$

2. *Energy Detector with dynamic threshold*

Noise uncertainty causes the decline in sensing sensitivity and also serious interference to authorized users. This should be avoided in dynamic spectrum access technology. Thus a new concept of dynamic threshold is presented in which, if a suitable dynamic threshold factor is selected, then the performance degradation due to noise

uncertainty can be overcome. Assume that the dynamic threshold factor is ρ' and $\rho' > 1$ and it lies in the interval $\xi' \in [\xi/\rho', \rho'\xi]$ instead of remaining constant.
The probabilities are expressed as

$$P_{fa} = Q\left(\frac{\rho'\xi - \sigma_n^2}{\sigma_n^2\sqrt{2/N}}\right) \quad (11)$$

$$P_d = Q\left(\frac{\xi/\rho' - (P+\sigma_n^2)}{(P+\sigma_n^2)\sqrt{2/N}}\right) \quad (12)$$

Eliminating threshold ξ and equating (11) and (12), we get [8]

$$N = 2[Q^{-1}(P_{fa}) - \rho'^2(1+SNR)Q^{-1}(P_d)]^2 * (\rho'^2 SNR + (\rho'^2 - 1))^{-2} \quad (13)$$

3. *Energy Detector under noise uncertainty with dynamic threshold*

After considering cases of dynamic threshold and noise uncertainty individually, we now give the expressions of false alarm probability and detection probability considering dynamic threshold and noise uncertainty jointly. The noise variance is in the interval $\sigma^2 \in [\sigma_n^2/\rho, \rho\sigma_n^2]$ and the threshold value is in the interval $\xi' \in [\xi/\rho', \rho'\xi]$.
The probabilities are expressed as [9]

$$P_{fa} = Q\left(\frac{\rho'\xi - \rho\sigma_n^2}{\rho\sigma_n^2\sqrt{2/N}}\right) \quad (14)$$

$$P_d = Q\left(\frac{\xi/\rho' - (P+\sigma_n^2)}{(P+\sigma_n^2)/\rho\sqrt{2/N}}\right) \quad (15)$$

Eliminating threshold ξ and equating (14) and (15), we get

$$N = 2\left[\left(\frac{\rho}{\rho'}\right)Q^{-1}(P_{fa}) - \rho'\left(\frac{1}{\rho} + SNR\right)Q^{-1}(P_d)\right]^2 * (\rho'^2 SNR + \rho'/\rho - \rho/\rho')^{-2} \quad (16)$$

B. *MATCH FILTER DETECTION ALGORITHM*

In the scenarios when, the signal X (n) is completely known to the receiver, the best optimal detector is the match filter detector, also called as coherent detector. This detector can be expressed as [9]

$$D(Y) = \frac{1}{N}\sum_{n=0}^{N-1} Y(n)X(n) \begin{array}{l} > \xi \quad H1 \\ < \xi \quad H0 \end{array} \quad (17)$$

where D(Y) is the decision variable, N is the number of samples and ξ is the decision threshold. If the noise variance is known completely, then according to the central limit theorem

$$D(Y|H0) \sim Normal(0, P\sigma_n^2/N) \quad (18)$$

$$D(Y|H1) \sim Normal(P, P\sigma_n^2/N) \quad (19)$$

where P is the average signal power and σ_n^2 is the noise variance. Thus expressions for the probabilities are

$$P_{fa} = Q\left(\frac{\xi}{\sqrt{P\sigma_n^2/N}}\right) \quad (20)$$

$$P_d = Q\left(\frac{\xi - P}{\sqrt{P\sigma_n^2/N}}\right) \quad (21)$$

From (20) and (21) we get relationship between N, SNR, Pd, and Pfa and threshold ξ also gets eliminated.

$$N = 2\left[Q^{-1}(P_{fa}) - Q^{-1}(P_d)\right]^2 (SNR)^{-1} \quad (22)$$

1. *Match Filter Detector under noise unceratainty*

Now, taking into account the case with uncertainty in the noise model, the limits of noise variance can be represented as $\sigma^2 \in [\sigma_n^2/\rho, \rho\sigma_n^2]$ where ρ is the noise uncertainty coefficient and $\rho > 1$.
Thus (5) and (6) are modified as

$$P_{fa} = Q\left(\frac{\xi}{\sqrt{\rho P\sigma_n^2/N}}\right) \quad (23)$$

$$P_d = Q\left(\frac{\xi - P}{\sqrt{\rho P\sigma_n^2/N}}\right) \quad (24)$$

Equating (23) and (24), we get

$$N = \rho\left[Q^{-1}(P_{fa}) - Q^{-1}(P_d)(1+SNR)\right]^2 (SNR)^{-1} \quad (25)$$

2. *Match Filter Detector with dynamic threshold*

Here we try to find effect of dynamic threshold on the detection performance. Assume that dynamic threshold factor is ρ' ($\rho' > 1$).The value of dynamic threshold can be in a single interval $\xi' \epsilon [\xi/\rho', \rho'\xi]$.
Thus (5) and (6) are revised to

$$P_{fa} = Q\left(\frac{\rho'\xi}{\sqrt{P\sigma_n^2/N}}\right) \quad (26)$$

$$P_d = Q\left(\frac{\rho'\xi - P}{\sqrt{P\sigma_n^2/N}}\right) \quad (27)$$

Eliminating threshold ξ we get,

$$N = \left[Q^{-1}(P_{fa}) - Q^{-1}(P_d)(1+SNR)\right]^2 (SNR)^{-1} \quad (28)$$

3. *Match Filter Detector under noise uncertainty with dynamic threshold*

This section discusses the detection performance expressions when consider noise uncertainty and dynamic threshold together. Noise uncertainty factor ρ and dynamic threshold factor ρ'.
The noise variance is in the interval $\sigma^2 \in [\sigma_n^2/\rho, \rho\sigma_n^2]$ and threshold value in the interval $\xi' \in [\xi/\rho', \rho'\xi]$ .Thus (5) and (6) modify as in [9]

$$P_{fa} = Q\left(\frac{\rho'\xi}{\sqrt{\rho P\sigma_n^2/N}}\right) \quad (29)$$

$$P_d = Q\left(\frac{\rho'\xi - P}{\sqrt{P\sigma_n^2/N\rho}}\right) \quad (30)$$

Thus solving (29) and (30) we get

$$N = \rho\left[Q^{-1}(P_{fa}) - Q^{-1}(P_d)(1+SNR)\right]^2 (SNR)^{-1} \qquad (31)$$

IV. RESULTS AND ANALYSIS

The analysis of energy detector and match filter detector algorithms based various parameters shows the following findings.

Fig. 1 shows the comparison of energy detector and match filter detector schemes using (7) and (22) when P_{fa} is varied from 0 to 0.9 for N=100 and different SNR values like -10dB, -12dB and -15dB. We find the performance of ED as well as MF increases as the SNR improves. We also find that for the same fixed values of parameters, MF algorithm performance is better than energy detection algorithm proving the fact that probability of correct detection Pd is inversely related to the square of SNR in ED and is inversely proportional to just the SNR in case of MF algorithm.

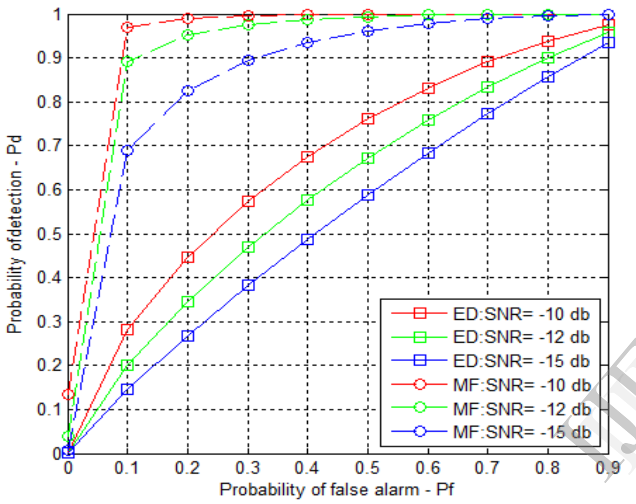

Fig. 1. ROC curves of Energy detector and Match filter detector for SNR=-10dB,-12dB,-15dB and N=100

Fig. 2 shows the comparison of energy detector and match filter detector methods again for (7) and (22) when P_{fa} is varied from 0 to 0.9, SNR = -20 dB and different values of N = 500, 1000 and 1500 are taken. We find the performance of ED as well as MF scheme improves as the number of samples are increased. We also find that for low SNR values like -20db the performance is not acceptable for energy detection technique in any case as P_{fa} and P_d are almost equal whereas we can see that, match filter algorithm, for the same number of samples has achieved a better performance on detection as compared to energy detection algorithm. Thus the advantage of match filter scheme over energy detector is that the match filter algorithm requires less number of samples (i.e. just 1/SNR) to meet a given probability of detection constraint and thus sensing time required is also less.

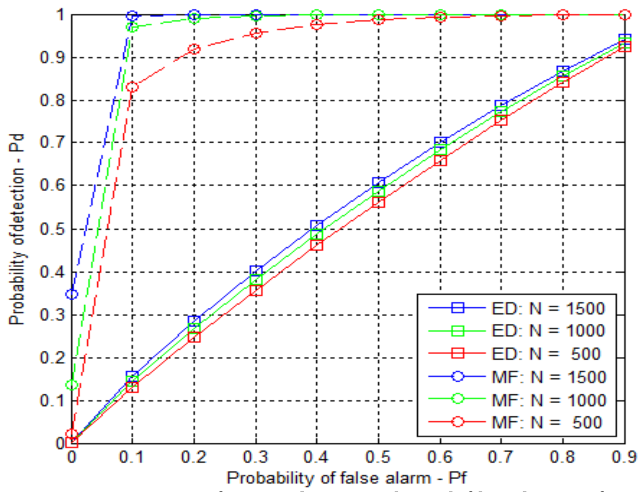

Fig. 2. ROC curves of Energy detector and Match filter detector, for N=500,1000,1500 and SNR= -20dB

Fig. 3 shows the comparison of energy detector and match filter detector algorithms using (7) and (22), where Pfa is set at a minimum value of 0.01 and considering values of number of samples as N =10,100,500 and 1000 ,graph for SNR variation from -30 dB to 20 dB has been plotted. It can be seen that for good SNR conditions above 0 dB, the difference between the values of probability of detection of the two methods is very less. But as SNR falls down, this difference increases. This shows that even at low SNR conditions, match filter detection algorithm performs well due to coherent detection but energy detection algorithm can perform well only in good SNR conditions.

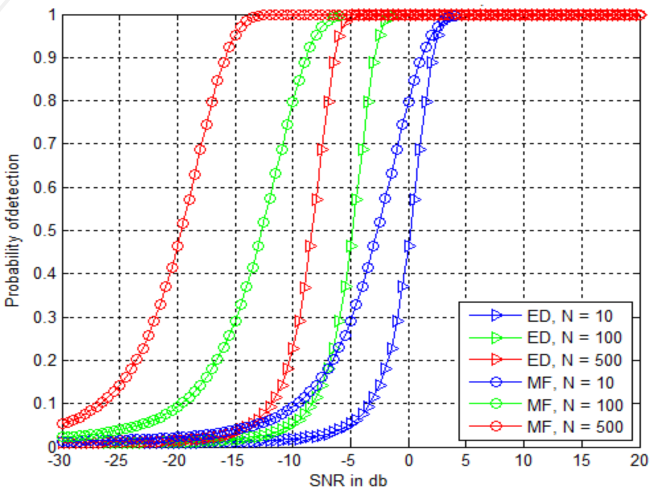

Fig. 3. Pd Vs SNR for Energy detector and Match filter detector for N=10,100,500 and 1000

Fig. 4 shows the comparison of energy detector and match filter detector algorithms using (10) and (25) when P_{fa} is varied from 0 to 0.9 for SNR = -12 dB, N = 100 and noise uncertainty values ρ=1.00, 1.01, 1.02 and 1.03. It can be seen that as noise uncertainty increases, the performance of energy detection scheme degrades, Pfa almost equaling Pd. This indicates that energy detection scheme is very sensitive to noise uncertainty and a very small variation of average noise power causes serious performance drop. At the same time there is no effect of uncertainty on the performance of match filter algorithm and thus we can deduce that matched filter detection scheme is not sensitive to noise uncertainty. Thus we can clearly infer that energy detection technique is sensitive to slight fluctuation in noise but match filter algorithm performance does not degrade when noise uncertainty is considered.

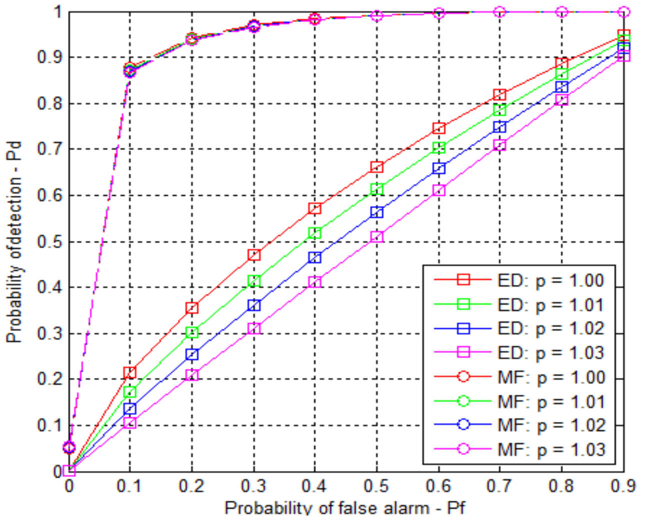

Fig. 4. ROC curves of Energy detector and Match filter detector, with noise uncertainty ρ = 1.00, 1.01,1.02,1.03 for N=100,SNR = -12dB

Fig. 5 shows energy detector's performance separately for (7), (10) and (13) with SNR= -15db and N=1500, when Pfa is varied from 0 to 0.9. It can be seen that with no noise uncertainty, the detection performance is a regular one but as noise uncertainty comes in picture, the plot shows decline in the performance where probability of false alarm and detection are nearly equal. But after incorporating dynamic threshold, algorithm, the performance increases again and gets back to normal. Thus it is observed that the degradation of detection performance caused by noise average power variation can be entirely eliminated with a choice of dynamic threshold factor.

Fig. 6 again shows energy detector's performance separately for (16), with SNR=-12db and N =500 when Pfa is varied from 0 to 0.9.The value of ρ=1.00 denotes no noise uncertainty considered and ρ′=1.00 denotes that threshold is fixed one. When noise uncertainty increases to 1.05, graph shows a serious decline in performance, Pfa exceeding Pd since threshold is kept fixed. But as threshold is increased to 1.03, the performance dramatically rises. Again when threshold is made 1.04, we still find more improvement in the performance. At threshold value 1.05, the energy detector's

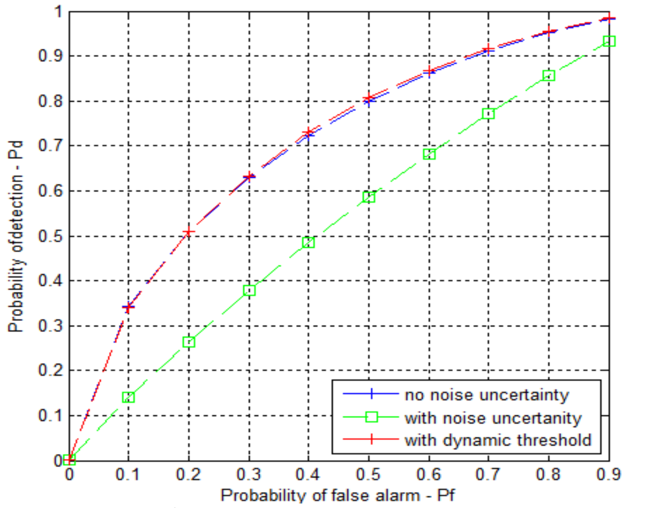

Fig. 5. ROC curves of Energy detector with no noise uncertainty, with noise uncertainty and with dynamic threshold

performance has become almost the same as with the no noise uncertainty case. Thus the plot indicates that a tiny fluctuation of average noise power leads to a sharp degradation in performance. But it improves significantly as the dynamic threshold factor increases. Thus if proper dynamic threshold factor is selected, the declining proportion of performance caused by noise uncertainty can be omitted.

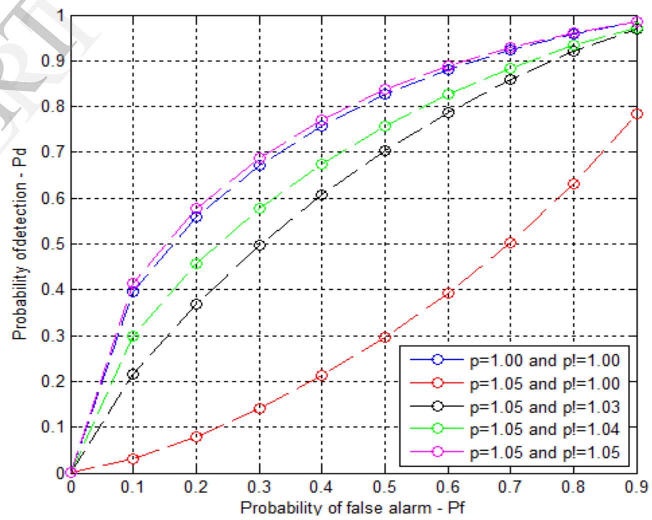

Fig. 6. ROC curves of Energy detector with different noise uncertainty ρ and dynamic threshold ρ′values

Fig. 7 shows the graph of probability of detection Vs SNR in the range of -30dB to 20dB for Enenrgy detctor and Match filter detector scheme. Here we have considered two types of input signals.In the first case an input cosine wave signal which is a amplitude modulated with various carrier frequencies, each forming a primary user transmitter signal is considered. To these different primary users, AWGN noise is added.These PU's are then received and detected using two algorithms: Energy detection and Match filter detection. In case of Energy detector, power spectrum density of received signal is calculated and it is compared with the threshold value to determine the presence of primary user signal using periodogram method. For match filter detection algorithm, coherent detection is used.

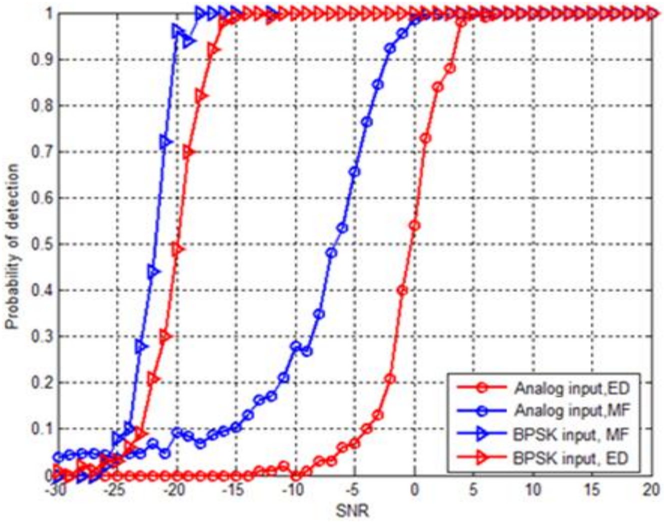

Fig. 7. Pd Vs SNR for Energy detector and Match filter detector for analog amplitude modulated input signal and digital BPSK modulated signal

The graph shows that energy detection technique is able to achieve the desired 100% probability of detection at around 5dB whereas match filter algorithm is able to achieve the same 100% probability of detection at 0dB itself. This is because of coherent detection present in match filter scheme. In the next case, the input signal is a BPSK signal and rest of the detection process is the same as mentioned in the first case. For BPSK input signal, energy detection scheme is able to achieve the desired 100% probability of detection at around

-15dB whereas match filter algorithm is able to achieve the same 100% probability of detection at -19dB itself. Clearly we can find the improvement in the detection performances when we switch over from analog input signal to digital one.

V. CONCLUSION

In this paper, the relationship of energy detection scheme as well as match filter detection scheme's performance with signal to noise ratio, the number of samples noise uncertainty and dynamic threshold is investigated. From simulation results, we can summarize that energy detector scheme gives improved performance as SNR goes on increasing even if numbers of samples are less. Also by increasing the number of sample points, the ED detection performance is much better even at lower SNR values. We also conclude that energy detection technique is very sensitive to uncertainty in noise and a very small variation of average noise power causes serious drop in the performance, especially with a lower SNR. This can be overcome by introduction of dynamic threshold algorithm which enhances the robustness of system by solving the noise uncertainty issue. On the other hand, noise uncertainty does not affect the performance of match filter algorithm and thus dynamic threshold is of no use. Thus it can perform well even in low SNR regimes. We also conclude that match filter technique requires less number of samples (i.e. just 1/SNR) as compared to energy detector (i.e. 1/ SNR^2) to meet a given probability of detection constraint and thus sensing time required for match filter scheme is also less. But it needs prior knowledge of the primary user's signaling type. Energy Detector requires minimum information about the primary user's signal. But it does not perform well in low SNR conditions. Thus each method has its own advantage and disadvantage. Lastly, simulations also infer that use of digital communication improves the overall performance of a system as compared to analog communication.

REFERENCES

[1] Khaled Ben Letaief, Fellow IEEE, and Wei Zhang, "Cooperative Communications for Cognitive Radio Networks", Proceedings of the IEEE, Vol. 97, No. 5, May 2009.
[2] Simon Haykin, "Cognitive Radio: Brain-Empowered Wireless Communications", IEEE Journal on selected areas in communications, Vol. 23, No. 2, February 2005.
[3] Anirudh M. Rao1, B. R. Karthikeya, Dipayan Mazumdar, Govind R. Kadambi "Energy detection technique for spectrum sensing in cognitive radio",SASTECH, Volume 9, Issue 1, April 2010.
[4] Rahul Tandra, Anant Sahai, "SNR walls for signal detection", IEEE Journal of selected topics in Signal Processing, 2:1(2008), 4-17.
[5] S. M. Kay, "Fundamentals of Statistical Signal Processing: Detection Theory". Englewood Cliffs: Prentice-Hall, 1998, vol. 2.
[6] Ashish Bagwari, Brahmjit Singh "Comparative performance evaluation of Spectrum Sensing Techniques for Cognitive Radio Networks" Fourth International Conference on Computational Intelligence and Communication Networks, 978-0-7695-4850-0/12 $26.00 © 2012 IEEE.
[7] Guicai Yu, Chengzhi Long and Mantian Xiang, "A Novel Spectrum Detection Scheme Based on Dynamic Threshold in Cognitive Radio Systems". Research Journal of Applied Sciences, Engineering and Technology, 4(21): 4245-4251, 2012.
[8] Chabbra, Kriti; Mahendru, Garima; Banerjee, P, "Effect of dynamic threshold & noise uncertainty in energy detection spectrum sensing technique for cognitive radio systems," Signal Processing and Integrated Networks (SPIN), 2014 International Conference on , vol., no., pp.377,361, 20-21 Feb. 2014.
[9] Gui-cai YU; Yu-bin Shao; Hua Long and Guang-xin Yue1 "Dynamic Threshold Based Spectrum Detection in Cognitive Radio Systems" 978-1-4244-3693-4/09 ©2009 IEEE.
[10] Digham, F.F.; Alouini, M.-S.; Simon, M.K., "On the energy detection of unknown signals over fading channels", ICC '03, IEEE International Conference on Communications, 3575 - 3579 vol.5, 2007.