# Computational Bayesian Approach to Dependency Assessment in System Reliability of Electro-Mechanical Actuator

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***Abstract*— Increased use of Electro-mechanical actuators (EMA) in high-power, flight-critical applications where hydraulics have traditionally been employed has increased the need for reliability assessment. The Bayesian network (BN) model is a proved powerful tool for the reliability estimation of EMA systems and provides significant advantages over traditional techniques like reliability block diagrams and fault trees in reliability estimation as they are not flexible enough to detect the uncertainties in the dependencies among system, subsystem and components. When Bayesian network model is combined with statistical Bayesian inference there is an advantage of combining information from multiple sources at multiple levels for finding the system reliability. In this work, it is proposed for developing the methodology to estimate the conditional probabilities in a Bayesian Network model of EMA system using Bayesian inference. Various component and system qualification and acceptance data of EMA system is to be collected and combined to estimate the system reliability. Three scenarios will be considered. a) In the first scenario the complete historical data set of states of the system/ subsystem and its components are available, b) failure summary is available, c) incomplete system/component level data. Assessing the posterior distribution of conditional probabilities is difficult to the understanding the structure of an EMA system. Hence computational Bayesian method, Markov chain Monte Carlo (MCMC) is used for quantifying system reliability structure with incomplete data. MATLAB and OpenBugs code will be developed to perform calculations and MCMC simulation.**

***Key words*— MCMC, Bayesian networks, OpenBugs, EMA**.

## I. INTRODUCTION

With the rapid development of the modem power electronics, Electro-Mechanical Actuator (EMA) is used more and more. Before, the control of missile system was mainly done by using the mechanical device like hydraulic actuator which is large, heavy and complex to realize exact position control. Also it is inflexible when there is any necessary sudden change. Also, the hydraulic drive system is complex and difficult for maintenance. Therefore, the Electro-Mechanical Actuator is substituted for mechanical device as a more effective actuator in missile system.

The demand for product functionality is becoming more and more complex over time. Hence the engineered products like EMA being discussed in this project are becoming more and more complex. For the EMA system used in the missile system, to estimate more consistent failure probability there is a need to consider that all the components, subsystems and system are dependent. Now here comes the real problem in assessing the system reliability.

Hence the traditional reliability assessment techniques like reliability block diagrams and fault trees analysis which are used for small systems are not consistent in analysing these complex systems. In this situation we have to develop new reliability model to integrate all the available information to precise prediction of system reliability.

While working for reliability estimation for a system we face many situations, some in which complete information may not be available about how a complex system fails in its operating conditions and environment. So there is a necessity to understand the relationship between the components, subsystems, and system. To do this we use Bayesian network for representing the probabilistic relation among the system, sub system and the component reliability. This is an extension of the relationship that is modelled typically by fault trees or block diagrams if the failure structure is clear and well understood.

## II. LITERATURE SURVEY

A system consists of subsystems and components or on functional wise, sub-functions and elementary functions, which are represented by node in the reliability topology of system. Na Wang et al. [1] presented a novel hybrid Three-Redundant Electro-Mechanical Actuator system which mainly consist of one manual BLDCM driver and two automatic brushless DC motor drivers. He gave a clear idea of system structure with block diagrams and also reliability prediction of each part of the system by adopting the failure models and reliability models, but this method is the primary method to any system that can estimate the reliabilities of the


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components, subsystems and also the system using the reliability block diagrams considering the components are independent to each other. But this method failed in calculating the dependencies among the component, sub-system and system. The Fault tree analysis is another method to consider the dependencies and calculate the reliability of the system using the probabilities of the components or sub-systems. Sohag Kabir [2] explained an overview of fault tree and its applications in the analysis of model based dependability. Karim Bourouni compared the reliability block diagram and fault tree for a reverse Osmosis plant. But this method uses deterministic relations like AND, OR relations. This method fails when the components, subsystem and system have non-deterministic relationship.

A. Bobbio et, al. [3] gives a brief description of mapping fault trees into Bayesian networks and establishing the conditional probability tables to the nodes having the parent nodes with pre-determined probabilistic relations like AND, OR relations. The methodology to combine multistate variables using Bayesian approach is modelled. Hamada M et al. [4] applied the same approach as Johnson et al. on the non-overlapping which is continuous failure time data extracted from basic and higher-level failure events in a fault tree. Graves Todd L et al. [5] further extended the research by using the fault trees for multistate. They used Dirchlet distribution for prior probabilities of multistate system. He also proposed a Bayesian approach to combine simultaneous multilevel data.

Martz and Waller [6] addressed the problem of combining the multilevel data from different levels of the system and also from the expert guesses about the parameters of the distribution and reliability of system components. These papers focused on series and parallel systems, where component failure data were modelled using binomial distributions and whereas beta distributions are used for the prior information at components, subsystems and system levels. Martz et al. also explained native prior and induced prior. Any one of the priors are used to find the posterior in the unavailability of one another. Both the priors can also be used in the case of availability.

Yontay et al. [7] discussed the situations that we do not have complete information about how a complex system would fail in its operating environment and also more about the interactions between the system and its components and how they work together. They used Bayesian network to represent the probabilistic relationship between the system and components, which is a natural extension of deterministic relationship typically modelled by block diagrams of fault trees when the failure structure is clear and well understood.

We used Bayesian networks to model the EMA system in three different scenarios of data availability. We divided the system based on the data availability and calculate the posterior probabilities separately to each part and then we have combined them using induced prior method. We have also found the system probability in case of limited data availability by using both the Bayesian and fault tree analysis. We found MCMC (Markov Chain Monte Carlo) is the best suited algorithm to find the posterior probabilities. The OpenBugs software found to be the best one to perform MCMC which uses Metropolis-Hastings algorithm to sample from the distribution.

## III. BAYESIAN NETWORKS AND FAILURE MODES OF EMA

### *A. Bayesian networks*

The Bayesian network model is an advanced tool that can provide many methodological advantages over traditional techniques in dependency assessment. Because the traditional methods like reliability block diagrams and fault trees are still not flexible enough to predict the uncertainties in the dependencies among system, subsystem and components. Bayesian networks allows components, systems and subsystems to be related with conditional probabilities. Where as in reliability block diagrams and fault trees are related with pre deterministic relations like AND, OR relations. One of the big advantages of Bayesian network analysis over traditional methods is it can combine the information from multiple levels and multiple sources when it is coupled with statistical Bayesian inference techniques.

Hence it is helpful when we combine the both Bayesian networks and Bayesian inference techniques. The requirement for the Bayesian network model is conditional probabilities. These conditional probabilities represents the complex failure relations in multilevel systems.

Now the conditional probabilities for a Bayesian networks has the ability to combine information from different sources like, objective information sources, such as older generation's failure records products, life tested data of components and available field data. This data comes with different structures and different types, therefore we may face difficulty in calculating the conditional probabilities from the data.

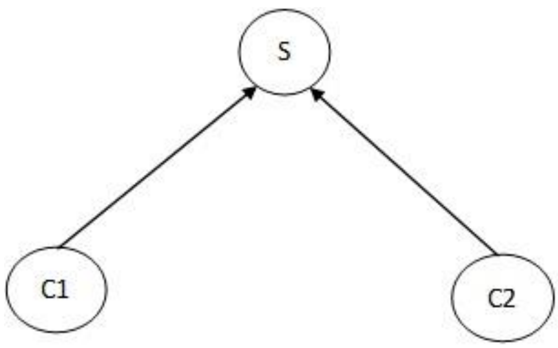


Fig.1: Bayesian network representation

Bayesian inference is statistical inference method through which we can estimate the model parameters by combining the prior and likelihood. That is we can combine the prior information and the information coming from different sources to obtain more precise estimation of Bayesian network model parameters. The Bayesian network representation as follows in fig.1.


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As the C1 (component 1) and C2 (component 2) are conditionally independent they have marginal probability distributions as follows.

| Table1: Marginal Probabilities | | |
|---|---|---|
| Component | Failure probability | Success probability |
| C1 | $P_1(C1)$ | $P_0(C1)$ |
| C2 | $P_1(C2)$ | $P_0(C2)$ |

Where suffix '1' represents the failure and suffix '0' represents the success. The system has the conditional probabilities as in the table2.

| Table2: Conditional Probabilities | | |
|---|---|---|
| Component combinations | System failure | System success |
| C1=0, C2=0 | $P_1(00)$ | $P_0(00)$ |
| C1=0, C2=1 | $P_1(01)$ | $P_0(01)$ |
| C1=1, C2=0 | $P_1(10)$ | $P_0(10)$ |
| C1=1, C2=1 | $P_1(11)$ | $P_0(11)$ |

To get the marginal failure probability of the system S we should find the joint probabilities of the four conditions given that system is failed and add the joint probabilities. The mathematical expression is as follows.

$$P(S=1) = \sum_{i=1}^{n}\sum_{j=1}^{n} P_1(i,j) * P_i(C1) * P_j(C2) \quad \ldots\ldots\ldots\ldots\ldots\ (1)$$

From the equation 1 we can find the marginal probability of the system [8].

The Bayesian equation to find the posterior as follows.

$$P(S|C1,C2) = \frac{P(C1,C2|S) \times P(S)}{P(C1,C2)} \ldots\ldots (2)$$

In the above equation (2)

$P(S|C1,C2) = Posterior$

$P(C1,C2|S) = Likelihood$

$P(S) = Prior$

$P(C1,C2) = Normalizing\ constant$

Most of the times the normalizing constant becomes analytically difficult to calculate hence we use MCMC (Mote Carlo Markov Chain) to find the posterior of the system as we do not need normalizing constant in this algorithm to calculate the posterior of the system.

Both the prior and posterior data looks in similar format but the probabilities will be updated based on the likelihood. Where likelihood is in the form of test data in our project.

## B. *Electro-Mechanical Actuator And Its Failure Modes*

The fault tree diagram of the available system is as follows in fig2. The available EMA system consists of following subsystems and components which are considered as potential sources of failure.

The following subsystems and their components are the sources of failure of the system.

1. Servo control (SC) unit failure
   a. Control PCB (CP) failure
   b. Power PCB (PP) failure
2. BLDCM (Brushless DC Motor) assembly failure
   a. Hall Sensor PCB (HP) failure
   b. Stator failure (S)
   c. Rotor failure (R)
3. Gear train assembly (G) failure
4. Resolver (RE) failure

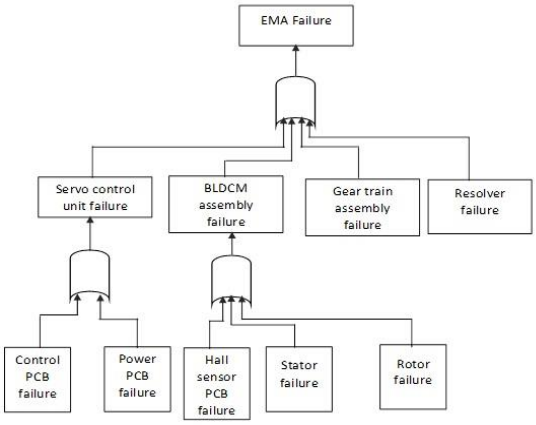


Fig.2: Fault tree diagram for EMA system.

1. *servo control (SC) unit failure*

The servo controller is the main part of the whole system. The servo control unit automates the process. The failure of servo control unit is due to the following failures.

a. *Control PCB (CP) failure*: The control PCB consists of advanced digital signal processor and resolver to digital circuits. This is a micro-electronics system.
b. *Power PCB (PP) failure*: The power circuit is based on switched mode power devices. This is a power electronic circuit.

Both the PCB failure occur due to environmental stress like dirt and high environmental temperatures and also manufacture problems like poor soldering and shorting out of closely placed traces accidentally.

2. *BLDCM Assembly Failure*

Brushless DC motor has many advantages over commutator-type dc and ac motors due to brush removal. The electromagnetic interference and its associated sparking is eliminated. The BLDCM failures are mainly due to

a. *Hall sensor PCB (HP) failure*: Hall sensor PCB can be used to measure the intensity of magnetic fields for measuring applications. Furthermore, the Hall sensors can distinguish the polarity of the magnetic field and the main advantage of hall sensor is there is no need to strip the cable. The failure in Hall sensor may be due to the


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manufacturing problems in the amplifiers and Integrated circuits used in it and also due to the poor health of the PCB.

b. *Stator failure (S)*: The stator is the static part of the motor. The stator failure will be mainly due to problems in winding.

c. *Rotor failure (R)*: The rotor is the rotating part of the motor that is connected to the load and the failures in the rotor are mainly due to the moisture, overloading and thermal stress.

3. *Gear Train Assembly (G) Failure*

Gear train is a combinations of gears of different specified dimensions for controlling the speed and motion of the electro-mechanical actuator. The failure of Gear train assembly is due to gear teeth faults while manufacturing.

4. *Resolver (RE) Failure*

Resolver is a feedback device with high precision and resolution. This resolver is a position sensor of the rotor. It sends the signals to the controller in order to calculate the accurate position. The failure in resolver may occur due to the breakage of circuit connections.

Keeping all these failure modes aside there are many undermined failure modes and causes which cannot be found normally. We have considered these unknown causes also while finding the failure probability of the system. These causes are non-deterministic and we can only find through Bayesian analysis.

## IV. METHODOLOGY

The goal of this research is to develop the methodology to estimate failure probabilities using Bayesian networks and inference for an Electro-Mechanical actuator system used in missile system in all the three possible cases of available data that we have faced. The Servo control unit, brushless DC motor assembly, gear train assembly and rotary potentiometer assembly are the components of the EMA system which are connected using BN and inferences are made by estimating the parameters of the distributions. Here we used conjugate priors to estimate the posterior of the components, subsystems and system and also non-conjugate priors to estimate the probabilities when incomplete data is available. We used Markov Chain Monte Carlo (MCMC) for simulation and also to sample from the posterior distribution. When the data is incomplete the model becomes complex to understand but it gives accurate results, the use of informative priors make the results more accurate.

For the servo control unit subsystem in the actuator complete data is available. Hence we model the subsystem with Bayesian networks and infer the posterior by taking beta distribution as prior and binomial as likelihood function.

In the second case failure summary is available which is used as likelihood. The mean and standard deviation of BLDCM subsystem are available which are used to find the prior to the BLDCM subsystem. The prior probabilities of the gear train assembly failure and resolver failure are not available hence we took Jeffrey's prior and we took pass-fail data as likelihood to find the posterior and then using the posteriors of the four subsystems we found the induced prior to the system. Where at the end the whole system is subjected to ground test and flight test and total test results are taken as likelihood and posterior of the system is found.

In the third case the series system of EMA and B are unchanged. Due to the redesign the series system of SC became uncertain. We are interested in finding the relation between the subsystem SC and components CP and PP.

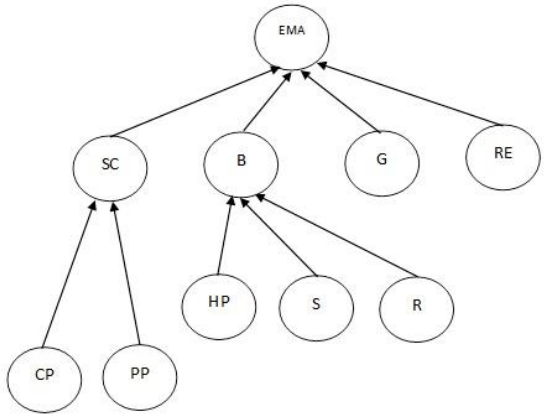


Fig.3: Bayesian network representation of EMA system.

*Scenario 1: Complete data is available*

In scenario 1 we find the failure probability of SC subsystem as all the data set of SC is available that is all the nodes of SC, CP and PP are monitored with the help of sensors. We tabulated the prior data of the SC subsystem based on the expert opinion. Each probability in the table1 follows beta distribution as shown. In all the tables the suffix of P denoted the state of the node, 1 is for failure state and 0 is for success state.

Here the data and the system structure has undergone changes so as to preserve the confidentiality of the organization.

Table3: prior marginal probabilities

| Component | Failure probability | Success probability |
|---|---|---|
| CP | $P_1(CP) \sim dbeta(2{,}10)$ | $1 - P_1(CP)$ |
| PP | $P_1(CP) \sim dbeta(1{,}10)$ | $1 - P_1(PP)$ |

Table4: prior conditional probabilities

| Component combinations | System failure | System success |
|---|---|---|
| CP=0, PP=0 | $P_1(00) \sim dbeta(2{,}10)$ | $1 - P_1(00)$ |
| CP=0, PP=1 | $P_1(01) \sim dbeta(2{,}10)$ | $1 - P_1(01)$ |
| CP=1, PP=0 | $P_1(10) \sim dbeta(2{,}10)$ | $1 - P_1(10)$ |
| CP=1, PP=1 | $P_1(11) \sim dbeta(2{,}10)$ | $1 - P_1(11)$ |


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Now we take likelihood follows Binomial distribution i.e,

$$F(node) \sim dbin(P, N)$$

Where, P is the prior probability and N is the total no of trials and F is the failure count or evidence.

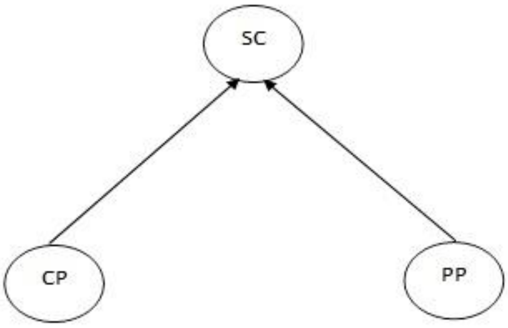


Fig.4: Bayesian network for subsystem SC

The evidence is in the form of test data as in the below table 5.

| Table5: test data | | |
|---|---|---|
| Components and their combinations | No. Of failures | Total no. Of tests |
| CP | 4 | 24 |
| PP | 5 | 30 |
| SC\|CP=0,PP=0 | 2 | 35 |
| SC\|CP=0,PP=1 | 16 | 28 |
| SC\|CP=1,PP=0 | 16 | 25 |
| SC\|CP=1,PP=1 | 18 | 20 |

We have written an openbugs code to find the posterior using the available prior and evidence. After simulating for 1000 iterations we have got the below results in table6.

| Table6: Posterior Probabilities Of SC | | | |
|---|---|---|---|
| | Mean | SD | Median |
| $P_1(CP)$ | 0.1623 | 0.05915 | 0.1564 |
| $P_1(PP)$ | 0.1447 | 0.05235 | 0.1403 |
| $P_1(00)$ | 0.08441 | 0.04081 | 0.07703 |
| $P_1(01)$ | 0.4516 | 0.07764 | 0.4487 |
| $P_1(10)$ | 0.4871 | 0.07898 | 0.4888 |
| $P_1(11)$ | 0.6236 | 0.08503 | 0.6277 |

Here the suffix of P that is '1' represents the failure and 0 for success. Where,

$$P_0(\quad) = 1 - P_1(\quad)$$

Now if we observe the above table6 there are four conditional probabilities for the system node depending on the parent nodes but we should have a single marginal probability for the system to go further with our methodology so we find the marginal posterior probability for subsystem SC with the help of equation (1).

In this case equation (1) becomes,

$$P(SC) = \sum_{i=0}^{1}\sum_{j=0}^{1} P_1(i,j) \times P_i(CP) \times P_j(CP) \ldots\ldots (3)$$

To calculate the above equation we developed a Matlab program. The result we have got is the failure probability of the subsystem SC.

$$P_1(SC) = 0.197$$

*Scenario2: failure summary is available*

In this scenario as the data of subsystem B is available in the form of summarized failure data. Hence we developed a method to extract the summarized failure data into the required likelihood form and use a common prior given by the expert to calculate the posterior failure probability of the system.

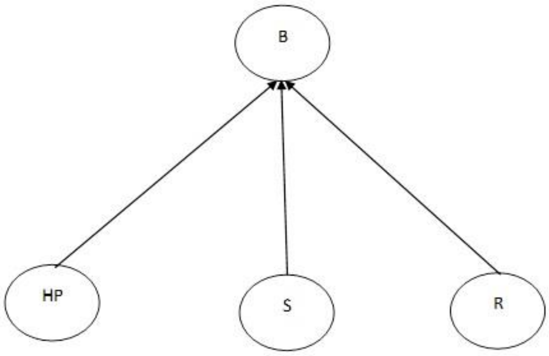


Fig.5: Bayesian network for B subsystem

We can see the available limited data format in table7.

| Table7: summarized subsystem (b) failure data | | | |
|---|---|---|---|
| BLDCM failures | HP | S | R |
| Failure event 1 | X | | |
| Failure event 2 | | | X |
| Failure event 3 | X | X | |
| Failure event 4 | | | X |
| Failure event 5 | X | | X |
| Failure event 6 | | X | |
| Failure event 7 | | | |
| Failure event 8 | | X | |
| Failure event 9 | | | X |
| Failure event 10 | | X | |
| Failure event 11 | X | X | X |
| Failure event 12 | | X | X |
| Failure event 13 | X | X | X |
| Failure event 14 | X | X | X |
| Failure event 15 | X | | |
| Failure event 16 | | X | X |


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| | | | |
|---|---|---|---|
| Failure event 17 | X | | |
| Failure event 18 | X | | X |
| Failure event 19 | X | | X |
| Failure event 20 | X | X | |
| Failure event 21 | X | X | |
| Failure event 22 | X | X | X |
| Failure event 23 | X | X | X |
| Failure event 24 | X | X | X |
| Failure event 25 | X | X | X |
| Failure event 26 | X | X | X |
| Failure event 27 | X | X | X |

The prior of the subsystem B is given by the expert in the form and mean and standard deviation of the failure probability distribution. I.e, Mean=0.1923 and standard deviation=0.088.

Now we fit a beta distribution to the subsystem prior failure probabilities with the help of methods of moment estimators,

If M is the mean and S is the standard deviation of the probability distribution then the methods of moment estimators says,

$$a = M\left(\left(\frac{(1-M)}{S^2}\right) - 1\right) \dots (4)$$

$$b = (1-M)\left(\left(\frac{(1-M)}{S^2}\right) - 1\right) \dots (5)$$

Where 'a' and 'b' are the parameters of the beta prior distribution, M is the mean and S is the standard deviation.

We generated a Matlab code for equation 4 and 5 and we got $a = 19.8646$ and $b = 83.4355$.

That is the failure prior of the subsystem B follows Beta (19.86, 83.4355).

Now we extract the counts of the combination from the table7 and we construct table8 as below.

Table8: Counts of combinations from table7

| Parent combinations | BLDCM Failure Counts |
|---|---|
| HP=0, S=0, R=0 | 1 |
| HP=0, S=0, R=1 | 3 |
| HP=0, S=1, R=0 | 3 |
| HP=0, S=1, R=1 | 2 |
| HP=1, S=0, R=0 | 3 |
| HP=1, S=0, R=1 | 3 |
| HP=1, S=1, R=0 | 3 |
| HP=1, S=1, R=1 | 5 |

The marginal probabilities of the components of the subsystem are available from the records as shown in the below table9.

**Table9: Marginal failure probabilities of components of the subsystem B**

| **Component** | **Marginal failure probabilities** |
|---|---|
| $P_1(HP)$ | 0.22 |
| $P_1(S)$ | 0.32 |
| $P_1(R)$ | 0.25 |

Now we have a prior distribution $B \sim dbeta(19.8646, 83.4355)$ and evidence for the likelihood is in the form of table8. We use binomial distribution as likelihood

i.e,

$$P_1(HP, S, R|B) \sim dbin(P, N)$$

Where N=total no. of observations = 27 from the table7.

We simulated probabilities with the help on Open Bugs statistical software for 1000 iterations and we got the results in the table10.

Table10: posterior failure probabilities of subsystem B.

| Conditional probabilities for B | Mean | Standard deviation | Median |
|---|---|---|---|
| $P_1(000)$ | 0.1631 | 0.0222 | 0.08387 |
| $P_1(001)$ | 0.1766 | 0.02403 | 0.09578 |
| $P_1(010)$ | 0.1791 | 0.023 | 0.09425 |
| $P_1(011)$ | 0.166 | 0.02304 | 0.0892 |
| $P_1(100)$ | 0.1728 | 0.02309 | 0.09613 |
| $P_1(101)$ | 0.1767 | 0.0219 | 0.09593 |
| $P_1(110)$ | 0.1753 | 0.02279 | 0.09505 |
| $P_1(111)$ | 0.1922 | 0.02624 | 0.1331 |


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Now we have eight conditional probabilities of subsystem B depending on their components. We find the marginal posterior distribution of the node B using the below equation.

$$P_1(B) = \sum_{i=0}^{1}\sum_{j=0}^{1}\sum_{k=0}^{1} P_1(i,j,k) \times P_i(HP) \times P_j(S) \times P_k(R) \dots\dots\dots\dots\dots\dots (6)$$

We generated a Matlab program for equation6 and we have got the posterior failure probability of the subsystem B is 0.1609.

Then we find the posterior for the remaining two subsystems G and RE. The prior information for the nodes G and RE are not available hence we use non-informative Jeffrey's prior that is Beta (0.5, 0.5).

The test data of G and RE are available in the table11.

**Table11: Test data for G and RE**

| Subsystems | No. of failures | Total no. of tests |
|---|---|---|
| Gear train assembly (G) | 2 | 42 |
| Resolver (RE) | 7 | 30 |

Using Jeffrey's prior and the test data in the table11 we have simulated the results for 1000 iterations in openbugs. We got the results in the table12.

**Table12: Posterior failure probabilities of G and RE**

| | Mean | SD | Median |
|---|---|---|---|
| $P_1(G)$ | 0.01151 | 0.01524 | 0.006138 |
| $P_1(RE)$ | 0.1063 | 0.05175 | 0.09884 |

We have found the marginal posterior probabilities of all the subsystems considering the dependencies. As our aim is to find the failure probability of the EMA system depending on the subsystems and components. That is by considering the multilevel data we had found the prior to the EMA system depending on the subsystem test and prior data using 'induced prior' method proposed by HF. Martz et.al.(1988) [6] and we find the beta parameters as follows,

$$a = \frac{[M^2(1-M) - VM]}{V} \dots\dots\dots (7)$$

$$b = \frac{[M^2(1-M)^2 - V(1-M)]}{V} \dots\dots\dots (8)$$

Where M=moment, V=variance

$$M = \prod_{i=1}^{7} \frac{F_i^0 + F_i + 1}{N_i^0 + N_i + 2} \dots\dots (9)$$

And

$$V = \prod_{i=1}^{7} \frac{(F_i^0 + F_i + 1)(F_i^0 + F_i + 2)}{(N_i^0 + N_i + 2)(N_i^0 + N_i + 3)} \dots. (10)$$

Where

$$F_i^0 = a\ value\ in\ beta\ parameter$$
$$F_i = failure\ count\ from\ pass\ fail\ data$$

$$N_i^0 = a + b$$
$$N_i = total\ no.of\ test\ in\ pass\ fail\ test$$

*For subsystem SC*: To get the pass- fail data we add all the failures of the SC conditioned on the components, then, we get,

$$F_1 = 63\ and\ N_1 = 162$$
$$F_1^0 = 11\ and\ N_1^0 = 60$$

*For subsystem B*: There are only system failure records observed but there are no N values (total no. of trials) hence we take the pass fail data as zero in this case

$$F_2 = 27\ and\ N_2 = 27$$
$$F_2^0 = 19.86\ and\ N_2^0 = 83.43$$

*For subsystem G*:

$$F_3 = 2\ and\ N_3 = 42$$
$$F_3^0 = 0.5\ and\ N_3^0 = 1$$

*For subsystem RE*:

$$F_4 = 7\ and\ N_4 = 30$$
$$F_4^0 = 0.5\ and\ N_4^0 = 1$$

By substituting the values in the above equations we got the induced prior to the system Beta (0.67, 1494.70). The test data of the system is available in table13.

Table13: Test data of EMA system

| System | No. of failures | Total no. of tests |
|---|---|---|
| EMA | 10 | 100 |

Fitting a beta distribution to the test data and using the obtained beta induced prior to the system we calculated the posterior failure probability of the system by simulating the results for 1000 iterations.

The obtained posterior failure probability of the EMA system is 0.03774 from table14.

Table14: The posterior results of the EMA

| | Mean | SD | Median |
|---|---|---|---|
| $P_1(EMA)$ | 0.02351 | 0.007059 | 0.02464 |

We can calculate the reliability of the system by complimenting$P_1(EMA)$. i.e,


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$Reliability\ of\ EMA = 1 - P_1(EMA).$

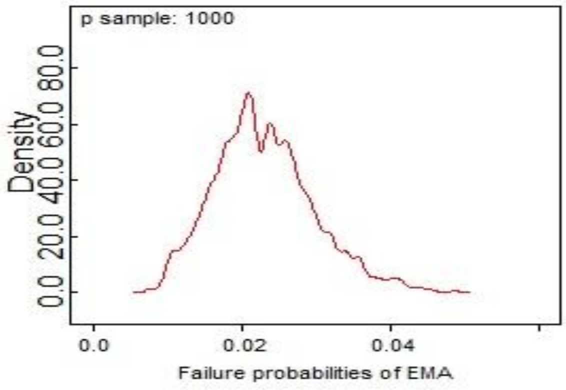


Fig.6: Posterior failure probability distribution of EMA

*Scenario3: Incomplete data is available*

In scenario3 we have a special case in which we have a very limited data here we use Bayesian networks for finding the failure probability of the required uncertain subsystem and then we apply fault tree analysis to rest of the system to find the failure probability of the system if needed but here we stop with inferring the reason for failure.

As shown in the fault tree in fig.6 the sensors are placed on the system, control PCB, Power PCB, Hall sensor PCB and rotor and stator. The experts claim that the servo control unit is not following the probabilistic relation of fault tree. Due to some unknown cause there is change in failure probability of SC and due to that there will be change in failure probability of whole system. We have calculated the change in failure probability of EMA system with the help of the available evidence, "among a series of 10 observed failure events there are 2 failures at sensor2 and 3 failures at sensor4 and no failures at sensors 3 and 5".

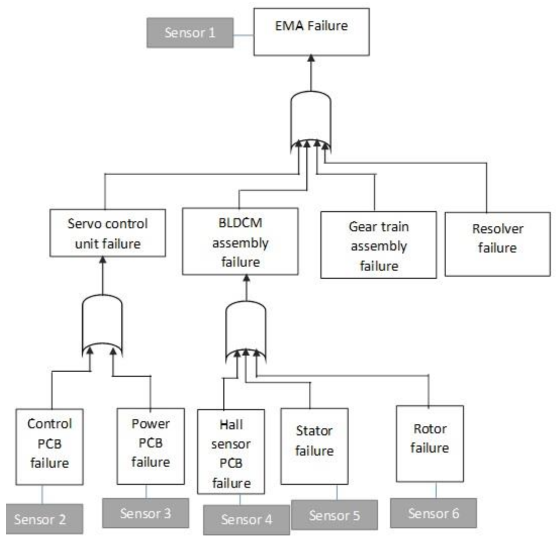


Fig.7: Fault tree of EMA with sensors

According to the claim there is uncertainty in the subsystem SC. The nodes CP, PP, H, S, and R are always stochastic nodes now as there is an uncertainty claim on SC, the node SC also becomes a stochastic node. Now we consider only stochastic nodes to find the failure probability of SC and the system.

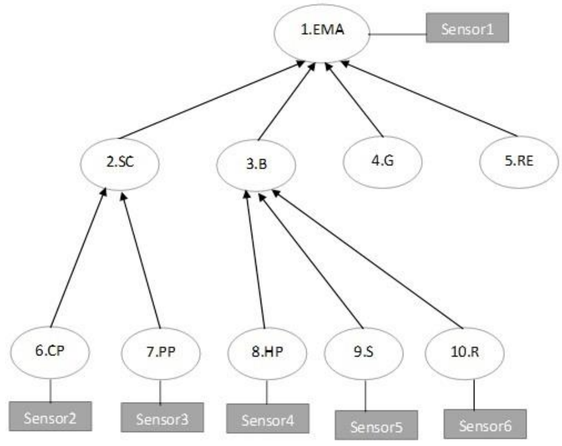


Fig.8: Bayesian network representation for EMA system with sensors

There are several steps to find the likelihood function to the subsystem SC. The steps are as follows [8].

*Step1*: Select the stochastic nodes and write the state combinations.

*Step2*: Treat each state combination as state vector and the n denote with $x1, x2, \dots \dots xn$.

*Step3*: Find the joint probabilities for all the state vectors and denote with $JP(1), JP(2), \dots \dots \dots JP(n)$ .

*Step3*: We generate count vectors from the given evidence.

*Step4*: The likelihood is the sum of count vectors following multinomial distribution with count vector as counts for the joint probabilities.

The step1, step2 and step3 are tabulated in table15. Now we should generate the count vector as per the step4. The count vector is the counts of the state vectors over a period of system use. For example if the state vector x1 has occurred twice and x2 occurred thrice we denote the counts as y1=2 and y2=3 similarly we write the counts for all the state vectors and combine them to form a count vector. We have generated count vectors from the evidence. The evidence says that among a series of 10 observed failure events there are 2 failures at sensor2 and 1 failures at sensor4 and no failures at sensors 3 and 5 and 6. Hence we write for both the states like this,

1) two {0,1,0,0,0,0} and two {1,1,0,0,0,0}
2) one {0,0,0,1,0,0} and one {1,0,0,1,0,0}
3) Seven {0,0,0,0,0,0} and five {1,0,0,0,0,0}


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i.e,

1) two times x33 or x49
2) One x5 or x37
3) Seven x1 or x33

We have to keep in mind that all the state vectors that we had extracted from the evidence are system failure combinations so total failure counts in all the possible count vectors should be equal to the total observed failures i.e, 10 failure from our available evidence.

From the above extraction of counts from the evidence, we have written the count vector as follows. Let the count vectors be denoted by Y and counts be y.

Table15: The state vectors and joint probabilities

| State vectors | SC (2) | CP (6) | PP (7) | HP (8) | S (9) | R (10) | Joint Probabilities | | Joint Probability Denotation |
|---|---|---|---|---|---|---|---|---|---|
| x1 | 0 | 0 | 0 | 0 | 0 | 0 | (1-P2(00000)) | (1-P6)(1-P7)(1-P8)(1-P9)(1-P10) | JP(1) |
| x2 | 0 | 0 | 0 | 0 | 0 | 1 | (1-P2(00001)) | (1-P6)(1-P7)(1-P8)(1-P9)P10 | JP(2) |
| x3 | 0 | 0 | 0 | 0 | 1 | 0 | (1-P2(00010)) | (1-P6)(1-P7)(1-P8)P9(1-P10) | JP(3) |
| x4 | 0 | 0 | 0 | 0 | 1 | 1 | (1-P2(00011)) | (1-P6)(1-P7)(1-P8)P9P10 | JP(4) |
| x5 | 0 | 0 | 0 | 1 | 0 | 0 | (1-P2(00100)) | (1-P6)(1-P7)P8(1-P9)(1-P10) | JP(5) |
| x6 | 0 | 0 | 0 | 1 | 0 | 1 | (1-P2(00101)) | (1-P6)(1-P7)P8(1-P9)P10 | JP(6) |
| x7 | 0 | 0 | 0 | 1 | 1 | 0 | (1-P2(00110)) | (1-P6)(1-P7)P8P9(1-P10) | JP(7) |
| x8 | 0 | 0 | 0 | 1 | 1 | 1 | (1-P2(00111)) | (1-P6)(1-P7)P8P9P10 | JP(8) |
| x9 | 0 | 0 | 1 | 0 | 0 | 0 | (1-P2(01000)) | (1-P6)P7(1-P8)(1-P9)(1-P10) | JP(9) |
| x10 | 0 | 0 | 1 | 0 | 0 | 1 | (1-P2(01001)) | (1-P6)P7(1-P8)(1-P9)P10 | JP(10) |
| ………. | …. | …. | …. | …. | …. | …. | ............ | …………….. | ………. |
| x64 | 1 | 1 | 1 | 1 | 1 | 1 | P2(11111) | P6P7P8P9P10 | JP(64) |

Then we have denoted the count vector as $Y = \{y1, y2, y3, \dots \dots yn\}$. Therefore the count vector from our evidence can be written as.

Y1= {7 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 2 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0}

There will be many combinations among the counts depending on the evidence. One more combination can be written as $Y2 =$
{6 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 2 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0}.

Similarly we got 48 combinations of the counts forming 48 count vectors [7]. We have generated the likelihood analytically as in equation 11.

$$likelihood = \sum_{Y_1}^{Y_{48}} N! \prod_{i=1}^{64} \frac{1}{y_{i!}} JP(i)^{y_i} \dots .(11)$$

Using the above likelihood function in equation 11 and non-informative uniform priors we found the posterior probabilities of the stochastic nodes we have selected.
The results are in table16.

Table16: Posterior probabilities of stochastic nodes.

| | Mean | SD | Median |
|---|---|---|---|
| $P_{00}$ | 0.5023 | 0.2895 | 0.5148 |
| $P_{01}$ | 0.4935 | 0.285 | 0.4952 |
| $P_{10}$ | 0.4969 | 0.2874 | 0.4897 |
| $P_{11}$ | 0.4758 | 0.2869 | 0.4686 |
| P6 | 0.2485 | 0.1255 | 0.2365 |
| P7 | 0.07807 | 0.06805 | 0.05812 |
| P8 | 0.1649 | 0.1003 | 0.1457 |
| P9 | 0.0853 | 0.07923 | 0.06165 |
| P10 | 0.09055 | 0.07837 | 0.07023 |


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## V. DISCUSSION

In first two cases of the project we updated the prior information depending present available test data. We have also considered Jeffrey's non-informative prior when the prior is unavailable. The probability of system after second case says that the system is highly reliable. Coming to the third scenario it is a special case in which no prior data is available. In this case we found the failure probabilities which totally because of likelihood or present available incomplete data. So that we can find the failure probabilities of the subsystem or system which has a claim of declining the deterministic relation of fault tree or which undergone changes. The third case gave us consistent probability values even when we have incomplete data. The results in the third case mainly depend on the likelihood function. OpenBugs software is used to find the posterior probabilities using Metropolis-Hastings algorithm. The use of Matlab made us easy to deal with mathematical equations and calculations. Matlab is used to write the joint probability statements in the Openbugs software to reduce the time for typing.

The state vectors and count vectors are generated as in [7]. In the future research we like to d-separate the subsystems and components depending on the evidence to find the likelihood so that process becomes easy for finding the likelihood.

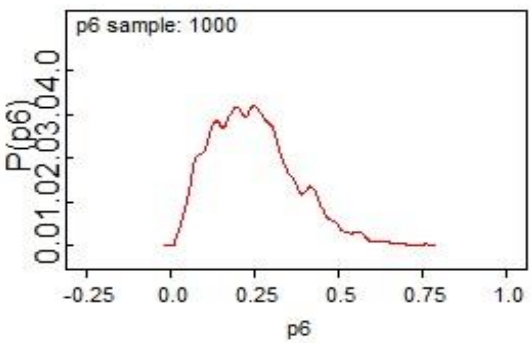


Fig.9: Density plot for P6

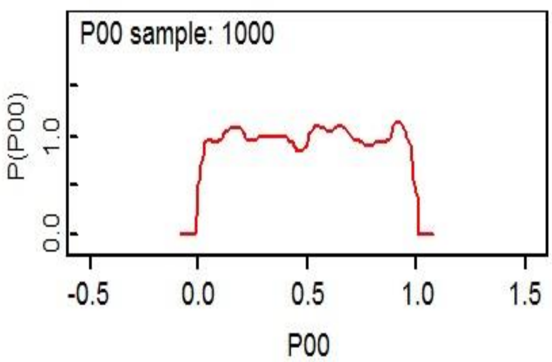


Fig.10: Density plot for P00

## VI. CONCLUSION

From the failure probability of the system after the scenario 2 we infer that the EMA system is highly reliable even in the case of partial failures among the components and subsystems. But failure probability of system is more when at least one component or subsystem is failed. In the third scenario the failure probability of the subsystem SC when both of its components are working is more that is the subsystem after replacement with a new design it is undergoing some unknown assembly or compatibility failures. We have reinvestigate the causes of the failures of the subsystem. The component CP also has the failure probability around 0.2 which is not negligible but from the results we can say that the component CP's failure is not effecting more on the subsystem SC.

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