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Connectedness and Punctual Space in Fiber Bundle Space

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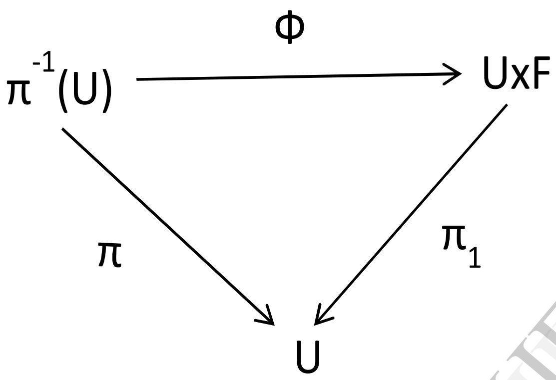
1.Introduction:

The topology of a fiber bundles intuitively a space E with locally looks like a product space B x F but globally may have a different topological structure. Specifically the similarity between fiber bundle E and the product space B x F is defined using a continuous surjection that satisfies a local trivially condition. Jeffrey M. Lee [7] defined as

Definition 1.1 [7]

Let F, B, and E be C^r manifolds and let π:E→B be a C^r-map. The quadruple(E, π, B, F) is called a (locally trivial) C^r-fiber bundle if for each point b ϵ B there is an open set U containing b and a C^r diffeomorphism $\Phi:\pi^{-1}(U)\rightarrow U \times F$ such that the following diagram commutes



In differential geometry, your attention is usually focused on C^∞ fiber bundle(that is smooth fiber bundles)

If (E,π,B,F) is a smooth fiber bundle, then E is called the total space,

π:E→B is called the bundle projection,

B: is called the base space,

F: is called the typical fiber,

For each b ϵ B, the $E_b\pi^{-1}(b)$ is called the fiber over b.

Examples of fiber bundles: 1.2:-

1. (**Vector bundle**): A real n-dimensional vector bundle is a fiber bundle with fiber $\mathbb{R}^n$ and structure group GL (n, ℝ). Similarly an n-dimensional complex vector bundle has fiber C^n and structure group GL (n, ℂ). By introducing a metric in each vector bundle. We may reduce the structure group to O(n) and U(n), respectively.
2. (**The tangent frame bundle of a C^p manifold M**): Let M be a n-dimensional manifold. By a frame at p ϵ M we mean on ordered basis $[V_1,V_2,\ldots V_n]$ of the tangent space T_pM. Denote by F_p the set of all frames at p. We also consider the space F(M) of all frames at all points of M. The general linear group GL(n, ℝ) acts on each fiber F_b on the right.
$F_b \times GL(n, \mathbb{R})\rightarrow F_b$, namely for a frame $u=[V_1,V_2,\ldots V_n] \in F_b$ and for a regular matrix
$g= (gij) \in GL(n, \mathbb{R})$, we set,
$w_i= \sum g_{ji} V_j$, $u_g=[w_1,\ldots,w_n]$.
3. **Covering manifold**: The covering map π : N→M of manifolds becomes a fiber bindle with zero dimensional manifold as fiber.
4. For a smooth manifold B and F. We have the projection π_1:E→B is a submersion and each fiber $\pi^{-1}(b)$ is a regular manifold which is diffeomorphic to F.

This discus shows that if both F and B are connected the E is connected.

A global smooth section of a fiber bundle ξ = (E, π, B, F) is a smooth map σ:B→E such that $\pi o\sigma=Id_B$ i.e. $\sigma(b) \in E_b$.

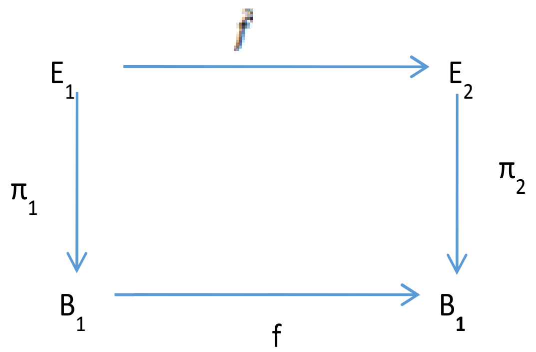
A local smooth section over an open set U is a smooth map σ:U→E such that $\pi o \sigma = Id_U$. The set of a smooth section of ξ is denoted by Γ(ξ)

Shigeyuki Morita [12] and [John Lee[6]] ,[Jeffrey Lee[7]] defined the isomorphic property of bundles as.

Definition 1.3

Let $\xi_i = (E_i, \pi_i, B_i, F)$ (i=1, 2) be two fiber bundles with the same fiber, by a bundle map from ξ_1 to ξ_2 we mean C^∞ maps

$\tilde{f}:E_1 \rightarrow E_2$. f: $B_1\rightarrow B_2$ such that the diagram



is commutative.

In this paper we extended the topological properties of fiber space. Lawrence Conlon [8] defined connected property of fiber space. We discussed this property in different ways like simply connected, weakly connected, strongly connected and densely connected. These properties on E, B spaces which are topological manifolds.

In Section 2 we recalled some basic concepts and definitions.

In Section 3 we redefined the connected property of fiber bundle particularly Base Space (B), Total space E with different concepts like globally connectedness on E, B, ξ(E, π, B, f)

Lastly we in introduced the cut point and punctured points on the fiber bundles on the Base Space B. B. Honari[1], D.K. Kamboj and Vinod kumar[2] discussed the cut points of 'Topological Manifold M. We discussed on fiber bundles. This concept also defined on ξ and it's structure group G. Naoyuki Monden[] introduced punctured surface and mapping which we discussed on ξ space (i.e. ξ = (E, B, F,π).

2.Basic Definitions

A fiber bundle is a topological manifold our general assumption is that the Base B is Topological Manifold.

Definition 2.1 :-

Let F be a C∞, manifold. Suppose there are C∞ manifolds E and B and a C∞-map π:E→B, we call ξ=(E, π,B, F)is differentiable fiber bundle (or a differentiable F bundle) if it satisfies the following conditions

(Local Trivially) For each point bϵB, there are open neighborhoods U and a diffeomorphism $\Phi:\pi^{-1}(U) \cong UxF$ such that for an arbitrary $u\epsilon\pi^{-1}(U)$.

We have $\pi(U) = \pi_1 o\Phi(U)$, where π_1:UxF→U denotes the projection onto the first components.

We call E – The total space

B – The Base Space

F – Fiber

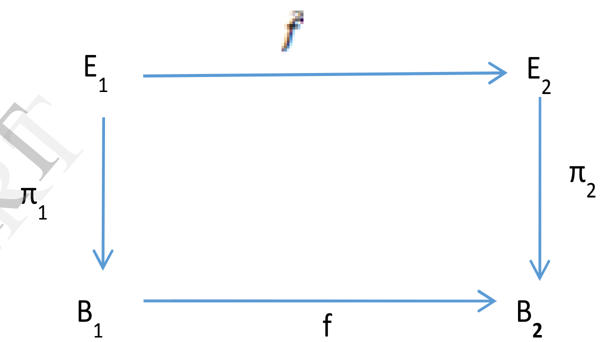
π – projection

We call $E_b = \pi^{-1}(b)$ the fiber over b. Instead of (E, π, B, f) we may ball π:E→B or simply E a fiber bundle or ξ is fiber bundles.

Definition 2.2 [S. Morita[12]]

Let $\xi_i = (E_i, \pi_i, B_i, F)$, i = 1, 2 be two fiber bundles with the same fiber. By a bundle map from ξ_1 to ξ_2 we mean C^∞-maps

$\tilde{f}:E_1 \to E_2$, $f:B_1 \to B_2$ such that the diagram



is commutative (ie $\pi_2 o\tilde{f} = (fo\pi_1)$ and arbitrary fiber $\pi^{-1}(b)$, b ϵB is a diffeomorphism. If further more f is a diffeomorphism so is $\tilde{f}$(vice versa). Also ($\tilde{f}^{-1}$, f^{-1}) is a bundle map.

Definition 2.3 [S.Morita[12]]

Two fiber bundles $\xi_i=(E_i,\pi_i,B,F)$ over the same base space B and with the same fiber F are said to be **isomorphic** is there exists a bundle map $\tilde{f}:E_1 \to E_2$ together with the identity map f:B→B we write $\xi_1 \cong \xi_2$. A bundle that is isomorphic to the product bundle BxF is called a **trivial** bundle.

Theorem:2.4 (Jhon Lee)(6)

Let X, Y be topological spaces and let f:X→Y be a continuous map. If X is connected, then f(X) is connected.

3. Connectedness in Fiber

In fiber bundle connectedness is the major concept of topological space. Here we define connectedness of E, B and F, as topological space.

In this section a general assumption is that connected and path connected are the same concepts.

Definition 3.1: Connectedness in Base space.

A topological space B of dimension is said to be connected if there exists continues map g: [0, 1] $\rightarrow$B such that g (1) =b_1 ,g (1)=b_n for all b_1,b_n ϵ B

Definition 3.2: Connectedness in total space.

A topological space E of dimension n is said to be connected if there exists a continuous map π^{-1} : B $\rightarrow$E, such that π^{-1} (b_1)= p_1 , π^{-1} (b_n) =p_2 for all p_1, p_2 ϵE, b_1,b_n ϵ B

The map g and π^{-1} satisfies the composition π^{-1}og[0]=p_1, π^{-1}og[1]=p_2

Definition 3.3: Simply (weakly)Connectedness in B.

Let B_i (i=1………..n) be sub bases of fiber bundle ξ, there exists a map between

Some B_i to B_j that is f:$B_i$$\rightarrow$$B_j$ such that the compositions $[0,1]\xrightarrow{g}B_i\xrightarrow{fi}B_j$

Are defined as

$f_i$og (0) = b_1 ϵ B_j

$f_i$og (1) = b_2 ϵ B_j, for a fixed j.

Definition 3.4: Strongy connected in B

Let B_i(i=1………n)be sub bases of fiber bundle .there exists a map f_j

$B_i$$\rightarrow$$B_j$ for all i and j such that the composition map f_j o g_i (0) = b_i ϵ B_j

f_j o g_i (1)=b_i ϵ B_j, for all i and j.

Definition 3.5: Simply (weakly) Connectedness in E.

Let E_i (i=1……….n) be sub spaces of E such that there exists a maps $\tilde{f}_I$;E_i $\rightarrow$ E_j such that

$\tilde{f}_I$;E_i $\rightarrow$ E_j is defined as $\tilde{f}_i$(p_i)=p_j, for all p_i ϵ E_i, p_j ϵ E_j.

it also satisfy the composition

$[0,1]\xrightarrow{g}E_i \rightarrow \tilde{f}_i\xrightarrow{fi}E_j$

f_i o g(0) =p_1 ϵ E_j ,f_i o g(1) =p_2 ϵ E_j for fixed j

Definition -3.6-Strongy connected in E:

Let E_i (i=1…….n) be sub space of E, there exists a maps $\tilde{f}_I$:E_i $\rightarrow$ E_j and

Continuous functions **g_i: [0,1] $\rightarrow$$E_i$**

Such that g_i (0)=p_1

g_i (i)=p_n

for each p_1,p_n ϵ E_i defines the composition $\tilde{f}_j$ og_i :[0,1] $\rightarrow$ $E_i$$\rightarrow$ E_j

$\tilde{f}_I$ o g_i [0] =p_1 $\epsilon$$E_j$

$\tilde{f}_I$ o g_i [1] =p_n ϵ E_j

Definition 3.7 :

In all the above definition the maps which are defined from $\tilde{f}_I$:E_i $\rightarrow$ E_j

are called bundle maps which are commutative in n-dimensionaly projection

π_i: E_i $\rightarrow$ B_i

π_j:$E_j$$\rightarrow$ B_j

These maps and projection are diffeomorphisms and n-projection of fiber bundle .

This shows that,

$\tilde{f}_i^{-1}$, f_i^{-1} are also bundle maps .

These concepts of bundle map forms a **densely connected space** in ε.

Definition 3.8: Densely connected **ε (fiber bundle)**

Let E_i ,B_i,π_i are the subtotal spaces , subbase space and projection maps respectively and a continuous map, g:[0,1] $\rightarrow$$E_i$ or B_i ,satisfies

g(0)=e_i ϵ E_i or b_i ϵ B_i for all i.

These bundle map and projections forms densely connection between them then such enrich structure is called **densely connected ε.**

Theorem 3.9

Let π:E → B be a projection map , if E is connected then B is connected .

Proof :

Let E and B are topological spaces. The map π:E →B be a continues map .

A total space E is connected, then there exists a continuous map g

Defined g:[0,1] →E such that

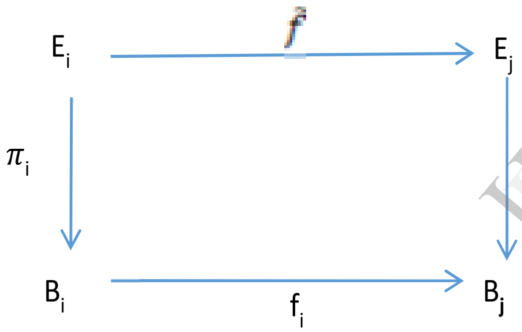
$g(0)=P_1=\pi^{-1}(b_1)$ and

$g(1)=P_2=\pi^{-1}(b_2)$ for all $P_1, P_2 \epsilon$ E and $b_1,b_2 \epsilon$ B.

We know π is projection map which is always continues map also bijective map

Therefore the image of connected space is connected under continuous map.(see theorem 2.4)

[For basic theorem see [Jhon Lee[6] [p-67] theorem :4,3.]



As E is connected and B be a topological space π:E → B which is continuous.

Therefore B is connected.

Corollary 3.10 :

Let E and B be a topological spaces and π:E →B is bijective map which is continuous if B is connected then E is connected.

Proof :

Let $E_1$and B are topological spaces. π:E →B is bijective map and continuous then π^{-1}:B→E is also continuous map .(see theorem2.3).

Then by (Jhon Lee[6] [p-67] theorem4.3 & theorem 3.9) E is connected.

Theorem 3.11 :

Let E and B be a topological space. π :E→ B is a bundle projection if E is simply connected then B is simply connected.

Proof :

Let E_i(i=1,……. n) be sub space of E such that there exists a map $\tilde{f}_i : E_i \to E_j$ defined as $\tilde{f}_i(P_i)=P_j$ $P_i,P_j \epsilon E_i ,E_j$ respectively

By definition of simply connected in E

The composition , $\tilde{f}_i \circ g[0]=P_2 \epsilon E_j$ for fixed j

$\tilde{f}_i \circ g[1]=P_n \epsilon E_j$

$[0,1]\xrightarrow{g} Ei \to \tilde{f}_i \_ \to E_j \approx E$

$\tilde{f}_i (g)(0)=P_2 = \tilde{f}_i (P_1)=P_2,$

$\tilde{f}_i\ g(1)= \tilde{f}_i (P_{n-1})=P_n$

These composition maps are continuous in E_j ‘s.

Now $E_j ,E_j \subseteq E$, consider the projections map generally

$\pi_i :E_i \to B_i \subseteq B$ are projections from E_i to B_i

Then the composition

$\pi_j \circ \tilde{f}_i \circ g(0)=b_j$, $\pi_j \circ \tilde{f}_i \circ g(1)=b_j$ for some i

Therefore these composition maps are continuous $\pi_j \circ \tilde{f}_i \circ g$,this shows the path between **some** points in B .i.e.$\pi_j \circ \tilde{f}_i$

i

This commutative diagram shows that

$\pi_j \circ \tilde{f}_i (P_i)=b_j$, $P_i \epsilon E_i$ $b_i \epsilon B_j$

similarly $\pi_j \circ \tilde{f}_i \circ g(0)=b_{j1}$ --------------(i)
for some n fixed j

$\pi_j \circ \tilde{f}_i og(1)=b_{jn}$ ---------------(ii)

These two equation gives the composition map

π_j o $\tilde{f}_i$ o g (0)& π_j o $\tilde{f}_i$ o g (1) given a path

between some point b_{j1} and b_{jn} respectively

➡ B is simply connected space

Theorem 3.12

Let E be total space and B –base space and the projection π:E→ B is bijective map.

If B is simply connected than E is also simply connected.

Proof:

Let B_i are sub bases space of B and E_i are sub space of E.

As B is simply connected by definition there exists a maps f_i :B_i → B_j

And the continuous map g:[0,1] →B_i ,the composition is defined

$f_i og (0)=b_{j1} \in B_j$,

i.e. $f_i [b_i]=b_{j1}$, for a fixed j.

$f_i og(1)=b_{jn}$,

i.e. $f_i[b_{in}]= b_{jn}$

consider the projection

π_i :E_i → B_i is bijective implies

π_i^{-1} :B_i → E_i is continuous

The following commutative map shows that

$\tilde{f}_i$ o π_i^{1} $(b_i)=P_j$, $b_i \in B_i$, $P_j \in E_j$, for fixed i

OR

π_i^{-1} o f_i^{-1} $(b_j)=P_i$, $P_i \in E_i$, b_j → B_j for fixed j

Since f_i^{-1} , $\tilde{f}_i$ are bundle maps

case ii

$\pi^{-1}_j f_i (b_i)=P_j$ $P_j \in E_j$, $b_j \in B_i$ for fixed i,

$\tilde{f}_i^{-1} \circ \pi^{-1}_j (b_j)=p_i$ for fixed j

These cases show that case i is considered with composition map of g,

g:[0,1]→B_i

and composition

$\tilde{f}_i$ o π^{-1}_io g (0) =P_{j1}

$\tilde{f}_i$o π^{-1}_io g (1)=P_{jn}

This shows that there exists a simple path between P_{j1} to P_{jn} in $E_j \subseteq E$

similarly case i

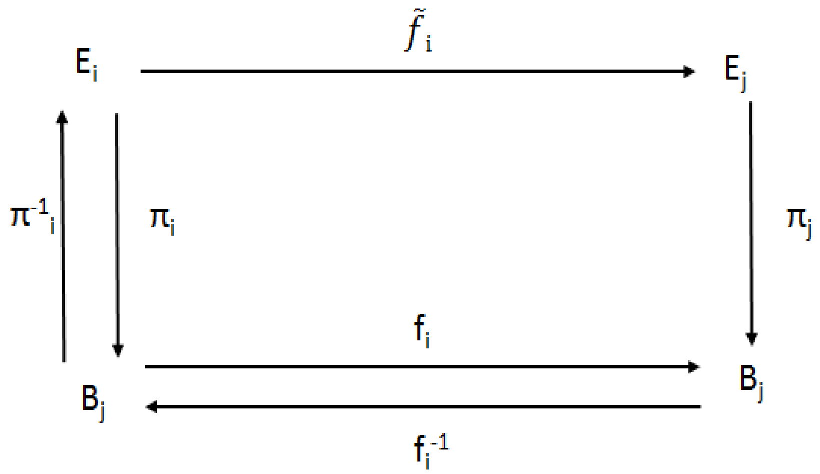
π^{-1}_i o f^{-1}_i (g(0))=P_{i1}

π^{-1}_i o f_i (g(1))=P_{in}

This shows that there exists path in $E_i \subseteq E$.

Similarly for case ii we will get the subspace E_i and E_j are simply connected.

Therefore E is simply connected space.



case i

Theorem 3.12

If π:E→B be a projection map and E is strongly connected then B is so.

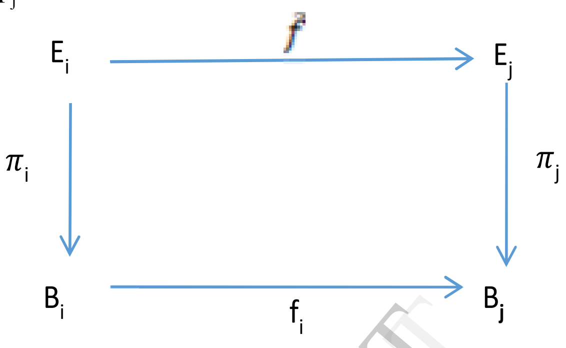
Proof

Let E_i (i=1….n)are subspaces of E. B_i are the spaces of B and the projection

π :E→B a bundle projection which is bijective and continuous map.

As E is strongly connected there exists map g:[0,1] →E and bundle maps $\tilde{f}_i$ from E_i to E_j that is

$\tilde{f}_i:E_i \to E_j$ defined as $\tilde{f}_i[P_i]=P_j$



The composition map $\tilde{f}_I$ og:[0,1]→E_j is a bundle map which is also continuous map

The commutative

By definition of a strongly connectedness in E.

π_j o$\tilde{f}_j[P_i]=b_j \in B_j$

π_i o$f_i[P_i]=b_j \in B_j$

Therefore the composition map

π_j o $\tilde{f}_i$: $E_i \to B_j$ which is continuous for any point $P_i \in E_i$

The image of connected space under continuous map $\tilde{f}_{I\,o}\,\pi_j$ is connected & a continuous map

g:[0,1]→E_i

π_j o $\tilde{f}_i$ o g [0]=$b_{j\,1} \in B_j$

π_j o $\tilde{f}_i$ o g [1]=$b_{j\,n} \in B_j$

OR

f_i oπ_i o g(0) =$b_{i\,1} \in B_j$

f_i oπ_i o g(1) =$b_{i\,n} \in B_j$

Therefore each element of base space connected by a path

Therefore this shows that all B_i 's are connected with continuous maps f_i

Therefore B is strongly connected.

Corollary 3.13

If π:E→B be a bijective map & B is strongly connected then E is so.

4. Cut Points and Punctured Points in Fiber Bundles.

A Topological spaces E and B are connected which contains at least two points. A point which becomes separate or disconnected is a cut point space with different dimensions of topological space like 1 dimension or more, which are defined in connectedness paper.

The concept of cut point and punctured point in fiber space are same concept as in topological space but they depend only on base space. We defined cut point on base space as…

Definition 4.1 Cut point in B

A point x ϵ B is said to be a cut point if B – {x} has a separation or B becomes disconnected. That is by removal of a point x from the B becomes disconnected as subspaces.

Lemma 4.2

Let B be a topologically connected space, x ϵ B be any point of B, then by removal of x from B, B becomes disconnected but disconnected set satisfies the following properties.

i) If B_1 and B_2 are two separated sets then $\{x_n\} \cup B_1 \cup B_2 = B$ and $B_1 \cap B_2 = x$

ii) The separated sets B_1 and B_2 are connected.

Theorem 4.3

Let B be connected topological space and x ϵ ct x ϵ B with condition $B_1, B_2 \subseteq B$ and $B_1 \cup B_2 \cup \{x\} = B$, $B_1 \cap B_2 = x$ then there exists a path between this to subspace B_1 and B_2 passing through x.

Proof

Let B be connected topological space x be a cut point of B which forms a separation of

$B = B_1/B_2$.

Separated space (sets) are connected.

Let x_i's ϵ B_1 , y_i's ϵ B_2 are points in separated sets for some i.

Consider a function g:[0, 1]→B defined as g(0) = x, g(1) = y.

The x_i's and y_i's are any other point in B, particularly in two compartments in B like B_1 and B_2.

As B is connected which implies B_1 and B_2 are also connected, interconnected by the point x_{ct} which is called a cut point of B.

B_1 and B_2 are interconnected then some points of B_1 and some points of B_2 are connected to a path γ.

γ:[0, 1]→B, $\gamma(0) = x \in B$, $\gamma(1) = y$

Let the family of paths $\gamma_i(1) = y_i$ for all $x_i \in B_1$ $y_i \in B_2$. But B_1 and B_2 are connected by a single point all paths γ_1's are passing through the point which is common in B_1 and B_2 which cut point of B set x_{ct}.

Therefore B is connected with cut point. There exists a path passing through cut point of B.

Corollary 4.4

If B is a path connected then B is connected.

Corollary 4.5

If B is connected with cut point then B is path connected.

Definition 4.6 Cutput in E

Let E be a total space, B be a base space for fiber space. The projection π:E→B is continuous and bijective, a point x ϵ B is cut point of B then $\pi^{-1}(x) \in E$ is called cut point of E.

That is the pre image of cut point of base space is cut point of total space E.

Definition 4.7 Punctured point in E and B

A point P(x) is said to a punctured point of B if removal of P(x) from B, B becomes hole space, but B is connected.

That is P(x) be a punctured point of B. Then {B – P(x)} is connected.

Theorem 4.8

Let B be connected a topological manifold (space). A point P(x) be a punctured point of B then {B-P(x)} is connected.

Proof

Let B be a topological base space which is connected. There exists a function f:[0, 1]→B satisfies f(0) = x, f(1) = y for all x, y ϵ B.

Let x_p be a point other than x and y of B is a punctured point of B. All the neighborhood point of x_p are connected with B. There exists a path between them.

Any two points except x_p of B are connected by a path.

By a removal of a point x_p from B the remaining points of B are connected by a path.

Therefore B is a path connected space and is connected.

Theorem 4.9

Let E topologically connected total space. A point P(x) ϵ B a punctured point of B then π^{-1}(Px) is punctured in E.

Proof

Let E be Topological connected total space. π:E→B a projection map which is continuous and bijection. $P(x_p)$ ϵ B be a punctured point of B then projection map π:E→B and π^{-1}:B→E also a continuous map. Satisfies $\pi^{-1}(x) = F_1$ for all $F_1 \in E$. for all x ϵ B

But P(x) is punctured point of B.

$\pi^{-1}(P(x))$ is punctured point of E.

That is F_1 is a punctured of E.

Theorem 4.10

If π:E→B is a projection map and continuous map and B is a path connected space then E is also.

Theorem 4.11

Let E be topological connected total space. A point P(x) ϵ B a punctured point of B and B is connected then there exists a punctured point π^{-1}(P(x)) ϵ E and E is connected as arc wise. (path connected).

Proof:

Let E be topological connected total space. A point P(x) ϵ B a punctured of B. B is connected, there exists a continuous function f:[0, 1]→B such that

f(0) = x, f(1) = y for all x, y ϵ B

π:E→B is continuous and bijective. This implies π^{-1}:B→E such that $\pi^{-1}(x) = F_1$, $\pi^{-1}(y) = F_2$ for $\pi(F_1) = x$, $\pi(F_2) = y$, for all F_1, F_2 ϵ E

As B is connected there exists a path γ:[0, 1]→B satisfies $\gamma(0) = x$, $\gamma(1) = y$.

If P(x) ϵ B be a punctured point of B.

There does not exists a path to punctured point. But its neighborhood points are connected with B which forms a boundary of punctured space (p(x)).

We have boundary of any space which is connected. There exists a path between any two points in neighborhood of P(x).

Therefore B is connected, B-P(x) is also connected by a path.

Therefore the continuous map π^{-1}:(P(x))→F_p is defined as P(x) = $\pi(F_p)$

But neighborhood of F_p are connected with E space as B is connected E is also connected.

By continuous map π:E→B

Therefore E is connected which implies every point between ant two points except punctured point F_p are connected by a path

Therefore E is path connected.

Corollary 4.12

Let E and B be a connected topological spaces. A point π^{-1}(x) ϵ E is punctured in E and π:E→B projection map then x ϵ B which need not be punctured point of B.

Conclusion:

Connectedness of base space, total space gives connectedness of fiber bundle which is base of further work, and application in Biology.

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