# Consistency in the Analytic Hierarchy Process: A Framework for Automobile Steering Technology

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***Abstract*— In this paper it is shown that the principal eigenvector is a necessary representation of the priorities derived from a positive reciprocal pairwise comparison judgment matrix. When providing numerical judgments, an individual attempts to estimate sequentially an underlying ratio scale and its equivalent consistent matrix of ratios. Near consistent matrices are essential because when dealing with intangibles, human judgment is of necessity inconsistent, and if with new information one is able to improve inconsistency to near consistency, then that could improve the validity of the priorities of a decision. If such perturbations were forced, they could be arbitrary and thus distort the validity of the derived priority vector in representing the underlying decision.**

**The analytic hierarchy process (AHP) provides a decision maker with a way of examining the consistency of entries in a pairwise comparison matrix and the hierarchy as a whole through the consistency ratio measure. In this paper researcher has tried to explain AHP model of technology competitiveness for automobile steering. Six criteria were used for evaluating steering technology: Cost Effectiveness, Performance, Controlling, Market, Serviceability, Maintenance and Reliability, Flexibility. Consistency has been evaluated through pairwise comparison judgement matrix for six criteria's of steering technology.**



***Keywords*— analytic hierarchy process; automobile steering; criteria ; matrix**

## I. Introduction

Various methods have been developed in the last three decades which use pairwise comparisons of the alternatives and criteria for solving multi-criteria decision-making (MCDM). The pairwise comparison analytic hierarchy process (AHP) method was introduced by Saaty (1977) that has increased in popularity amongst other MCDM. AHP main strength is its ability to structure a problem and then relate both tangible/intangible factors in a common framework. In the pairwise comparison AHP method, criteria and alternatives are presented in pairs of one or more referees (e.g. experts or decision makers).

It is necessary /essential to appraise individual alternatives, deriving weights for the criteria, constructing the overall rating of the alternatives and identifying the best one. This method uses a reciprocal decision matrix obtained by pairwise comparisons so that the information is given in a linguistic form.[1]

Various methods have been developed in the last three decades which use pairwise comparisons of the alternatives and criteria for solving multi-criteria decision-making (MCDM). The pairwise comparison analytic hierarchy process (AHP) method was introduced by Saaty (1977) that has increased in popularity amongst other MCDM. Researcher found AHP method is appropriate for his current research work.

Finding priority for various steering technology criteria is not an easy task because every experts/person has his own perception about technology. In AHP method Saaty's original 1-to-9 scale used for deciding priority for criteria values of steering technology. AHP main strength is its ability to structure a problem and then relate both tangible/intangible factors in a common framework. It is an effective tool to generalised/tested various criteria for the steering technology. Categorisation of various parameters is necessary to understand/evaluate steering technology in depth AHP is used for the same and Hierarchical structure of steering mechanism is formulated to achieve technology competitiveness.

## II. Analytic Hierarchy Process (AHP)

This Ideal Mode AHP was later accepted by Saaty (1980, 1994)[2] and is widely considered to be the most reliable MCDM method.[3] (Triantaphyllou and Mann (1995)[4],(Liberatore and Nydick 2008)[5]

### A. The Nature of Consistency

Impeccable consistency rarely occurs in practice. In the AHP the pairwise comparisons in a judgment matrix are considered to be adequately consistent if the corresponding consistency ratio (CR) is less than 10% (Saaty, 1980). Thomas L. Saaty the originator of AHP, defines inconsistency as

$$consistency\ Index\ (CI) = \frac{\lambda\ Max - n}{n - 1}$$

Where $\lambda_{Max}$ is the largest principal eigenvalue of a positive reciprocal pairwise comparison matrix of size n. If the paired comparisons are perfectly consistent, then $\boldsymbol{\lambda_{Max}}$ is equal to the size of the matrix and the consistency index is zero.

People often have difficulty in achieving a consensus on certain issues. The judgements may be inconsistent, and how to measure inconsistency and improve the judgements it is a big challenge. Appropriate consistency evaluation of elicited preferences for steering technology is seen as important principally because the achievement of an acceptable consistency level is desirable. The more consistent are the preferences of a decision maker, equally if judgements are far from consistency, i.e. they are heavily contradictory, it is likely that they were given with poor competence and care.
In current research consistence of priorities matrix is measured for steering technology criteria. In this matrix, six criteria i.e. Cost Effectiveness, Performance, Controlling, Market, Serviceability, Maintenance and Reliability, Flexibility is used for evaluation of steering technology. Matrix of 6 X 6 is formed and AHP is used for checking consistence. By 6 X 6 priorities matrix weightage of six criteria is calculated. Hear researcher try to get more consistence matrix so result accuracy of the steering technology weightage could be amplified.

The consistency ratio tells us how consistent judgment matrix is. A higher number means less consistent,

whereas a lower number means that more consistent. In general, if the consistency ratio is 0.10 or less, the decision maker's answers are relatively consistent. For a consistency ratio that is greater than 0.10, the researcher should seriously consider re-evaluating responses during the pairwise comparisons that were used to obtain the original matrix of pairwise comparisons. Table: 1 shows value of Random Consistency index (RCI).

$$consistency\ Ratio\ (CR) = \frac{CI}{Mean\ Randome\ CI}$$

TABLE 1: RCI VALUES FOR DIFFERENT VALUES OF N[6]

| n | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|
| RCI | 0 | 0 | 0.58 | 0.90 | 1.12 | 1.24 | 1.32 | 1.41 | 1.45 |

*B. Decision Making through AHP Hierarchical Modeling*

The AHP model of technology competitiveness of automobile steering mechanism shown in Fig. 1, it consists of three levels. Level one shows object/goal and level two display Criteria considered and in level three affecting factors of criteria displayed.

Cost is vital factor for technology competitiveness, for steering mechanism Criteria of Cost effectiveness includes Steering cost and maintenance cost Factors. Performance is related to the functioning of steering. It includes: System Efficiency, System control technology, Vehicle Width and durability and comfort.

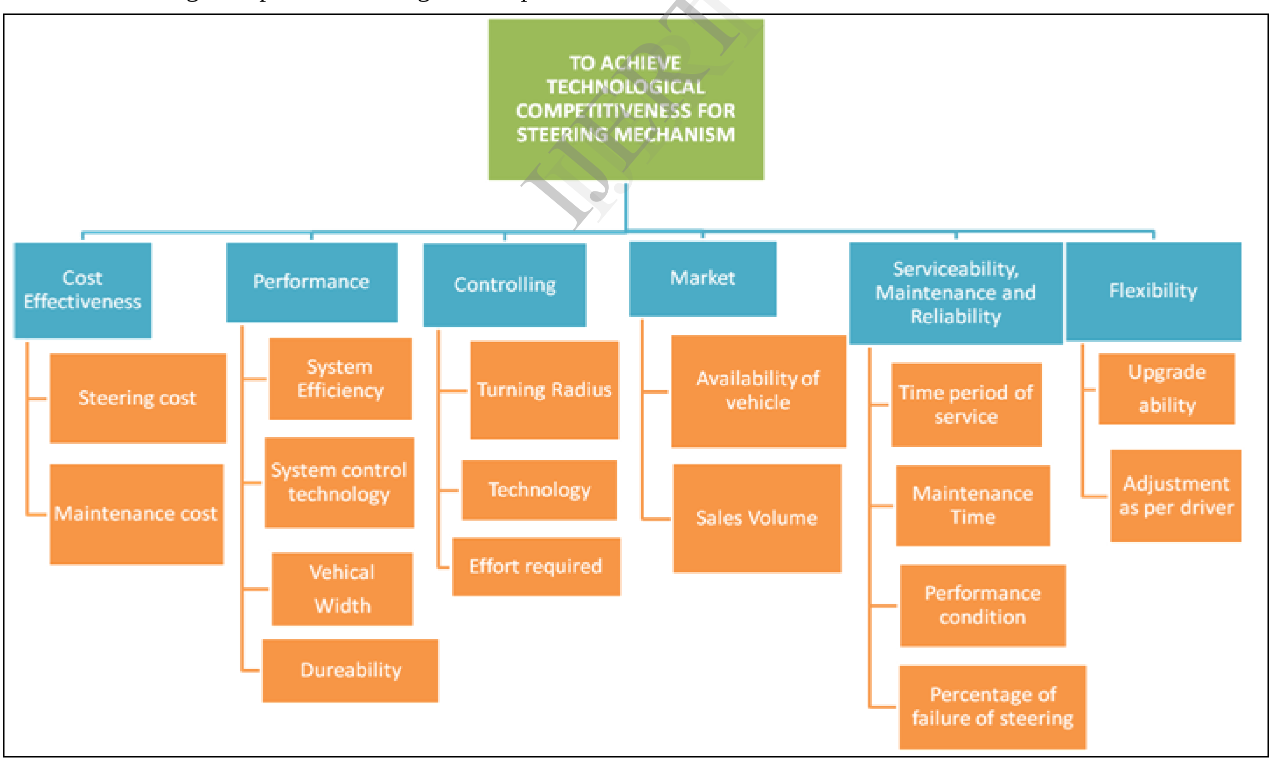


FIGURE 1 : HIERARCHICAL STRUCTURE TO ACHIEVE TECHNOLOGY COMPETITIVENESS FOR STEERING MECHANISM

The Controlling criterion include: turning radius and effort required and market criterion includes availability of car at dealer place and the number of vehicles sold in the market. Serviceability, Maintenance and Reliability is also a major part of steering technology. The customer always looks for reliable product with less maintenance and servicing. It includes preventive maintenance schedule of service, time required to rectify/repair, Performance condition and frequency of repair/failure of steering. Flexibility in technology is extra added advantage it includes: option to upgrade technology, adjustment as per driver ergonomic requirement. Figure 1 describes the criteria and corresponding factors affecting each criterion in achieving the objective technological competitiveness for automobile steering. Six criteria and factors associated with each criterion on the measure of effectiveness were finalized.

To compare the elements within hierarchy, a decision maker would be asked three questions:

• How important is "cost effectiveness" as compared to "performance of steering"?

• How important is "cost effectiveness" as compared to "controlling"?

• How important is "controlling" as compared to "performance"?

If the response to the first question is "Weak or slight", to the second "strong importance" and to the third "Very strong importance" the A matrix would be structured as shown in Table 2

TABLE 2 : PAIR WISE COMPARISON OF THE HIERARCHY USING VERBAL RESPONSES CORRESPONDING TO SAATY'S LINEAR 1-TO-9 SCALE.

| | cost effectiveness | Controlling | performance of steering |
|---|---|---|---|
| cost effectiveness | 1 | Moderate importance | strong importance |
| controlling | | 1 | Very strong importance |
| performance of steering | | | 1 |

*C. Scale Selection*

The decision-maker expresses his/her opinion regarding the relative importance of the criteria and preferences among the alternatives by making pairwise comparisons using a nine-point system ranging from 1 (the two choice options are equally preferred) to 9 (one choice option is extremely preferred over the other) (Table 3). The concept of scale types is received by various researcher/psychologist S. S. Stevens (1946, 1951, 1975).[7] The AHP scoring system is a ratio scale where the ratios between values indicate the degree of preference. The nine-point scale has been the standard rating system used for the AHP (Saaty, 2000).

**Table 3: Explanation of the standard nine-point preference scoring system used for the AHP (Adapted from Saaty, 2000)**

| Preference score | Explanation of numerical preference score |
|---|---|
| 1 | Two attributes preferred equally |
| 2 | Judgment indicates weak favoring of one attribute over another |
| 3 | Judgment slightly favors one attribute over another |
| 4 | Judgment moderately favors one attribute over another |
| 5 | Judgment strongly favors one attribute over another |
| 6 | Judgment slightly more than strongly favors one attribute over another |
| 7 | Judgment very strongly favors one attribute over another |
| 8 | Judgment very, very strongly favors one attribute over another |
| 9 | Extreme preference of one attribute over another |

*D. Pair wise comparison of hierarchy*

A matrix is shown in Table 4. The second column represents "controlling" as compared to "cost effectiveness" in the first row. Here, "cost effectiveness" was given moderate importance over "controlling", a value of 3 using Saaty's original 1-to-9 scale, so the inverse value, 1/3 is entered in the first column of the second row. The second column of the second row compares "controlling" to itself so a value of 1 is entered. For the third column of the second row the result of "controlling" compared to "performance of steering" is recorded with a value of 7, representing the decision maker's view that "controlling" has Very strong importance over "performance of steering".

The decision- The AHP scoring system is a ratio scale where the ratios between values indicate the degree of preference. The nine-point scale has been the standard rating system used for the AHP (Saaty, 2000). Its use is based upon research by psychologist George Miller, (1956) which indicated that decision makers were unable to consistently repeat their expressed gradations of preference finer than 'seven plus or minus two.'[8]

TABLE 4 : PAIR WISE COMPARISON OF HIERARCHY USING SAATY'S LINEAR 1-TO-9 SCALE TO CONVERT THE VERBAL RESPONSES GIVEN IN TABLE 3 TO NUMERIC VALUES

| | cost effectiveness | Controlling | performance of steering |
|---|---|---|---|
| cost effectiveness | 1 | 3 | 5 |
| controlling | 1/3 | 1 | 7 |
| performance of steering | 1/5 | 1/7 | 1 |



## III HOW TO BUILD A CONSISTENT MATRIX

Proposed methodology intends to materialize an augmentation to overcome the above mentioned lacuna to some extent. Procedure outlined to implement the proposed methodology is as follows:

**Step 1**: A decision-maker should first rank all the n attributes to be weighed, according to their importance in the Hierarchical structure. (Figure: 1)

**Step 2:** Exercise (n−1) comparisons among the consecutive criteria using the Saaty scale (Table 3).

**Step 3:** Priorities for remaining pairs (non-consecutive) can easily be computed logically as follows: If B be prioritized r times to A and C is prioritize s times to B, then C is prioritized r×s times to A. Objective ratings to all potential pair wise comparisons can be provided in this manner and represented in a matrix form to provide weights to given set or criteria. It is conspicuous to mention here that priorities within a given pair of attributes are self-reciprocal, i.e. if B be prioritized q times to A then preference of A over B is 1/q times.

**Step 4:** The procedure results in perfectly consistent comparison matrix supported by the fact $\lambda max = n$ and hence $CI = 0$. Eigenvector corresponding to this maximum eigenvalue provides the requisite criteria weights. Geometric mean or weighted geometric mean of individual judgments may be taken to accomplish aggregated matrices for the set of criteria at various levels of hierarchy.

## IV EXAMPLE OF STEERING TECHNOLOGY

Criteria in the left vertical column are compared with the criteria in the top row and the comparisons scored with the 1–9 system. A comparison is assigned a reciprocal score if the item in the left vertical column is preferred less than that in the top row. Each criterion compared with itself results in a diagonal of 1s (i.e. equal preference). Experts provided their comparative judgments for each pair of criteria and factors. These Pairwise comparison matrices show criteria Weightage with respect to the goal in table 5 and 6.

*Calculation steps of CI*

- Step A: Adding the columns in the judgment matrix1 i.e. columns sum Shown in Table: 7
- Step B: Normalizing columns sum by dividing each cell of column 1of matrix1 by column sum of first columns, then this process applied for remaining columns/cells Shown in Table: 8
- Step C: To apply Step B matrix1 it converted in normalized matrix
- Step D: Using formula [Ax = $\lambda_{Max}$ x] to calculate $\lambda_{Max}$, AX obtained by Multiplying first column of normalized matrix with matrix1 Shown in Table: 9
- Step E: $\lambda_{Max}$=Average of {AX/X}
- Step F: The CI value is calculated by using the formula: CI = ($\lambda_{Max}$ - n)/ (n - 1).
- Step G: consistency ratio CR is obtained by dividing the CI value by the Random Consistency index (RCI) as given in table 1.

C1: Cost Effectiveness, C2: Performance, C3: Controlling, C4: Market, C5: Serviceability, Maintenance and Reliability, C6: Flexibility

TABLE 5: MATRIX 1 PAIRWISE COMPARISON OF STEERING MAIN CRITERIA MATRIX WITH RESPECT TO THE GOAL [9]

| | C1 | C2 | C3 | C4 | C5 | C6 |
|---|---|---|---|---|---|---|
| C1 | 1 | 5/3 | 7/2 | 3 | 7/2 | 3 |
| C2 | 3/5 | 1 | 3/5 | 2 | 3/7 | 2 |
| C3 | 2/7 | 5/3 | 1 | 3 | 2/9 | 3/2 |
| C4 | 1/3 | ½ | 1/3 | 1 | 2/7 | 2 |
| C5 | 2/7 | 7/3 | 9/2 | 7/2 | 1 | 7 |
| C6 | 1/3 | ½ | 2/3 | ½ | 1/7 | 1 |

TABLE 6 : COLUMNS SUM OF JUDGMENT MATRIX1

| | C1 | C2 | C3 | C4 | C5 | C6 |
|---|---|---|---|---|---|---|
| C1 | 1 | 5/3 | 7/2 | 3 | 7/2 | 3 |
| C2 | 3/5 | 1 | 3/5 | 2 | 3/7 | 2 |
| C3 | 2/7 | 5/3 | 1 | 3 | 2/9 | 3/2 |
| C4 | 1/3 | ½ | 1/3 | 1 | 2/7 | 2 |
| C5 | 2/7 | 7/3 | 9/2 | 7/2 | 1 | 7 |
| C6 | 1/3 | ½ | 2/3 | ½ | 1/7 | 1 |
| **Column sums** | **2.83** | **7.66** | **10.60** | **13.00** | **5.57** | **16.5** |

TABLE 7 : NORMALIZED COLUMNS OF MATRIX1

| | C1 | C2 | C3 | C4 | C5 | C6 |
|---|---|---|---|---|---|---|
| C1 | 0.352 | 0.217 | 0.330 | 0.231 | 0.627 | 0.182 |
| C2 | 0.211 | 0.130 | 0.057 | 0.154 | 0.077 | 0.121 |
| C3 | 0.101 | 0.217 | 0.094 | 0.231 | 0.040 | 0.091 |
| C4 | 0.117 | 0.065 | 0.031 | 0.077 | 0.051 | 0.121 |
| C5 | 0.101 | 0.305 | 0.425 | 0.269 | 0.179 | 0.424 |
| C6 | 0.117 | 0.065 | 0.063 | 0.038 | 0.026 | 0.061 |
| **sums** | **1** | **1** | **1** | **1** | **1** | **1** |



TABLE 8 : CALCULATION OF Λ MAX AND AX

| **A** | | | | | | | | **X** | | **AX** |
|---|---|---|---|---|---|---|---|---|---|---|
| | C1 | C2 | C3 | C4 | C5 | C6 | | | | |
| C1 | 1 | 5/3 | 7/2 | 3 | 7/2 | 3 | | 0.352 | | 2.219 |
| C2 | 3/5 | 1 | 3/5 | 2 | 3/7 | 2 | | 0.211 | | 0.922 |
| C3 | 2/7 | 5/3 | 1 | 3 | 2/9 | 3/2 | * | 0.101 | = | 1.039 |
| C4 | 1/3 | ½ | 1/3 | 1 | 2/7 | 2 | | 0.117 | | 0.682 |
| C5 | 2/7 | 7/3 | 9/2 | 7/2 | 1 | 7 | | 0.101 | | 2.140 |
| C6 | 1/3 | ½ | 2/3 | ½ | 1/7 | 1 | | 0.117 | | 0.475 |

***Step E:*** *$\lambda_{Max}$=Average of {AX/X}*

$$\lambda max = Average\ of \left\{\frac{2.219}{0.352}, \frac{0.922}{0.211}, \frac{1.039}{0.101}, \frac{0.682}{0.117}, \frac{2.140}{0.101}, \frac{0.475}{0.117}\right\}$$

***$\lambda_{Max}$=7.985***

***Step F:*** *The CI value is calculated by using the formula:*

$$consistency\ Index\ (CI) = \frac{\lambda\ Max - n}{n-1}$$

$$consistency\ Index\ (CI) = \frac{7.985-6}{6-1}$$

$$consistency\ Index\ (CI) = 0.397$$

***Step G:*** *consistency ratio CR is obtained by dividing the CI value by the Random Consistency index (RCI) as given in table 1.*

$$consistency\ Ratio\ (CR) = \frac{CI}{Mean\ Randome\ CI}$$

$$consistency\ Ratio\ (CR) = \frac{0.39}{1.24}$$

Table 9 : Matrix 1 with eigenvalue for criteria 1 to criteria 6

| | C1 | C2 | C3 | C4 | C5 | C6 | Eigenvalue |
|---|---|---|---|---|---|---|---|
| C1 | 1 | 5/3 | 7/2 | 3 | 7/2 | 3 | 0.34 |
| C2 | 3/5 | 1 | 3/5 | 2 | 3/7 | 2 | 0.11 |
| C3 | 2/7 | 5/3 | 1 | 3 | 2/9 | 3/2 | 0.12 |
| C4 | 1/3 | ½ | 1/3 | 1 | 2/7 | 2 | 0.08 |
| C5 | 2/7 | 7/3 | 9/2 | 7/2 | 1 | 7 | 0.29 |
| C6 | 1/3 | ½ | 2/3 | ½ | 1/7 | 1 | 0.06 |

***$\lambda_{Max}$=7.985 CI=0.397 CR=0.320***

**Saaty only accepts a matrix as a consistent one if CR < 0.1.**Saaty suggests that if that ratio exceeds 0.1 the set of judgments may be too inconsistent to be reliable. Consistence Ratio of 0 means judgements is perfectly consistent. Consistency of matrix1 is not in the range (0.320) as per suggested by Saaty, so researcher incorporated view of some more experts for their comparative judgments and form a new matrix i.e. matrix2 as shown in Table: 10. In this matrix 2 Weightage of some criteria has altered and calculation of **$\lambda_{Max}$** CI and CR has been done by researcher using same method discusses earlier. . In matrix 2 CR is 0.064 which is accepted limit and provides results with all the attributes having provided with prioritized weights. Comparing CR values of matrix 2 and matrix1, matrix 2 is more consistent then matrix one. Further researcher compare eigen values of matrix 1 and matrix2 (Shown in table: 12) and found slide changes in the eigen value but no change in priority of the criteria.

Table 10: Matrix 2 Pairwise comparison of steering main criteria matrix with respect to the Goal

| | C1 | C2 | C3 | C4 | C5 | C6 | Eigenvalue |
|---|---|---|---|---|---|---|---|
| C1 | 1 | 6/3 | 8/7 | 8/2 | 7/4 | 6 | 0.30 |
| C2 | 3/6 | 1 | 3/8 | 4/2 | 6/9 | 3 | 0.13 |
| C3 | 7/8 | 8/3 | 1 | 3/2 | 7/6 | 4/2 | 0.22 |
| C4 | 2/8 | 2/4 | 2/6 | 1 | 2/7 | 2 | 0.07 |
| C5 | 4/7 | 9/6 | 6/7 | 7/2 | 1 | 7 | 0.23 |
| C6 | 1/6 | 1/3 | 2/4 | 1/2 | 1/7 | 1 | 0.05 |

**$\lambda_{Max}$=6.398 CI=0.08 CR=0.064**



TABLE 11: WEIGHTAGE/EIGEN VALUES OF STEERING TECHNOLOGY CRITERIA

| | Criteria | Eigen Value/ Weightage Of Matrix 1 | Eigen Value/ Weightage Of Matrix 2 |
|---|---|---|---|
| C1 | Cost Effectiveness | 0.34 (1) | 0.30 (1) |
| C2 | Performance | 0.11(4) | 0.13 (4) |
| C3 | Controlling | 0.12 (3) | 0.21 (3) |
| C4 | Market | 0.08 (5) | 0.08 (5) |
| C5 | Serviceability,Maintenance and Reliability | 0.29 (2) | 0.23 (2) |
| C6 | Flexibility | 0.06 (6) | 0.05 (6) |

## V CONCLUDING REMARKS

The AHP provides a convenient approach for solving complex MCDM problems in engineering. However, as this paper demonstrated with some illustrative examples, its use to engineering problems should be a cautious one.

The examples in this paper, along with the previous research of the authors in this area strongly suggest that when some alternatives appear to be very close with each other, then the decision-maker needs to be very cautious. An apparent remedy is to try to consider additional decision criteria which, hopefully, can assist in drastically discriminating among the alternatives.

In this research Steering technology criteria’s weightage with respect to each other have been determined by using AHP pair wise comparison. Weightage of all six criteria calculated and compared with consistence/inconsistence matrices, Eigen value of criteria’s has been changed although order of priorities has not changed. Significant changes were observed in controlling criteria. In other criteria, slight/insignificant changes have been observed. This result of consistent matrix is more accurate and further used in other sub factor analysis.

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