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CONTACT STRESS ANALYSIS OF SPUR GEAR

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Abstract

Contact stress is generally the deciding factor for the determination of the requisite dimensions of gears. Research on gear action has confirmed fact that beside contact pressure, sliding velocity, viscosity of lubricant as well as other factors such as frictional forces, contact stresses also influence the formation of pits on the tooth surface. So thorough study of contact stress developed between the different matting gears are mostly important for the gear design. Gearing is one of the most critical components in mechanical power transmission systems. Current Analytical methods of calculating gear contact stresses use Hertz's equations, which were originally derived for contact between two cylinders. So for CONTACT STRESSES it's necessary to develop and to determine appropriate models of contact elements, and to calculate contact stresses using ANSYS and compare the results with Hertzian theory.

2. Introduction

Gearing is one of the most effective methods for transmitting power and rotary motion from the source to its application with or without change of speed or direction. Gears will prevail as a critical machine element for transmitting power in future machines due to their high degree of reliability and compactness. The rapid development of heavy industries such as vehicle, shipbuilding and aircraft industries require advanced application of gear technology.

A gearbox consists of a set of gears, shafts and bearings that are mounted in an enclosed lubricated housing. They are available in a broad range of sizes, capacities and speed ratios. Their function is to convert the input provided by the prime mover into an output with lower speed and corresponding higher torque. In this thesis, contact stress analysis of Spur gear is studied using finite element analysis.

The crucial requirement of effective power transmission in various machines, automobiles, elevators, generators, etc has created an increasing demand for more accurate analysis of the characteristics of gear systems. For instance, in automobile industry highly reliable and lightweight gears are essential. Furthermore the best way to diminution of noise in engine requires the fabrication of silence gear system. Noise reduction in gear pairs is especially critical in the rapidly growing today's technology since the working environment is badly influenced by noise. The most successful way of gear noise reduction is attained by decreasing the vibration related with them. The reduction of noise by vibration control can be achieved through a research endeavour by an expert in the field.

The finite element method is proficient to supply this information but the time required to generate proper model is a large. Therefore to reduce the modelling time a pre-processor method that builds up the geometry required for finite element analysis may be used, such as Pro/Engineer. Pro/Engineer can generate three dimensional models of gears. In Pro/Engineer the generated model geometry is opened in ANSYS for analysis.

The major cause of vibration and noise in a gear system is the transmission error between the meshing gears. By definition transmission error is the difference between the theoretical and the actual position between driving gear and the driven gear. It can be defined also as the amount by which the ratio at a given point in a revolution departs from the correct ratio. For this reason, with prior knowledge of the operating conditions of the gear set it is possible to design the gears with minimum vibration and noise.

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Gear analysis can be performed using analytical methods which required a number of assumptions and simplifications which aim at getting the maximum stress values only but gear analyses are multidisciplinary including calculations related to the tooth stresses and to failures like wear. In this thesis, an attempt is made to analyze contact stress using Hertz theory.

2. Hertz Contact Stress (Involutes Gear Tooth Contact Stress Analysis)

One of the main gear tooth failure is pitting which is a surface fatigue failure due to many repetition of high contact stresses occurring in the gear tooth surface while a pair of teeth is transmitting power. Contact failure in gears is currently predicted by comparing the calculated Hertz contact stress to experimentally determined allowable values for the given material. The method of calculating gear contact stress by Hertz's equation originally derived for contact between two cylinders. Contact stresses between cylinders are shown in figure1 and figure 2.

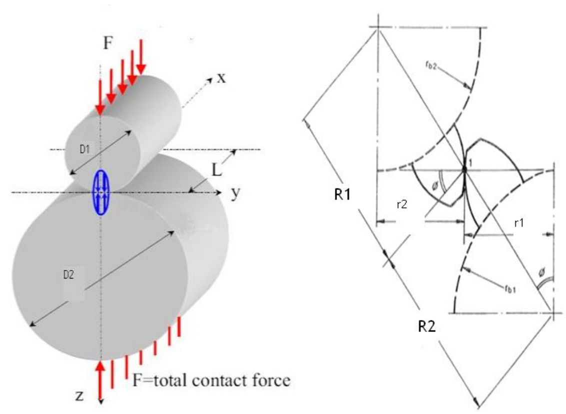


Fig 1 Cylinders in contact under compression

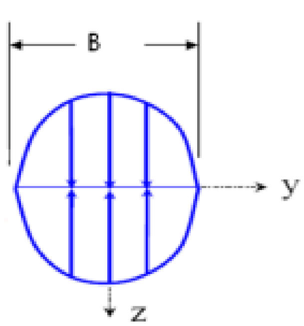


Fig 2 Elliptical stress distribution across the width

In machine design, problems frequently occurs when two members with curved surfaces are deformed when pressed against one another giving rise to an area of contact under compressive stresses. Of particular interest to the gear designer is the case where the curved surfaces are of cylindrical shape because they closely resemble gear tooth surfaces.

In Fig.1 two gear teeth are shown in mating condition at the pitch point. Referring to Fig.2, the area of contact under load is a narrow rectangle of width B and length L. The stress distribution pattern is elliptical across the width as shown in figure 3.

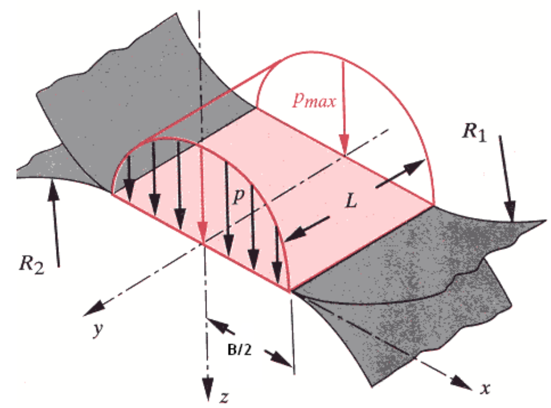


Fig 3 Ellipsoidal–prism pressure distribution

Value is given by,

$$p_{c\,max} = \frac{4\times F}{\Pi\times B\times L} \qquad 3.1$$

Where,

$$B=\sqrt{\frac{8\times F}{\Pi\times L}\times\frac{\frac{1-v1^2}{E_1}+\frac{1-v2^2}{E_2}}{\frac{1}{D_1}+\frac{1}{D_2}}}$$

Here, F is the applied force, $v1$ and $v2$are Poison's ratio of the two materials of the cylinders with diameters D_1 andD_2, and E_1 and E_2 are the respective modulii of elasticity.

Putting the values of B and assuming a value of 0.3 to poison's ratio in Esq. 3.1, and by replacing diameters by respective radii,

$$p_{c\,max} = \sqrt{0.35\times\frac{F}{L}\times\frac{\frac{1}{R_1}+\frac{1}{R_2}}{\frac{1}{E_1}+\frac{1}{E_2}}} \qquad 3.2$$

The Hertz equations discussed so far can be utilised to calculate the contact stresses which prevail in case of tooth surfaces of two mating spur gears. Though an approximation, the contact aspects of such gears can be taken to be equivalent to those of cylinders having the same radii of curvature at the contact point as the load transmitting gears have. Radius of curvature changes continuously in case of an involutes curve, and it changes sharply in the vicinity of the base circle.

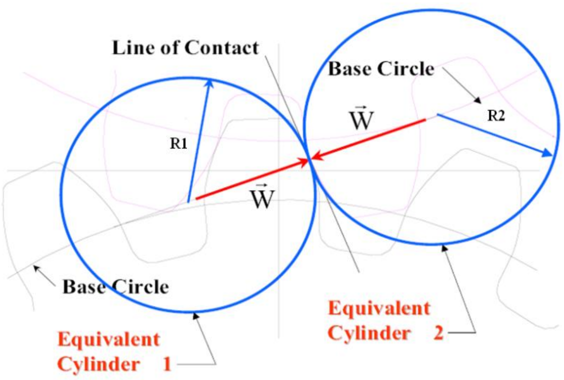


Fig 4 Equivalent contacting cylinder

Now by putting following equations in Esq. 3.1

$$F=\frac{F_t}{\cos\alpha},\ L = b,\ R_1=\frac{d_1\times\sin\alpha}{2},\ R_2=\frac{d_2\times\sin\alpha}{2}$$

Where F_t is the tangrntial force or transmitted load, b is the tooth width, R_1 and R_2 are the radii of curvature at pitch point, and d_1 and d_2 are the pitch circle diameters of the gears.

Putting,

$$E=\frac{2\times E_1\times E_2}{E_1+E_2} \text{ and } u=\frac{d_2}{d_1}$$

We get

$$\frac{1}{R_1}+\frac{1}{R_2}=\frac{2}{\sin\alpha}\times\left(\frac{1}{d_1}+\frac{1}{d_2}\right)=\frac{2}{\sin\alpha}\times\left(\frac{1}{d_1}+\frac{1}{u\times d_1}\right)=\frac{2}{d_1\times\sin\alpha}\times\left(\frac{u+1}{u}\right) \qquad 3.3$$

Inserting these values in Esq. 3.2 we get the expression for the maximum contact pressure at the pitch point

$$p_p=\sqrt{0.35\times\frac{F_t}{\cos\alpha}\times\frac{1}{b}\times\frac{2}{\sin\alpha}\times\frac{1}{d_1}\times\frac{u+1}{u}\times\frac{E}{2}}$$

$$=\sqrt{0.35\times\frac{F_t}{b}\times\frac{E}{d_1}\times\frac{u+1}{u}\times\frac{1}{\cos\alpha\times\sin\alpha}}$$

Now by considering service pressure angle α_w,

$$p_p=\sqrt{0.35\times\frac{F_t}{b}\times\frac{E}{d_1}\times\frac{u+1}{u}\times\frac{1}{cos^2\alpha\times\tan\alpha_w}} \qquad 3.4$$

To simplify calculations, Esq. 3.4 is written in the form

$$p_p=y_m\times y_p\sqrt{\frac{F_t}{b\times d_1}\times\frac{u+1}{u}} \qquad 3.5$$

Allowable Maximum $P_p=p_p/\text{FOS}$

From the above equation which is 3.5, Pressure or Stress is obtained at pitch point in which factor of safety will be included by dividing p_p with factor of safety which can be taken from ANSYS result or may be from other FOS tables. After that the allowable max. Pressure or Stress at pitch point answer will be obtained. Ultimately minimum factor of safety is taken from ANSYS result in order to get an accurate result of allowable max. Pressure or Stress at pitch point.

y_m Is the material coefficient and y_p is the pitch point coefficient, which are given by

$$y_m=\sqrt{0.35\times\frac{2\times E_1\times E_2}{E_1+E_2}}$$

$$=\sqrt{0.35\times E}$$

$$y_p=\sqrt{\frac{1}{\cos\alpha\times\tan\alpha_w}}$$

3. Parametric Modelling of Spur Gear

Description	Pinion	Gear
Number of teeth	Z_1	Z_2
Pitch circle diameter	$d_1=Z_1\times m$	$d_1=Z_2\times m$
Transmission ratio	$i=\frac{n_1}{n_2}=\frac{Z_2}{Z_1}$	
Pitch cone angle	$\tan\delta_1=\frac{d_1}{d_2}$	$\tan\delta_2=\frac{d_2}{d_1}$
Shaft angle	$\Psi=\delta_1+\delta_2$	
Tip circle diameter	$d_{a1}=d_1+2m\cos\delta_1$	$d_{a2}=d_2+2m\cos\delta_2$
Middle circle diameter	$d_{m1}=d_1-b\sin\delta_1$	$d_{m2}=d_2-b\cos\delta_2$
Face width	$b=\frac{R}{3}$	
Middle module	$m_m=\frac{d_{m1}}{Z_1}=\frac{d_{m2}}{Z_2}$	
Addendum	$h_{a1}=h_{a2}=m$	
Dedendum	$h_{f1}=h_{f2}=1.2\ m$	
Addendum angle	$\tan\Theta_{a1}=\tan\Theta_{a2}=\frac{m}{R}$	
Dedendum angle	$\tan\Theta_{f1}=\tan\Theta_{f2}=1.2\frac{m}{R}$	

Table 1 Dimensions of Spur Gear

Parametric modeling allows the design engineer to let the characteristic parameters of a product drive the design of that product. During the gear design, the main parameters that would describe the designed gear such as module, pressure angle, root radius, and tooth thickness, number of teeth could be used as the parameters to define the gear. But, the parameters do not have to be only geometric. They can also be key process information such as case hardening specifications, Quality of grades, metallurgical properties and even load classifications for the gear being designed.

In this paper work module, pressure angle, numbers of teeth of Spur pinion and numbers of teeth Spur gear are taken as input parameters. Pro/Engineer uses these parameters, in combination with its features to generate the geometry of the Spur gear and all essential information to create the model. By using the relational equation in Pro/Engineer, the accurate three dimensional Spur gear model is developed. The parametrical process can increase design accuracy, reduce lead times and improve overall engineering productivity.

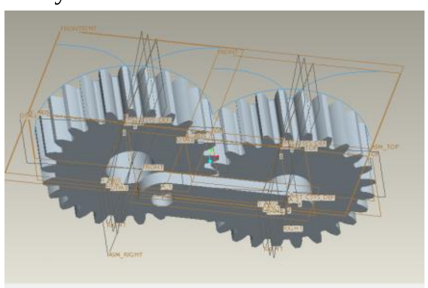


Fig 5 Assembly of Parametric Spur Gear Model

Module	2 mm
Number of teeth of pinion	25
Number of teeth of gear	25
Pressure angle	20^0

Table 2 Key geometrical parameters of Spur gear

4. Analytical Contact Stress Analysis of Spur Gear

Input Parameters

Sr. No	Input Parameters	Symbol	Value
1.	Module	m	2 mm
2.	Shaft angle	Θ	180^0
3.	Nominal input power	P	2 KW
4.	Transmission ratio	i	1
5.	Pinion speed	n_1	50 rpm
6.	No of pinion teeth	Z_1	25
7.	Pressure angle	α	20^0
8.	Material for pinion	4o Cr 4	
9.	Material for gear	45 C 8	

Table 3 Input parameters for the sample calculation

Sample Calculation for Module Size of 2

Z_2= i × Z_1= 1 × 25 = 25

Nominal torque on the pinion shaft $= 9550\times \frac{P}{n_1}$

$= 9550\times \frac{2}{50}$

= 382 N m

= 382000 N mm

Torque (T) = Force (F) × Radius (R_a)

382000 N mm = Force (F) × 40 mm {40mm is the radius of the Lever}

Force (F) = 9550 N

The Hertzian contact stress is given by,

$$P_p = Y_m Y_p \sqrt{\frac{F_t}{b*d_1} * \frac{u+1}{u}}$$

Where, Y_m is the material Co-efficient

$$Y_m = \sqrt{0.35 * \frac{2\,E1\,E2}{E1+E2}}$$

$$= \sqrt{0.35 * \frac{2*2*10^5*2*10\char`^5}{2*10^5+2*10\char`^5}}$$

$$= \sqrt{70000}$$

= 264.5

Where, Y_p is the Pitch point Co-efficient.

$$Y_p = \sqrt{\frac{1}{cos^2\alpha * tan\alpha_w}}$$

$$= \sqrt{\frac{1}{cos^2 20 * tan20}}$$

= 1.76

D_1 = Module (m) * Number of the teeth (Z_1)

=2 * 25

= 50

D_2 = Module (m) * Number of the teeth (Z_2)

=2 * 25

= 50

u = D_1/D_2

= 1(Unity)

$$P_p = Y_m Y_p \sqrt{\frac{F_t}{b*d_1} * \frac{u+1}{u}}$$

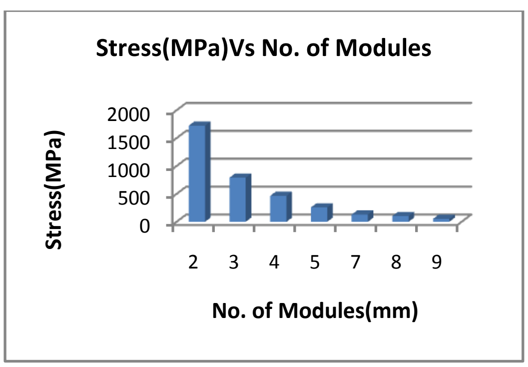
$$= 210.4 \times 1.2 \sqrt{\frac{9550}{20*50} * \frac{1+1}{1}}$$

= 2034.48 Mpa/ 1.18

= 1724.13 MPa [For Module = 2], Including FOS=1.18 from ANSYS result which is clearly indicated in Fig. 6

Sr. No.	Module(mm)	Max.Contact Stress(MPa)
1.	2	1724.13
2.	3	791.02
3.	4	465.56
4.	5	257.34
5.	7	129.61
6.	8	102.85
7.	9	52.39

Table 4 Hertzian analytical results for different modules



Graph 1 Graph for Hertzian analytical results for different modules

5. FEM Results for Contact Stress of Spur Gear by ANSYS 13.0

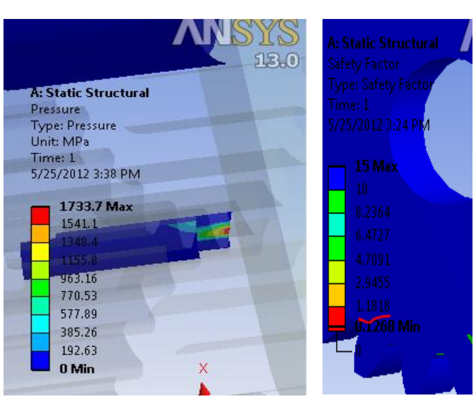


Fig 6.FEM Result for Module 2

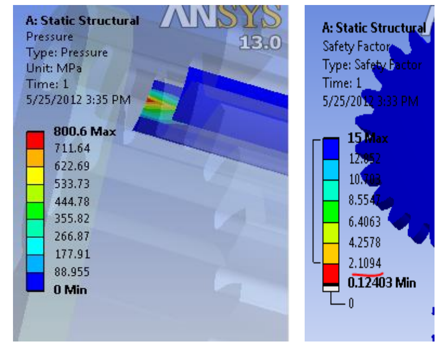


Fig.7.Result for Module 3

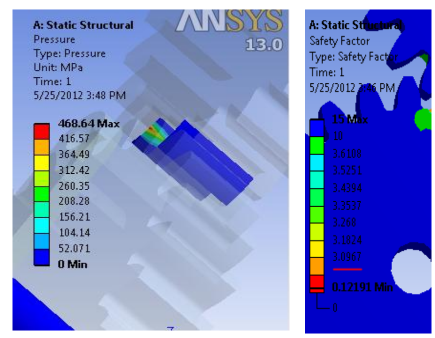


Fig.8.Result for Module 4

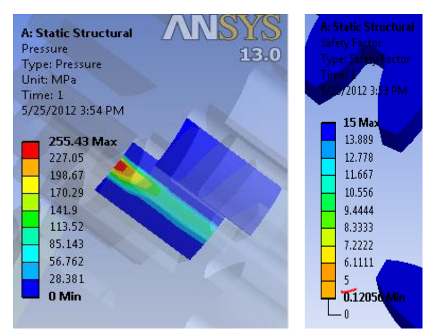


Fig.9.Result for Module 5

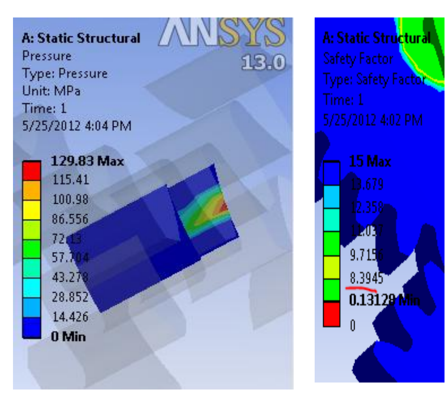


Fig.10.Result for Module 7

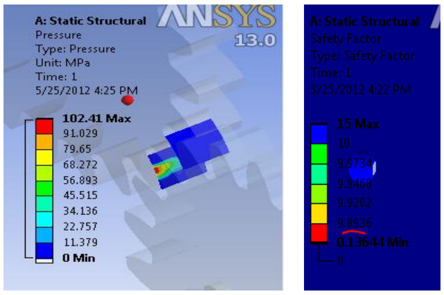


Fig.11.Result for Module 8

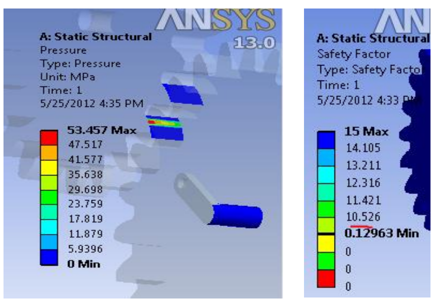
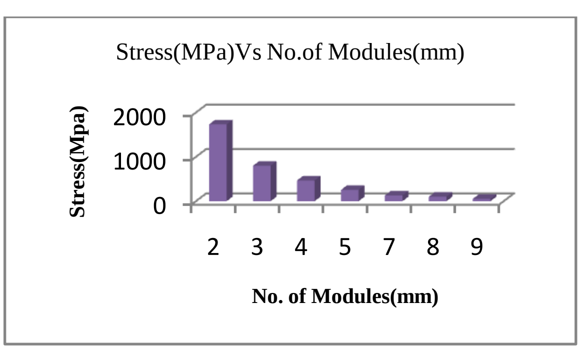


Fig.12.Result for Module 9

Sr. No.	Module(mm)	Max.Contact Stress(MPa)
1.	2	1724.13
2.	3	791.02
3.	4	465.56
4.	5	257.34
5.	7	129.61
6.	8	102.85
7.	9	52.39

Table 5 ANSYS FEM results for different modules



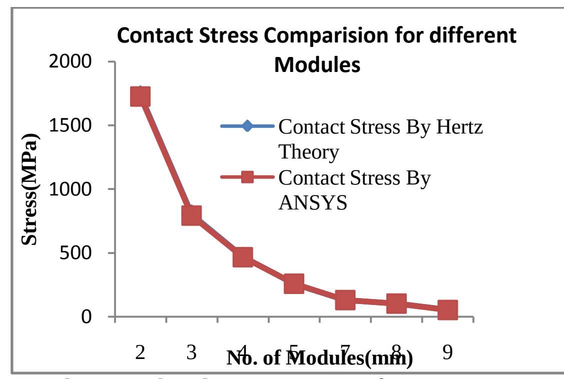
Graph 2 Graph for ANSYS FEM results for different modules

6. Comparison

In this paper comparison between the results of Hertz Theory and ANSYS 13.0 is indicated which is as follow

Sr. No.	Module (mm)	$P_{p(ANSYS)}$ (MPa)	$P_{p(Hertzian\ stress)}$ (MPa)	Differences [%]
1	2	1733.7	1724.13	0.5
2	3	800.6	791.02	1.19
3	4	468.64	465.56	0.65
4	5	255.43	257.34	0.74
5	7	129.83	129.61	0.16
6	8	102.41	102.85	0.43
7	9	53.457	52.39	1.97

Table 6 Comparison of peak values of the contact stresses by considering different modules



Graph 3 Graphical representation for contact stresses comparison for different modules

7. Conclusion

It is also shown that the development of finite element analysis model of the Spur gear assembly to simulate the contact stress between two gears reasonably and obtained result is compared with the Hertzian theoretical equation.

Based on the result from the contact stress analysis the hardness of the gear tooth profile can be improved to resist pitting failure: phenomena in which small particles are removed from the surface of the tooth that is because of the high contact stresses that are present between mating teeth.

The module is important geometrical parameter during the design of gear. As it is expected, in this work the maximum contact stress decreases with increasing module and it will be higher at the pitch point. As a result, based on this finding if the contact stress minimization is the primary concern and if the large power is to be transmitted then a Spur gear with higher module is preferred.

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