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Decay of Energy of MHD Turbulence For Four-Point Correlation

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Abstract

In this study we consider the decay of energy of MHD fluid turbulence for four-point correlations prior to the ultimate phase. Three and four point correlation equations are obtained. The correlation equations are converted to spectral form by their Fourier-transform .By neglecting the quintuple correlations in comparison to the second, third and fourth order correlation terms. Finally integrating the energy spectrum over all wave numbers and we obtained the energy decay law of MHD turbulence for magnetic field fluctuations.

1. Introduction

The idea of magneto hydrodynamics is that magnetic fields can induce currents in a moving conductive fluid, which create forces on the fluid. In magneto hydrodynamics we study the dynamics of electrically conducting fluids. The examples of such fluids include plasmas, liquid metals and salt water. The electrical field effects are neglected as is usually done in MHD.Taylor introduced correlation coefficients between the quantities. Chandrasekhar [1] studied the invariant theory of isotropic turbulence in magneto-hydrodynamics. S. Corrsin [2] discussed on spectrum of isotropic temperature fluctuations in isotropic turbulence. Deissler [3, 4] developed "A theory decay of homogeneous turbulence for times before the final period". Using Deissler's theory Kumar and Patel [5] studied the first order reactant in homogeneous turbulence before the final period for the case of multipoint and single time consideration. Loeffler and Deissler [6] studied the decay of temperature fluctuation in homogeneous turbulence. Patel [7] extended the problem [5] for the case of multipoint and multi-time concentration correlations. Islam and Sarker [8] studied the decay of dusty fluid MHD turbulence before the final period in a rotating system. Sarker and Kishore [9] studied the decay of MHD turbulence before the final periods. Azad, Aziz and Sarker [10] studied the first order reactant in magneto-hydrodynamic turbulence before the final period of decay in presence of dust particles. They considered the two and three point correlation equations and solved these equations after neglecting the fourth and higher order correlation terms.

In this paper, the turbulence for three point correlations is generalized to some extent in order to analyze the four- point turbulence at higher Reynolds numbers. In this case, the quadruple correlation terms in the three-point correlation are retained and in addition, a four-point correlation equation is considered. Following Deisslers approach we studied the decay of energy of MHD turbulence for four- point correlation system. The decay law comes out to be in the form

$$\langle h^2 \rangle = A(t-t_0)^{-3/2} + B(t-t_0)^{-5} + C(t-t_1)^{-15/2} + D(t-t_1)^{-17/2},$$

where $\langle h^2 \rangle$ denotes the total energy and t is the time, A, B, C and D are arbitrary constants determined by initial conditions.

2. Four-point correlation and spectral equations

We take the momentum equation of MHD turbulence at the point p and the induction equation of magnetic field fluctuation four point correlation and equations at p', p'' and p''' as

$$\frac{\partial u_l}{\partial t} + u_k \frac{\partial u_l}{\partial x_k} - h_k \frac{\partial h_l}{\partial x_k} = -\frac{\partial \omega}{\partial x_l} + \nu \frac{\partial^2 u_l}{\partial x_k \partial x_k} \qquad (2.1)$$

$$\frac{\partial h_i'}{\partial t} + u_k' \frac{\partial h_i'}{\partial x_k'} - h_k' \frac{\partial u_i'}{\partial x_k'} = \frac{\nu}{p_M} \frac{\partial^2 h_i'}{\partial x_k' \partial x_k'} \qquad (2.2)$$

$$\frac{\partial h_j''}{\partial t} + u'' \frac{\partial h_j''}{\partial x_k''} - h'' \frac{\partial u_j''}{\partial x_{k\ k}''} = \frac{\nu}{p_M} \frac{\partial^2 h_j''}{\partial x_k'' \partial x_k''} \qquad (2.3)$$

$$\frac{\partial h_m'''}{\partial t} + u_k''' \frac{\partial h_m'''}{\partial x_k'''} - h_k''' \frac{\partial u_m'''}{\partial x_k'''} = \frac{\nu}{p_M} \frac{\partial^2 h_m'''}{\partial x_k'''} \qquad (2.4)$$

Where $\omega = \frac{P}{\rho} + \frac{1}{2}|\overline{h}|^2$ is the total MHD pressure $\rho(x,t)$ is the hydrodynamic pressure, ρ is the fluid density, $P_M = \frac{\nu}{\lambda}$ is the Magnetic Prandtl, number ν is the kinematics viscosity, λ is the magnetic diffusivity, $h_i(x,t)$ is the magnetic field fluctuation,

$u_k(x,t)$ is the turbulent velocity ,t is the time, x_k is the space co-ordinate and repeated subscripts are summed from 1 to 3 .

Multiplying equation (2.1) by $h_i' h_j'' h_m'''$ (2.2) by $u_l h_j'' h_m'''$ (2.3) by $u_l h_i' h_m'''$ (2.4) by $u_l h_i' h_j''$ and adding the four equations, we than taking the space or time averages and they are denoted by $\overline{\ldots\ldots}$ or $\langle\ldots\ldots\ldots\rangle$. We get

$$\frac{\partial}{\partial t}(\overline{u_l h_i' h_j'' h_m'''}) + \frac{\partial}{\partial x_k}(\overline{u_l u_k h_i' h_j'' h_m'''}) - \frac{\partial}{\partial x_k}(\overline{h_k h_l h_i' h_j'' h_m'''}) +$$

$$\frac{\partial}{\partial x_k'}(\overline{u_l u_k h_i' h_j'' h_m'''}) - \frac{\partial}{\partial x_k'}(\overline{u_l u_i' h_k' h_j'' h_m'''}) + \frac{\partial}{\partial x_k''}(\overline{u_l u_k'' h_i' h_j'' h_m'''}) -$$

$$\frac{\partial}{\partial x_k''}(\overline{u_l u_j'' h_i' h_k'' h_m'''}) + \frac{\partial}{\partial x_k'''}(\overline{u_l u_k''' h_i' h_j'' h_m'''}) - \frac{\partial}{\partial x_k'''}(\overline{u_l u_j'' h_i' h_j'' h_m'''}) =$$

$$-\frac{\partial}{\partial x_l}(\overline{w h_i' h_j'' h_m'''}) + \frac{\partial^2}{\partial x_k \partial x_k}(\overline{u_l h_i' h_j'' h_m'''}) + \frac{\nu}{p_M}[\frac{\partial^2}{\partial x_k' \partial x_k'}(\overline{u_l h_i' h_j'' h_m'''}) +$$

$$\frac{\partial^2}{\partial x_k'' \partial x_k''}(\overline{u_l h_i' h_j'' h_m'''}) + \frac{\partial^2}{\partial x_k''' \partial x_k'''}(\overline{u_l h_i' h_j'' h_m'''})] \qquad (2.5)$$

Using the transformations

$$\frac{\partial}{\partial x_k''} = \frac{\partial}{\partial r_k'}, \; \frac{\partial}{\partial x_k'} = \frac{\partial}{\partial r_k'}, \; \frac{\partial}{\partial x_k} = -(\frac{\partial}{\partial r_k'} + \frac{\partial}{\partial r_k'} + \frac{\partial}{\partial r_k''})$$

into equations (2.5) we get,

$$\frac{\partial}{\partial t}(\overline{u_l h_i' h_j'' h_m'''}) + (1+p_M)\frac{\partial^2}{\partial r_k' \partial r_k'}(\overline{u_l h_i' h_j'' h_m'''}) +$$

$$(1+p_M)\frac{\partial^2}{\partial r_k'' \partial r_k''}(\overline{u_l h_i' h_j'' h_m'''}) + 2p_M \frac{\partial^2}{\partial r_k \partial r_k'}(\overline{u_l h_i' h_j'' h_m'''})$$

$$+2p_M \frac{\partial^2}{\partial r_k' \partial r_k''}(\overline{u_l h_i' h_j'' h_m'''}) + 2p_M \frac{\partial^2}{\partial r_k \partial r_k''}(\overline{u_l h_i' h_j'' h_m'''}) =$$

$$\frac{\partial}{\partial r_k}(\overline{u_l u_k h_i' h_j'' h_m'''}) + \frac{\partial}{\partial r_k'}(\overline{u_l u_k h_i' h_j'' h_m'''}) + \frac{\partial}{\partial r_k''}(\overline{u_l u_k h_i' h_j'' h_m'''})$$

$$-\frac{\partial}{\partial r_k}(\overline{h_l h_k h_i' h_j'' h_m'''}) - \frac{\partial}{\partial r_k'}(\overline{h_l h_k h_i' h_j'' h_m'''}) - \frac{\partial}{\partial r_k''}(\overline{h_l h_k h_i' h_j'' h_m'''})$$

$$-\frac{\partial}{\partial r_k}(\overline{u_l u_k' h_i' h_j'' h_m'''}) + \frac{\partial}{\partial r_k}(\overline{u_l u_i' h_k' h_j'' h_m'''}) - \frac{\partial}{\partial r_k'}(\overline{u_l u_k'' h_i' h_j'' h_m'''})$$

$$+\frac{\partial}{\partial r_k'}(\overline{u_l u_j'' h_i' h_k'' h_m'''}) - \frac{\partial}{\partial r_k''}(\overline{u_l u_k''' h_i' h_j'' h_m'''}) + \frac{\partial}{\partial r_k''}(\overline{u_l u_m''' h_i' h_j'' h_k'''}) +$$

$$\frac{\partial}{\partial r_l}(\overline{w h_i' h_j'' h_m'''}) + \frac{\partial}{\partial r_l'}(\overline{w h_i' h_j'' h_m'''}) + \frac{\partial}{\partial r_l''}(\overline{w h_i' h_j'' h_m'''})$$

(2.6)

In order to write the equation (2.6) to spectral form, we can define the following nine dimensional Fourier transforms

$$\langle u_l h_i'(\hat{r}) h_j''(\hat{r}') h_m'''(\hat{r}'') \rangle = \int\int\int_{-\infty}^{\infty} \langle \phi_l \gamma_i'(\hat{k}) \gamma_j''(\hat{k}') \gamma_m'''(\hat{k}'') \rangle \exp[i(\hat{k}.\hat{r} + \hat{k}'.\hat{r}' + \hat{k}''.\hat{r}'')] d\hat{k} d\hat{k}' d\hat{k}'' \qquad (2.7)$$

$$\langle u_l u_k' h_i'(\hat{r}) h_j''(\hat{r}') h_m'''(\hat{r}'') \rangle = \int\int\int_{-\infty}^{\infty} \langle \phi_l \phi_k'(\hat{k}) \gamma_i'(\hat{k}) \gamma_j''(\hat{k}') \gamma_m'''(\hat{k}'') \rangle \exp[i(\hat{k}.\hat{r} + \hat{k}'.\hat{r}' + \hat{k}''.\hat{r}'')] d\hat{k} d\hat{k}' d\hat{k}'' \qquad (2.8)$$

$$\langle u_l u_i' h_i'(\hat{r}) h_j''(\hat{r}') h_m'''(\hat{r}'') \rangle = \int\int\int_{-\infty}^{\infty} \langle \phi_l \phi_i'(\hat{k}) \gamma_i'(\hat{k}) \gamma_j''(\hat{k}') \gamma_m'''(\hat{k}'') \rangle \exp[i(\hat{k}.\hat{r} + \hat{k}'.\hat{r}' + \hat{k}''.\hat{r}'')] d\hat{k} d\hat{k}' d\hat{k}'' \qquad (2.9)$$

$$\langle u_l u_k'' h_i'(\hat{r}) h_j''(\hat{r}') h_m'''(\hat{r}'') \rangle = \int\int\int_{-\infty}^{\infty} \langle \phi_l \phi_k''(\hat{k}') \gamma_i'(\hat{k}) \gamma_j''(\hat{k}') \gamma_m'''(\hat{k}'') \rangle \exp[i(\hat{k}.\hat{r} + \hat{k}'.\hat{r}' + \hat{k}''.\hat{r}'')] d\hat{k} d\hat{k}' d\hat{k}'' \qquad (2.10)$$

$$\langle u_l u_j'' h_i'(\hat{r}) h_k''(\hat{r}') h_m'''(\hat{r}'') \rangle = \int\int\int_{-\infty}^{\infty} \langle \phi_l \phi_j''(\hat{k}') \gamma_i'(\hat{k}) \gamma_j''(\hat{k}') \gamma_m'''(\hat{k}'') \rangle \exp[i(\hat{k}.\hat{r} + \hat{k}'.\hat{r}' + \hat{k}''.\hat{r}'')] d\hat{k} d\hat{k}' d\hat{k}'' \qquad (2.11)$$

$$\langle u_l u_k h_i'(\hat{r}) h_j''(\hat{r}') h_m'''(\hat{r}'') \rangle = \int\int\int_{-\infty}^{\infty} \langle \phi_l \phi_k \gamma_i'(\hat{k}) \gamma_j''(\hat{k}') \gamma_m'''(\hat{k}'') \rangle \exp[i(\hat{k}.\hat{r} + \hat{k}'.\hat{r}' + \hat{k}''.\hat{r}'')] d\hat{k} d\hat{k}' d\hat{k}'' \qquad (2.12)$$

$$\langle u_l u_i' h_i'(\hat{r}) h_j''(\hat{r}') h_m'''(\hat{r}'') \rangle = \int\int\int_{-\infty}^{\infty} \langle \phi_l \phi_i'(\hat{k}') \gamma_i'(\hat{k}) \gamma_j''(\hat{k}') \gamma_m'''(\hat{k}'') \rangle \exp[i(\hat{k}.\hat{r} + \hat{k}'.\hat{r}' + \hat{k}''.\hat{r}'')] d\hat{k} d\hat{k}' d\hat{k}'' \qquad (2.13)$$

$$\langle w h_i'(\hat{r}) h_j''(\hat{r}') h_m'''(\hat{r}'') \rangle = \int\int\int_{-\infty}^{\infty} \langle \delta \gamma_i'(\hat{k}) \gamma_j''(\hat{k}') \gamma_m'''(\hat{k}'') \rangle \exp[i(\hat{k}.\hat{r} + \hat{k}'.\hat{r}' + \hat{k}''.\hat{r}'')] d\hat{k} d\hat{k}' d\hat{k}'' \qquad (2.14)$$

Interchange of points p' *and* p'', p' and p''' the subscripts i and k; i and j results in the relations

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$$\overline{u_l u'''_k h'_i h''_j h'''_m} = \overline{u_l u'_k h'_i h''_j h'''_m};\ \overline{u_l u'''_k h'_i h''_j h'''_m} = \overline{u_l u'_k h''_i h''_j h'''_m};\ \overline{u_l u'''_m h'_i h''_j h'''_m} = \overline{u_l u'_i h'_i h'_k h''_j h'''_m};\ \overline{u_l u''_j h'_i h''_k h'''_m} = \overline{u_l u'_i h'_i h'_k h''_j h'''_m};$$

By use of these facts and equations (2.7) to (2.14), one can write equation (2.6) in the form

$$\frac{\partial}{\partial t}\overline{(\phi_l \gamma'_i \gamma''_j \gamma'''_m)} + \frac{\nu}{p_M}[(1+P_M)K^2 + (1+p_M)K'^2 + (1+p_M)K''^2 + 2p_M KK' + 2p_M KK'' + 2p_M KK'']$$

$$\overline{(\phi_l \gamma'_i \gamma''_j \gamma'''_m)} = \mathrm{i}(K_k + K'_k + K''_k)\,\overline{(\phi_l \phi_k \gamma'_i \gamma''_j \gamma'''_m)} - \mathrm{i}(K_k + K'_k + K''_k)\,\overline{(\gamma_l \gamma_k \gamma'_i \gamma''_j \gamma'''_m)} - \mathrm{i}(K_k + K'_k + K''_k)\,\overline{(\phi_l \phi'_k \gamma'_i \gamma''_j \gamma'''_m)} + \mathrm{i}(K_k + K'_k + K''_k)\,\overline{(\phi_l \phi'_i \gamma'_k \gamma''_j \gamma'''_m)} + \mathrm{i}(K_k + K'_k + K''_k)\,\overline{(\delta \gamma'_i \gamma''_j \gamma'''_m)} \tag{2.15}$$

The tensor equation (2.15) can be converted to the scalar equation by contraction of the indices i and j ;

$$\frac{\partial}{\partial t}\overline{(\phi_l \gamma'_i \gamma''_i \gamma'''_m)} + \frac{\nu}{p_M}[(1+P_M)K^2 + (1+p_M)K'^2 + (1+p_M)K''^2 + 2p_M KK' + 2p_M KK'' + 2p_M KK'']$$

$$\overline{(\phi_l \gamma'_i \gamma''_i \gamma'''_m)} = \mathrm{i}(K_k + K'_k + K''_k)\,\overline{(\phi_l \phi_k \gamma'_i \gamma''_i \gamma'''_m)} - \mathrm{i}(K_k + K'_k + K''_k)\,\overline{(\gamma_l \gamma_k \gamma'_i \gamma''_i \gamma'''_m)} - \mathrm{i}(K_k + K'_k + K''_k)\,\overline{(\phi_l \phi'_k \gamma'_i \gamma''_i \gamma'''_m)} + \mathrm{i}(K_k + K'_k + K''_k)\,\overline{(\phi_l \phi'_i \gamma'_k \gamma''_i \gamma'''_m)} + \mathrm{i}(K_k + K'_k + K''_k)\,\overline{(\delta \gamma'_i \gamma''_i \gamma'''_m)} \tag{2.16}$$

If we take the derivative with respect to x_l of the momentum equation (2.1) at p, we have,

$$-\frac{\partial^2 w}{\partial x_l \partial x_l} = \frac{\partial^2}{\partial x_l \partial x_l}(u_l u_k - h_l h_k) \tag{2.17}$$

Multiplying equation (2.17) by $h'_i h''_j h'''_m$, taking time averages and writing the equation in terms of the independent variables $\vec{r}, \vec{r}', \vec{r}''$ we have,

$$-[\frac{\partial^2}{\partial r_l \partial r_l} + \frac{\partial^2}{\partial r'_l \partial r'_l} + \frac{\partial^2}{\partial r''_l \partial r''_l} + 2\frac{\partial^2}{\partial r_l \partial r'_l} + 2\frac{\partial^2}{\partial r'_l \partial r''_l} + 2\frac{\partial^2}{\partial r_l \partial r''_l}]\,\overline{(w h'_i h''_j h'''_m)} = \frac{\partial^2}{\partial r_l \partial r_k} + \frac{\partial^2}{\partial r_l \partial r'_k} + \frac{\partial^2}{\partial r'_l \partial r_k} + \frac{\partial^2}{\partial r'_l \partial r'_k} + \frac{\partial^2}{\partial r'_l \partial r''_k} + \frac{\partial^2}{\partial r''_l \partial r'_k} + \frac{\partial^2}{\partial r_k \partial r''_l} + \frac{\partial^2}{\partial r'_k \partial r''_l} + \frac{\partial^2}{\partial r''_l \partial r''_k}](\overline{u_l u_k h'_i h''_j h'''_m} - \overline{h_l h_k h'_i h''_j h'''_m}) \tag{2.18}$$

$$-\overline{(\delta \gamma'_i \gamma''_j \gamma'''_m)} = \frac{(K_l K_k + K_l K'_k + K'_l K'_k + K''_l K'_k + K_l K''_k + K'_l K''_k + K''_l K_k + K''_l K_k + K''_l K'_k + K''_l K''_k)}{K_l K_l + K'_l K'_l + K''_l K''_l + 2K_l K'_l + 2K'_l K''_l + 2K_l K''_l}$$

$$\overline{(\phi_l \phi_k \gamma'_i \gamma''_j \gamma'''_m} - \overline{\gamma_l \gamma_k \gamma'_i \gamma''_j \gamma'''_m)} \tag{2.19}$$

Equation (2.19) can be used to eliminate $\overline{(\delta \gamma'_i \gamma''_j \gamma'''_m)}$ from equation (2.16) if we take contraction of the indices i and j in equation (2.19).

Equations (2.16) and (2.19) are the spectral equation corresponding to the four –point correlation equation.

3. Three-point correlation and spectral equations

The spectral equations corresponding to the three-point correlation equations by contraction of the indices i and j are

$$\frac{\partial}{\partial t}\overline{(\phi_l \beta'_i \beta''_i)} + \frac{\nu}{p_M}[(1+P_M)(K^2 + K'^2) + 2p_M KK']\,\overline{(\phi_l \beta'_i \beta''_i)} = \mathrm{i}(K_k + K'_k)\,\overline{(\phi_l \phi_k \beta'_i \beta''_i)} - (K_k + K'_k)\,\overline{(\beta_l \beta_k \beta'_i \beta''_i)} - \mathrm{i}(K_k + K'_k)\,\overline{(\phi_l \phi'_k \beta'_i \beta''_i)} + \mathrm{i}(K_k + K'_k)\,\overline{(\phi_l \phi'_i \beta'_i \beta''_i)} + \mathrm{i}(k_l + k'_l)\overline{\gamma \beta'_i \beta''_i} \tag{3.1}$$

and

$$-\overline{(\gamma\, \beta'_i \beta''_j)} = \frac{(K_l K_k + K'_l K_k + K_l k'_k + K'_l K'_k)}{(K_l^2 + K_l'^2 + 2K_l K'_l)}\,\overline{(\phi_l \phi_k \beta'_i \beta''_i} - \overline{\beta_l \beta_k \beta'_i \beta''_j)} \tag{3.2}$$

Here the spectral tensors are defined by

$$\langle u_l h_i'(\hat{r}) h_j''(\hat{r}')\rangle = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\langle \phi_l \beta_i'(\hat{k})\beta_j''(\hat{k}')\rangle \exp[i(\hat{k}.\hat{r}+\hat{k}'.\hat{r}')d\hat{k}d\hat{k}' \quad (3.3)$$

$$\langle u_l u_k'(\hat{r}) h_i'(\hat{r}) h_j''(\hat{r}')\rangle = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\langle \phi_l \phi_k'(\hat{k})\beta_i'(\hat{k})\beta_j''(\hat{k}')\rangle \exp[i(\hat{k}.\hat{r}+\hat{k}'.\hat{r}')d\hat{k}d\hat{k}' \quad (3.4)$$

$$\langle u_l u_i'(\hat{r}) h_i'(\hat{r}) h_j''(\hat{r}')\rangle = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\langle \phi_l \phi_i'(\hat{k})\beta_i'(\hat{k})\beta_j''(\hat{k}')\rangle \exp[i(\hat{k}.\hat{r}+\hat{k}'.\hat{r}')d\hat{k}d\hat{k}' \quad (3.5)$$

$$\langle u_l h_i'(\hat{r}) h_j''(\hat{r}')\rangle = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\langle \phi_l \beta_i'(\hat{k})\beta_j''(\hat{k}')\rangle \exp[i(\hat{k}.\hat{r}+\hat{k}'.\hat{r}')d\hat{k}d\hat{k}' \quad (3.6)$$

$$\langle u_l h_k(\hat{r}) h_i'(\hat{r}) h_j''(\hat{r}')\rangle = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\langle \phi_l \beta_k(\hat{k})\beta_i'(\hat{k})\beta_j''(\hat{k}')\rangle \exp[i(\hat{k}.\hat{r}+\hat{k}'.\hat{r}')d\hat{k}d\hat{k}' \quad (3.7)$$

$$\langle w h_i'(\hat{r}) h_j''(\hat{r}')\rangle = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\langle \gamma \beta_i'(\hat{k})\beta_j''(\hat{k}')\rangle \exp[i(\hat{k}.\hat{r}+\hat{k}'.\hat{r}')d\hat{k}d\hat{k}' \quad (3.8)$$

A relation between $\phi_l\phi_k'\beta_i'\beta_j'' and\ \phi_l\gamma_i'\gamma_j''\gamma_m'''$ can be obtained by letting $\hat{r}''=0$ in equation (2.7) and comparing the result with equation (3.4)

$$\langle \phi_l \phi_k'(\hat{k})\beta_i'(\hat{k})\beta_j''(\hat{k}')\rangle = \int_{-\infty}^{\infty}\langle \phi_l \gamma_i'(\hat{k})\gamma_j''(\hat{k}')\gamma_m'''(\hat{k}'')\rangle \exp[i(\hat{k}.\hat{r}+\hat{k}'.\hat{r}'+\hat{k}''.\hat{r}'')d\hat{k}'' \quad (3.9)$$

4. Two-point correlation and spectral equations

The spectral equation corresponding to the two point correlation equation taking contraction of the indices is

$$\frac{\partial}{\partial t}\langle \varphi_i\varphi_i'(\hat{k})\rangle + \frac{2\nu}{p_M}k^2\langle \varphi_i\varphi_i'(\hat{k})\rangle = 2ik_k[\langle \alpha_i\varphi_k\varphi_i'(\hat{k})\rangle - \langle \alpha_k\varphi_i\varphi_i'(-\hat{k})\rangle] \quad (4.1)$$

where,

$\varphi_i\varphi_i' and \alpha_i\phi_k\varphi_i'$ Are defined by

$$\langle h_i h_i'(\hat{r})\rangle = \int_{-\infty}^{\infty}\langle \varphi_i\varphi_i'(\hat{k})\rangle \exp[i(\hat{k}.\hat{r})]d\hat{k} \quad (4.2)$$

and

$$\langle u_i h_i h_i'(\hat{r})\rangle = \int_{-\infty}^{\infty}\langle \alpha_i\varphi_k\varphi_i'(\hat{k})\rangle \exp[i(\hat{k}.\hat{r})d\hat{k} \quad (4.3)$$

The relation between $\alpha_i\varphi_k\varphi_j'(\hat{k})$ and $\phi_l\beta_i'\beta_j''$ is obtained by letting $\hat{r}'=0$ in equation (3.3) and comparing the result with equation (4.3),
Then

$$\langle \alpha_i\varphi_k\varphi_i'(\hat{k})\rangle = \int_{-\infty}^{\infty}\langle \phi_l\beta_i'(\hat{k})\beta_i''(\hat{k}')\rangle d\hat{k}' \quad (4.4)$$

5. Solution neglecting quintuple correlations

As it stands the set of linear equations (2.15), (3.1), (3.2), (3.5), (3.6) and (4.4) is indeterminate as it contains more unknowns than equations in equation (2.16). Neglecting all the terms on the right side of equation (2.16), the equation can be integrated between t_1 and t to give

$$\langle \phi_l\gamma_i'\gamma_j''\gamma_m'''\rangle = \langle \phi_l\gamma_i'\gamma_j''\gamma_m'''\rangle_1 \exp\left[\left\{\frac{-\nu}{p_M}(1+p_M)(k^2+k'^2+k''^2+2kk'+2k'k''+2kk'\right\}(t-t_1)\right] \quad (5.1)$$

where $\langle \phi_l\gamma_i'\gamma_j''\gamma_m'''\rangle_1$ is the value of $\langle \phi_l\gamma_i'\gamma_j''\gamma_m'''\rangle$ at t= t_1 that is stationary value for small values of k, k' and k'' when the quintuple correlations are negligible. Equation (3.9) and (5.1) can be converted to scalar form by contracting the indices *i* and *j*. Equation (3.1) have been contracted already. Substituting of equation (3.2), (3.9), (5.1) in equation (3.1), Deissler, [3, 4]

We get

$$\frac{\partial}{\partial t}\overline{(k_k\phi_l\beta_i'\beta_i'')} + \frac{\nu}{p_M}[(1+P_M)(K^2+K'^2)+2p_M KK']\overline{(k_k\phi_l\beta_i'\beta_i'')} =$$

$$[a]_1\int_{-\infty}^{\infty}\exp[-\frac{\nu}{p_M}(t-t_1)\{(1+p_M)(k^2+k'^2+k''^2)+2p_M(kk'+k'k''+k''k)\}]dk''$$

$$+[b]_1\int_{-\infty}^{\infty}\exp[-\frac{\nu}{p_M}(t-t_1)\{(1+p_M)(k^2+k'^2+k''^2)+2p_M kk'-2p_M k''k)\}]dk''$$

$$+[c]_1\int_{-\infty}^{\infty}\exp[-\frac{\nu}{p_M}(t-t_1)\{(1+p_M)(k^2+k'^2+k''^2)+2p_M kk'-2p_M k''k')\}]dk'' \quad (5.2)$$

At t_1, γ'^s have been assumed independent of; that assumption is not, made for other times. This is one of several assumptions made concerning the initial conditions, although continuity equation satisfied the conditions. The complete specification of initial turbulence is difficult; the assumptions for the initial conditions made here in are partially on the basis of simplicity.

Substituting $dk'' = dk_1'' dk_2'' dk_3''$ *and* integrating with respect to k_1'', k_2'' and k_3'', we get,

$$\frac{\partial}{\partial t}\overline{(k_k \phi_l \beta_i' \beta_i'')} +$$

$$\frac{\nu}{p_M}[(1+P_M)(K^2+K'^2)+2p_M KK']\,\overline{(k_k\phi_l\beta_i'\beta_i'')} =$$

$$\left(\frac{\pi p_M}{\nu(t-t_1)(1+p_M)}\right)^{\frac{3}{2}}[a]_1 \exp$$

$$[\frac{\nu(t-t_1)(1+p_M)}{p_M}\{\frac{(1+2P_M)(k^2+k'^2)}{(1+p_M)^2}+\frac{2p_M kk'}{(1+p_M)^2}\}]+$$

$$\left(\frac{\pi p_M}{\nu(t-t_1)(1+p_M)}\right)^{\frac{3}{2}}[b]_1 \exp$$

$$[\frac{\nu(t-t_1)(1+p_M)}{p_M}\{\frac{(1+2P_M)(k^2)}{(1+p_M)^2}+\frac{2p_M kk'}{(1+p_M)}+k'^2\}]$$

$$+\left(\frac{\pi p_M}{\nu(t-t_1)(1+p_M)}\right)^{\frac{3}{2}}[c]_1 \exp[-$$

$$\frac{\nu(t-t_1)(1+p_M)}{p_M}\{k^2+\frac{(1+2P_M)(k'^2)}{(1+p_M)^2}+\frac{2p_M kk'}{(1+p_M)}\}] \qquad (5.3)$$

Integration of equation (5.3) with respect to time, and in order to simplify calculations, we will assume that $[a]_1 = 0$; That is we assume that a function sufficiently general to represent the initial conditions can obtained by considering only the terms involving $[b]_1$ *and* $[c]_1$. The substituting of equation (4.4) in equation (4.1) and setting $H = 2\pi k^2 \varphi_i \varphi_i'$, result in

$$\frac{\partial H}{\partial t}+\frac{2\nu k^2}{p_M}H = G \qquad (5.4)$$

where,
G=

$$k^2\int_{-\infty}^{\infty} 2\pi . i[\left\langle k_k\phi_l\beta_i'\beta_i''(\hat{k},\hat{k}')\right\rangle - \left\langle k_k\phi_l\beta_i'\beta_i''(-\hat{k},-\hat{k}')\right\rangle]_0 .$$

$$\cdot \exp[-\frac{\nu}{p_M}(t-t_0)\{(1+p_M)(k^2+k'^2)+2p_M kk'\}]dk'$$

$$+$$

$$k^2\int_{-\infty}^{\infty}\frac{2p_M.\pi^{\frac{5}{2}}}{\nu}i[(\hat{k}.\hat{k}')-b(-\hat{k}.-\hat{k}')_1].$$

$$\{-\omega^{-1}\exp\left[(\omega^2\{\frac{(1+2p_M)k^2}{(1+p_M)^2}+\frac{2p_M kk'}{(1+p_M)}+k'^2\}\right]$$

$$+k\exp[(-\omega^2)((1+p_M)(k^2+k'^2)+2p_M kk')]$$

$$\int_0^{\frac{\omega k}{2}}\exp(x^2)dx\}dk'+$$

$$k^2\int_{-\infty}^{\infty}\frac{2p_M.\pi^{\frac{5}{2}}}{\nu}i[(\hat{k}.\hat{k}')-c(-\hat{k}.-\hat{k}')_1].$$

$$\{-\omega^{-1}\exp\left[(-\omega^2)\{k^2+\frac{(1+2p_M)k'^2}{(1+p_M)^2}+\frac{2p_M kk'}{(1+p_M)}\}\right]+$$

$$k'\exp[-\omega^2((1+p_M)(k^2+k'^2)+2p_M kk')].$$

$$\int_0^{\frac{\omega k'}{2}}\exp(x^2)dx\}dk' \qquad (5.5)$$

where H is the magnetic energy spectrum function, which represents contributions from various wave numbers (or eddy sizes) to the energy and G is the energy transfer function, which is responsible for the transfer of energy between wave numbers. In order to make further calculations, an assumption must be made for the forms of the bracketed quantities with the subscripts 0 and 1 in equation (5.5) which depends on the initial conditions.

$$(2\pi)^2[\left\langle k_k\phi_l\beta_i'\beta_i''(\hat{k},\hat{k}')\right\rangle - \left\langle k_k\phi_l\beta_i'\beta_i''(-\hat{k},-\hat{k}')\right\rangle]_0 =$$

$$-\xi_0(k^2k'^4 - k^4k'^2) \qquad (5.6)$$

where ξ_0 is a constant depending on the initial conditions. For the other bracketed quantities in equation (5.5), we get,

International Journal of Engineering Research & Technology (IJERT)
ISSN: 2278-0181
Vol. 1 Issue 9, November- 2012

$$\frac{4p_M.\pi^{\frac{7}{2}}}{\nu}i\left[(\hat{k}.\hat{k}') - b(-\hat{k}.-\hat{k}')\right]_{\underline{1}} =$$

$$\frac{4p_M.\pi^{\frac{7}{2}}}{\nu}i\left[(\hat{k}.\hat{k}') - c(-\hat{k}.-\hat{k}')\right]_{\underline{1}} \qquad (5.7)$$

$$= -2\xi_1(k^4k'^6 - k^6k'^4)$$

Remembering that $d\hat{k}' = -2\pi.\hat{k}'^2 d(\cos\theta)$ and $kk' = kk'\cos\theta$, θ is the angle between $\hat{k}$ and $\hat{k}'$ and carrying out the integration with respect to θ, we get,

G=-

$$\int_0^\infty [\frac{\xi_0(k^2k'^4 - k^4k'^2)kk'}{\nu(t-t_0)}\{.$$

$$\exp[-\frac{\nu}{p_M}(t-t_0)\{(1+p_M)(k^2+k'^2) - 2p_M kk'\}] -$$

$$\exp[-\frac{\nu}{p_M}(t-t_0)\{(1+p_M)(k^2+k'^2) + 2p_M kk'\}]\}$$

$$.+\frac{\xi_1(k^4k'^6 - k^6k'^4)kk'}{\nu(t-t_0)}$$

$$(\omega^{-1}\exp[-\omega^2\left(\frac{(1+2p_M)k^2}{(1+p_M)^2} - \frac{2P_M kk'}{1+p_M} + k'^2\right)] -$$

$$\omega^{-1}\exp[-\omega^2\left(\frac{(1+2p_M)k^2}{(1+p_M)^2} + \frac{2P_M kk'}{1+p_M} + k'^2\right)] + \omega^{-1}$$

$$\exp[-\omega^2\left(k^2 - \frac{2P_M kk'}{1+p_M} + \frac{(1+2p_M)k'^2}{(1+p_M)^2}\right)]$$

$$-\omega^{-1}\exp[-\omega^2\left(k^2 + \frac{2P_M kk'}{1+p_M} + \frac{(1+2p_M)k'^2}{(1+p_M)^2}\right)]$$

$$+\{k\, exp[-\omega^2(\,(1+p_M)(k^2+k'^2) - 2p_M kk')]$$

-k.exp

$$-\omega^2(\,(1+p_M)(k^2+k'^2) + 2p_M kk')]\}$$

$$\int_0^{\frac{\omega k}{2}} \exp(x^2)dx + \{k' \exp$$

$$[-\omega^2(\,(1+p_M)(k^2+k'^2) - 2p_M kk')]$$

-k/exp

$$[-\omega^2(\,(1+p_M)(k^2+k'^2) + 2p_M kk')]\}$$

$$\int_0^{\frac{\omega k'}{2}} \exp(x^2)dx)]\,dk' \qquad (5.8)$$

where, $\omega = \left(\frac{\nu(t-t_1)(1+p_M)}{p_M}\right)^{\frac{1}{2}}$.

Integrating equation (5.8) with respect to k'.

We have,

$$G = G_\beta + G_\gamma \qquad (5.9)$$

where,

$$G_\beta = -\frac{\pi^{\frac{1}{2}}\xi_0 p_M^{\frac{5}{2}}}{\nu^{\frac{3}{2}}(t-t_0)^{\frac{3}{2}}(1+p_M)^{\frac{5}{2}}}\exp\left\{-\frac{\nu(t-t_0)(1+2p_M)k^2}{p_M(1+p_M)}\right\}$$

$$.\left[\frac{15p_M k^4}{4\nu^2(t-t_0)^2(1+p_M)} + \left\{\frac{5p_M^2}{(1+p_M)^2\nu(t-t_0)} - \frac{3}{2\nu(t-t_0)}\right\}k^6 + \frac{p_M}{1+p_M}\left\{\frac{p_M^2}{(1+p_M)^2} - 1\right\}k^8\right]$$

(5.10)

and,

$$G_\gamma = G_{\gamma_1} + G_{\gamma_2} + G_{\gamma_3} + G_{\gamma_4} \qquad (5.11)$$

where,

$$G_{\gamma_1} = \frac{\xi_1\pi^{\frac{1}{2}}p_M^5}{8\nu^2(t-t_1)^2(1+p_M)^5}\exp$$

$$\left(\frac{-\nu(t-t_1)(1+2p_M - p_M^2)}{p_M(1+p_M)}\right)k^2.$$

$$[\frac{90p_M k^6}{\nu^4(t-t_1)^4(1+p_M)} + 3$$

$$\left\{\frac{4p_M}{\nu^2(t-t_1)^2(1+p_M)} + \frac{2p^2_M}{\nu^3(t-t_1)^3(1+p_M)^2} - \frac{1}{\nu^3(t-t_1)^3}\right\}k^8$$

$$+\left\{\frac{64p^2_M}{\nu(t-t_1)(1+p_M)^2} + \frac{10p^3_M}{\nu^2(t-t_1)^2(1+p_M)^3} - \frac{40}{\nu(t-t_1)}\right\}k^{10}$$

ISSN: 2278-0181
Vol. 1 Issue 9, November- 2012

$$+8\left\{\left(\frac{p_M}{1+p_M}\right)^2-\left(\frac{p_M}{1+p_M}\right)\right\}k^{12}] \tag{5.12}$$

$$G_{\gamma_2}=\frac{\xi_1\pi^{\frac{1}{2}}p_M{}^5(1+p_M)^4}{8\nu^2(t-t_1)^2(1+2p_M)^{\frac{9}{2}}}\exp\left(\frac{-\nu(t-t_1)(1+p_M)(1+2p_M-p_M{}^2)}{p_M(1+p_M)}\right)k^2$$

$$.[\frac{90p_M(1+p_m)}{\nu^4(t-t_1)^4(1+2p_M)}k^6+$$

$$\left\{\frac{120p_M(1+p_m)}{\nu^2(t-t_1)^2(1+2p_M)}+\frac{2p^2{}_M(1+p_m)^2}{\nu^3(t-t_1)^3(1+2p_M)^2}-\frac{1}{\nu^3(t-t_1)^3}\right\}k^8$$

$$+\left\{\frac{64p^2{}_M(1+p_m)^2}{\nu(t-t_1)(1+2p_M)^2}-\frac{40}{\nu(t-t_1)}+\frac{10p^3{}_M(1+p_M)^3}{\nu^2(t-t_1)^2(1+2p_M)^3}\right\}k^{10}+$$

$$\left\{\frac{8p^3{}_M(1+p_M)^3}{(1+2p_M)^3}-\frac{p_M(1+p_M)}{(1+2p_M)}\right\}k^{12}] \tag{5.13}$$

$$G_{\gamma_3}=\frac{\xi_1\pi^{\frac{1}{2}}p_M{}^{\frac{9}{2}}}{8\nu^{\frac{3}{2}}(t-t_1)^{\frac{3}{2}}(1+p_M)^8}\exp$$

$$\left(\frac{-\nu(t-t_1)(1+2p_M)}{p_M}\right)k^2\cdot[\frac{90p_M}{\nu^4(t-t_1)^4(1+p_M)^2}k^7$$

$$\left\{\frac{120p_M}{\nu^2(t-t_1)^2}+\frac{60p^2{}_M}{\nu^3(t-t_1)^3(1+p_M)^2}-\frac{30}{\nu^3(t-t_1)^3}\right\}k^9+$$

$$\left\{\frac{64p^2{}_M}{\nu(t-t_1)}+\frac{10p^3{}_M}{\nu^2(t-t_1)^2(1+p_M)^2}-\frac{40(1+p_M)^2}{\nu(t-t_1)}\right\}k^{11}+$$

$$\left\{p^2{}_M-p_M(1+p_M)^2\right\}k^{13}]\int_0^{\frac{\omega_1}{2}}\exp(y^2)dy \tag{5.14}$$

where,

$$\omega_1=\left(\frac{\nu(t-t_1)(1+p_M)}{p_M}\right)^{\frac{1}{2}}k$$

$$G_{\gamma_4}=\frac{\xi_1\pi^{\frac{1}{2}}p_M{}^{\frac{15}{2}}}{2^8\nu(t-t_1)(1+p_M)^{\frac{29}{2}}}\exp$$

$$\left(\frac{-\nu(t-t_1)(1+2p_M)}{p_M}\right)k^2[$$

$$\frac{7560(1+p_M)^3}{\nu^4(t-t_1)^4p_M{}^2}k^6+\left\{\frac{20160(1+p_M)^5}{\nu^3(t-t_1)^3p_M}-\frac{4233600(1+p_M)^7}{\nu^3(t-t_1)^3p_M{}^3}\right\}k^8+$$

$$\left\{\frac{12096(1+p_M)^5}{\nu^2(t-t_1)^2}-\frac{3360(1+p_M)^7}{\nu^2(t-t_1)^2p_M{}^2}\right\}k^{10}+\left\{\frac{2304(1+p_M)^5p_M}{\nu(t-t_1)}-\frac{1344(1+p_M)^9}{p_M{}^2}\right\}k^{12}+$$

$$\left\{128(1+p_M)^5p^2{}_M-128(1+p_M)^7\right\}k^{14}+\ldots\ldots\ldots\ldots] \tag{5.15}$$

The integral expression in equation (5.9), The quantity G_β represents the transfer function arising owing to consideration of magnetic field at three point correlation equation; G_γ arises from consideration of the four –point equation. Integration of equation (5.9) over all wave number shows that

$$\int_0^\infty G.d\vec{k}=0 \tag{5.16}$$

Indicating that the expression for G satisfies the conditions of continuity and homogeneity, physically, it was to be expected, Since G is a measure of transfer of energy and the numbers must be zero. From (5.4), we get,

$$H=\exp\left[-\frac{2\nu k^2(t-t_0)}{p_M}\right]\int G\exp\left[-\frac{2\nu k^2(t-t_0)}{p_M}\right]dt+J(k)\exp\left[-\frac{2\nu k^2(t-t_0)}{p_M}\right]$$

where, $J(k)=\frac{N_0k^2}{\pi}$ is a constant of integration and can be obtained as by Corrsin, [2]

$$H=$$

$$\frac{N_0k^2}{\pi}\exp\left[\frac{-2\nu k^2(t-t_0)}{p_M}\right]+\exp\left[\frac{-2\nu k^2(t-t_0)}{p_M}\right]\int[G_\beta+(G_{\gamma_1}+G_{\gamma_2}+G_{\gamma_3}+G_{\gamma_4})]$$

$$\exp\left[\frac{-2\nu k^2(t-t_0)}{p_M}\right]dt \tag{5.17}$$

where,

$$G=G_\beta+G_{\gamma_1}+G_{\gamma_2}+G_{\gamma_3}+G_{\gamma_4} \tag{5.18}$$

after integration equation (5.17) becomes

$$H=\frac{N_0k^2}{\pi}\exp\left[-\frac{2\nu k^2(t-t_0)}{p_M}\right]+$$

$$H_\beta+[H_{\gamma_1}+H_{\gamma_2}+H_{\gamma_3}+H_{\gamma_4}] \tag{5.19}$$

where,

ISSN: 2278-0181
Vol. 1 Issue 9, November- 2012

$$H_{\beta}=\frac{\xi_0\pi^{\frac{1}{2}}p_M^{\frac{5}{2}}}{8\nu^{\frac{3}{2}}(1+p_M)^{\frac{7}{2}}}\exp\left(\frac{-\nu(t-t_0)(1+2p_M)}{p_M(1+p_M)}\right)k^2$$

where,

$$\mathrm{F}(\omega)=\exp(-\omega^2)\int_0^{\omega}\exp(x^2)dx,\omega=\left[\frac{\nu(t-t_0)}{p_M(1+p_M)}\right]^{\frac{1}{2}}k$$

and,

$$H_{\gamma_1}=-\frac{\xi_1\pi^{\frac{1}{2}}p_M{}^5}{8\nu^2(1+p_M)^5}\exp\left(\frac{-\nu(t-t_1)(1+2p_M-p_M{}^2)}{p_M(1+p_M)}\right)k^2$$

$$[\left(\frac{18p_M}{\nu^4(1+p_M)(t-t_1)^5}\right)k^6+\left(\frac{15-6p_M+21p^2{}_M}{4\nu^3(1+p_M)^2(t-t_1)^4}+\frac{4p_M}{\nu^2(1+p_M)(t-t_1)^3}\right)k^8$$

$$+\left(\frac{15-6p_M+36p^2{}_M-6p^3{}_M+61p^4{}_M}{12\nu^2p_M(1+p_M)^3(t-t_1)^3}+\frac{14p^2{}_M-40p_M-18}{\nu(1+p_M)^2(t-t_1)^2}\right)k^{10}$$

$$+\left(\begin{array}{l}\frac{(1+p_M)^2(75-30p_M+180p^2{}_M-30p^3{}_M+305p^4{}_M)}{120\nu p^2{}_M(1+p_M)^4(t-t_1)^2}+\\ \frac{14p^4{}_M-56p^3{}_M-12p^2{}_M-40p_M-18}{p_M(1+p_M)^3(t-t_1)}\end{array}\right)k^{12}$$

$$+$$

$$\left(\frac{(1+p_M)^2(75-3p_M+90p^2{}_M-30p^3{}_M+215p^4{}_M)}{120p^3{}_M(1+p_M)^5(t-t_1)}\right)k^{14}-$$

$$(\frac{\nu(1+p^2{}_M)(14p^4{}_M-56p^3{}_M-12p^2{}_M-40p_M-18)k^{14}}{p^2{}_M(1+p_M)^4}$$

$$+\frac{\nu(1+p^2{}_M)^3(75-3p_M+90p^2{}_M-30p^3{}_M+215p^4{}_M)k^{16}}{120p^4{}_M(1+p_M)^6})\exp(-\omega_2)Ei(\omega_2)$$

where

$$Ei(\omega_2)=\int\exp(\frac{\nu(1+p^2{}_M)tk^2}{p_M(1+p_M)})/(t-t_1)dt]$$

$$H_{\gamma_2}=-\frac{\xi_1\pi^{\frac{1}{2}}p_M{}^5(1+p_M)^4}{8\nu^2(1+2p_M)^{\frac{9}{2}}}\exp\left(\frac{-\nu(t-t_1)(1+2p_M-p_M{}^2)}{p_M(1+p_M)}\right)k^2$$

$$.[\left\{\frac{18p_M(1+p_M)}{\nu^4(1+2p_M)(t-t_1)^5}\right\}k^6+$$

$$\left\{\frac{17+32p_M-2p^2{}_M+4p^3{}_M+20p^4{}_M}{4\nu^3(1+2p_M)^2(t-t_1)^4}+\frac{120p_M(1+p_M)}{3\nu^2(1+2p_M)(t-t_1)^3}\right\}k^8$$

$$+$$

$$\left\{\begin{array}{l}\frac{17+49p_M+13p^2{}_M+13p^3{}_M+98p^4{}_M+134p^5{}_M+104p^6{}_M+60p^7{}_M}{4\nu^3(1+2p_M)^2(t-t_1)^4}+\\ \frac{52p^4{}_M+64p^3{}_M-48p^2{}_M-40p_M}{\nu(1+2p_M)^2(t-t_1)^2}\end{array}\right\}k^{10}+$$

$$+\left\{\begin{array}{l}(1+p_M-p^2{}_M+p^3{}_M)^2\\ \frac{.(17+49p_M+13p^2{}_M-13p^3{}_M+98p^4{}_M+134p^5{}_M+104p^6{}_M+60p^7{}_M)}{24p^2{}_M(1+2p_M)^5(t-t_1)}\end{array}\right\}k^{14}$$

$$-\left\{\begin{array}{l}\nu(1+p_M-p^2{}_M+p^3{}_M).\\ \frac{(-40p_M-89p^2{}_M+51p^3{}_M+124p^4{}_M-40p^5{}_M+36p^6{}_M+60p^7{}_M)}{p^2{}_M(1+2p_M)^4}k^{14}+\\ \nu(1+p_M-p^2{}_M+p^3{}_M)^3\\ \frac{.(17+49p_M+13p^2{}_M-13p^3{}_M+98p^4{}_M+134p^5{}_M+104p^6{}_M+60p^7{}_M)}{24p^4{}_M(1+2p_M)^6}k^{16}\end{array}\right\}.$$

$$\exp(\omega_3)Ei(\omega_3)]$$

where, $Ei(\omega_3)=$

$$\int\frac{\exp\left(\frac{-\nu(t-t_1)(1+2p_M-p_M{}^2)t}{p_M(1+2p_M)}\right)k^2}{(t-t_1)}dt$$

and, $\omega_3=\left(\frac{-\nu(t-t_1)(1+2p_M-p_M{}^2)t}{p_M(1+2p_M)}\right)k^2$

$$H_{\gamma_3}=-\frac{\xi_1\pi^{\frac{1}{2}}p^4{}_M}{16\nu(1+p_M)^{\frac{15}{2}}}\exp\left(\frac{-\nu(t-t_1)(1+2p_M)}{p_M}\right)k^2$$

$$[\frac{45p_M}{2\nu^4(1+p_M)^2(t-t_1)^4}k^8+$$

$$\left\{\frac{(20p^2{}_M-70p_M-5)}{2\nu^3(1+p_M)^2(t-t_1)^3}+\frac{60p_M}{\nu^2(t-t_1)^2}\right\}k^{10}+$$

International Journal of Engineering Research & Technology (IJERT)
ISSN: 2278-0181
Vol. 1 Issue 9, November- 2012

$$\left\{\frac{(20p^4_M - 40p^3_M + 160p^2_M - 60p_M - 5)}{4\nu^2 p_M (1+p_M)^2 (t-t_1)^2} + \frac{(24p^2_M - 200p_M + 20)}{\nu(t-t_1)}\right\}k^{12} +$$

$$\left\{\frac{(1-2p_M)(20p^4_M - 40p^3_M + 160p^2_M - 60p_M - 5)}{4\nu p^2_M (1+p_M)^2 (t-t_1)}\right\}k^{14}$$

$$- \left\{\begin{array}{l} \frac{(20 - 240p_M + 424p^2_M - 48p^3_M)}{p_M}k^{14} + \\ \frac{(1-2p_M)^2 (20p^4_M - 40p^3_M + 160p^2_M - 60p_M - 5)}{4p^3_M (1+p_M)^2}k^{16} \end{array}\right\}.\exp(-\omega_4)Ei(\omega_4) \quad]$$

$$\omega_4 = \left(\frac{\nu(1-2p_M)t}{p_M}\right)k^2 \text{ and}$$

$$\mathrm{Ei}(\omega_4) = \int \frac{\exp\left(\frac{\nu(1-2p_M)t}{p_M}\right)k^2}{(t-t_1)}dt$$

$$H_{\gamma_4} = - \frac{\xi_1 \pi^{\frac{1}{2}} p^{\frac{9}{2}}_M}{2^8 \nu (1+p_M)^{\frac{11}{2}}} \exp \left(\frac{-\nu(t-t_1)(1+2p_M)}{p_M}\right)k^2$$

$$[\left\{\frac{1890p_M}{\nu^4(1+p_M)^6(t-t_1)^4}\right\}k^6 +$$

$$\left\{\frac{-4231710 - 16938180p_M - 25381440p^2_M - 16894080p^3_M - 4213440p^4_M}{\nu^3(1+p_M)^6(t-t_1)^3}\right\}k^8 +$$

$$\left\{\frac{-2115855 - 4237380p_M + 4245780p^2_M + 16927680p^3_M + 14783328p^4_M + 4218816p^5_M + 4368p^6_M}{\nu^2(1+p_M)^6 p_M (t-t_1)^2}\right\}k^{10} +$$

$$\left\{\frac{-2115855 - 5670p_M + 12720540p^2_M + 8436120p^3_M - 19072032p^4_M - 25347840p^6_M - 4128p^7_M + 2304p^8_M}{\nu(1+p_M)^6 p^2_M (t-t_1)}\right\}k^{12}$$

$$- \left\{\begin{array}{l} \frac{-2115855 + 4226040p_M + 12731880p^2_M - 17004960p^3_M - 35944272p^4_M + 12796224p^5_M + 42264592p^6_M + 16857280p^7_M + 9920p^8_M - 4864p^9_M}{(1+p_M)^6 p^3_M}k^{14} \\ + 1344p_M k^{12} \end{array}\right\}$$

$$.\exp(\omega_5)Ei(\omega_5)]$$

where, $\omega_5 = \exp\frac{\nu(1-2p_M)tk^2}{p_M}$

$$\mathrm{Ei}(\omega_5) = \int \frac{\exp\frac{\nu(1-2p_M)tk^2}{p_M}}{(t-t_1)}dt$$

From equation (5.19), we get,

$$H = H_1 + H_2 \tag{5.20}$$

where,

$$H_1 = \frac{N_0 k^2}{\pi}\exp\left[-\frac{2\nu k^2(t-t_0)}{p_M}\right] + H_\beta \text{ and}$$

$$H_2 = (H_{\gamma_1} + H_{\gamma_2} + H_{\gamma_3} + H_{\gamma_4}) \; ;$$

In equation (5.20) H_1 and H_2 magnetic energy spectrum arising from consideration of the three and four –point correlation equations respectively. Equation (5.20) can be integrated over all wave numbers to give the total magnetic turbulent energy. That is

$$\frac{\langle h_i h_i' \rangle}{2} = \int_0^\infty H dk \tag{5.21}$$

where,

$$\int_0^\infty H_1 dk = \frac{N_0 p^{\frac{3}{2}}_M \nu^{-\frac{3}{2}} (t-t_0)^{-\frac{3}{2}}}{8\sqrt{2\pi}} + \xi_0 Q \nu^{-6}(t-t_0)^{-5},$$

$$\int_0^\infty H_2 dk = \xi_1 [R\nu^{-\frac{17}{2}}(t-t_1)^{-\frac{15}{2}} + S\nu^{-\frac{19}{2}}(t-t_1)^{-\frac{17}{2}}],$$

$$R = Q_2 + Q_4 + Q_6 + Q_7, S = Q_1 + Q_3 + Q_5$$

and Q's values are

$$Q = \frac{\pi p^6_M}{(1+p_M)(1+2p_M)^{\frac{5}{2}}}..$$

$$\left\{\frac{9}{16} + \frac{5p_M(7p_M - 6)}{(1+2p_M)} - \frac{35p_M(3p^2_M - 2p_M + 3)}{8(1+2p_M)^2} + \frac{8p_M(3p^2_M - 2p_M + 3)}{3.2^6.(1+2p_M)^3} + \ldots\ldots\ldots\right\}$$

$$Q_1 = -\frac{\pi p^{\frac{19}{2}}_M}{(1+p_M)^{\frac{5}{2}}(1+2p_M - p^2_M)^{\frac{7}{2}}}$$

$$[\frac{15.9}{2^6} + \frac{15.7(15 - 6p_M + 21p^2_M)}{2^{10}(1+2p_M - p^2_M)} +$$

ISSN: 2278-0181
Vol. 1 Issue 9, November- 2012

$$\frac{15.7.3(15-6p_M+36p_M^2-6p_M^3+61p_M^4)}{2^{11}(1+2p_M-p_M^2)^2}+$$

$$\left(\frac{11.9.7(1+p_M^2)(75-30p_M+180p_M^2-30p_M^3+305p_M^4)}{2^{13}(1+2p_M-p_M^2)^3}\right)$$

$$+\left(\frac{13.11.9.7(1+p_M^2)^2(75-3p_M+90p_M^2-30p_M^3+15p_M^4)}{2^{14}(1+2p_M-p_M^2)^4}\right)-\ldots\ldots\ldots\ldots$$

$$Q_2=-\frac{\pi p_M^{\frac{21}{2}}}{(1+p_M)^{\frac{3}{2}}(1+2p_M-p_M^2)^{\frac{9}{2}}}.$$

$$\left[\frac{15.7}{2^6}+\frac{15.9.7(14p_M^2-18-40p_M)}{2^9(1+2p_M-p_M^2)}+\frac{15.11.9.7(14p_M^4-56p_M^3-12p_M^2-40p_M-18)}{2^{10}(1+2p_M-p_M^2)^2}-\ldots\ldots\ldots\ldots\ldots\ldots\right]$$

$$Q_3=\frac{\pi p_M^{\frac{19}{2}}(1+p_M)^{\frac{1}{2}}}{(1+2p_M)^2(1+2p_M-p_M^2)^{\frac{7}{2}}}.$$

$$\frac{9.15}{2^6}+\frac{15.7(17+32p_M-2p_M^2+4p_M^3+20p_M^4)}{2^{10}(1+p_M)^2(1+2p_M-p_M^2)}$$

$$+\frac{9.7.5(17+49p_M+13p_M^2-13p_M^3+98p_M^4+134p_M^5+104p_M^6+60p_M^7)}{2^{11}(1+p_M)^3(1+2p_M-p_M^2)^2}+$$

$$\left(\frac{11.9.7.5(1+p_M-p_M^2+p_M^3)(17+49p_M+13p_M^2-13p_M^3+98p_M^4+134p_M^5+104p_M^6+60p_M^7)}{2^{13}(1+p_M)^4(1+2p_M-p_M^2)^3}\right)+$$

$$\left(\frac{13.11.9.7.5(1+p_M-p_M^2+p_M^3)^2(17+49p_M+13p_M^2-13p_M^3+98p_M^4+134p_M^5+104p_M^6+60p_M^7)}{2^{14}(1+p_M)^5(1+2p_M-p_M^2)^4}\right)$$

$$-\ldots\ldots\ldots\ldots]$$

$$Q_4=-\frac{\pi p_M^{\frac{21}{2}}}{(1+p_M)^{\frac{1}{2}}(1+2p_M)(1+2p_M-p_M^2)^{\frac{9}{2}}}.$$

$$\left[\frac{25.7.3}{2^5}+\frac{15.9.7(-40p_M-48p_M^2+64p_M^3+52p_M^4)}{2^9(1+p_M)^2(1+2p_M-p_M^2)}+\frac{15.11.9.7(-40p_M-89p_M^2+51p_M^3+124p_M^4-40p_M^5+36p_M^6+60p_M^7)}{2^{10}(1+p_M)^3(1+2p_M-p_M^2)^2}-\ldots..\right]$$

$$Q_5=-\frac{\pi p_M^{\frac{19}{2}}}{(1+p_M)^{\frac{19}{2}}(1+2p_M)^{\frac{9}{2}}}.$$

$$\left\{\frac{45.7.5.3}{2^{10}}+\frac{9.7.5.3(20p_M^2-70p_M-5)}{2^{11}(1+2p_M)}+\frac{11.9.7.5.3(20p_M^4-40p_M^3+160p_M^2-60p_m-5)}{2^{13}(1+2p_M)^2}+\frac{13.11.9.7.5.3(1-2p_M)(20p_M^4-40p_M^3+160p_M^2-60p_M-5)}{2^{14}.(1+2p_M)^3}-\ldots\ldots\ldots\ldots\right\}$$

$$Q_6=-\frac{\pi p_M^{\frac{21}{2}}}{(1+p_M)^{\frac{15}{2}}(1+2p_M)^{\frac{11}{2}}}.$$

$$\left\{\frac{15.9.7.5.3}{2^8}+\frac{11.9.7.5.3(24p_M^2-200p_M+20)}{2^{11}(1+2p_M)}-\ldots\ldots\ldots\ldots\ldots\ldots\right\}$$

$$Q_7=-\frac{\pi p_M^9}{(1+p_M)^{\frac{23}{2}}(1+2p_M)^{\frac{7}{2}}}.$$

$$\left\{\frac{9.7.5.3}{2^{11}}+7.5.3\left(\frac{-423170-16938180p_M-25381440p_M^2-16894080p_M^3-4213440p_M^4}{2^{13}(1+2p_M)}\right)-\ldots\right\}+$$

$$\left\{\frac{9.7.5.3(-2115855-4237380p_M+4245780p_M^2+16927680p_M^3+14783328p_M^4+4218816p_M^5+4218816p_M^6}{2^{14}(1+2p_M)^2}\right\}+$$

$$\left\{\frac{11.9.7.5.3(-2115855-5670p_M+12720540p_M^2+8436120p_M^3-190720032p_M^4-25347840p_M^6-4128p_M^7+2304p_M^8)}{2^{15}(1+2p_M)^3}-\ldots.\right\}$$

Therefore, from equation (5.21)

$$\frac{\langle h_i h_i'\rangle}{2}=\frac{N_0 p_M^{\frac{3}{2}}\nu^{\frac{-3}{2}}(t-t_0)^{\frac{-3}{2}}}{8\sqrt{2\pi}}+\xi_0.Q.\nu^{-6}.(t-t_0)^{-5}$$

$$+[\xi_1 R\nu^{-\frac{17}{2}}(t-t_1)^{-\frac{15}{2}}+\xi_1 S\nu^{-\frac{19}{2}}(t-t_1)^{-\frac{17}{2}}] \qquad (5.22)$$

Also, we can write equation (5.22) of the form

$$\langle h^2\rangle=A(t-t_0)^{-3/2}+B(t-t_0)^{-5}+C(t-t_1)^{-15/2}+D(t-t_1)^{-17/2}, \qquad (5.23)$$

This is the energy decay law of MHD turbulence for four point correlations.

where,

$\langle h^2\rangle=\langle h_i h_i'\rangle$, $A=\frac{N_0 p_M^{\frac{3}{2}}\nu^{-\frac{3}{2}}}{4\sqrt{2\pi}}$, B=2$\xi_0 Q\nu^{-6}$, C=2ξ_1, R$\nu^{-\frac{17}{2}}$ and D=2$\xi_1 S\nu^{-\frac{19}{2}}$.

ISSN: 2278-0181
Vol. 1 Issue 9, November- 2012

If R=0 and S=0 that is C=0 and D=0 in equation (5.23), than we get,

$$\left\langle h^2 \right\rangle = A(t-t_0)^{-3/2} + B(t-t_0)^{-5} \qquad (5.24)$$

This is the energy decay of MHD turbulence in three-point correlations.

6. Result and discussions

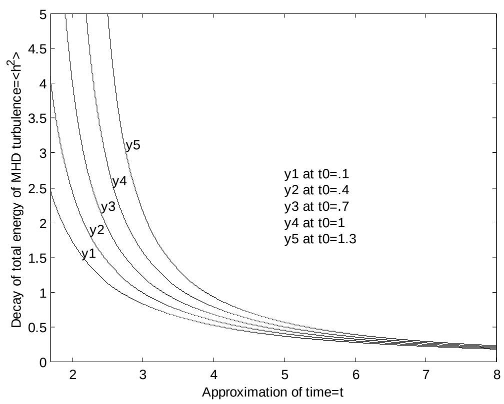

Fig. 6.1: Decay of energy of MHD turbulence for three-point correlation.

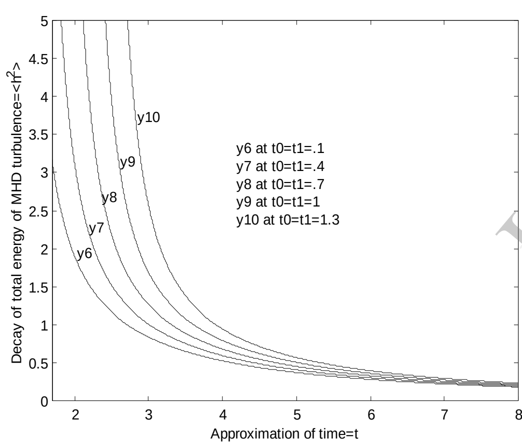

Fig. 6.2: Decay of energy of MHD turbulence for four-point correlation.

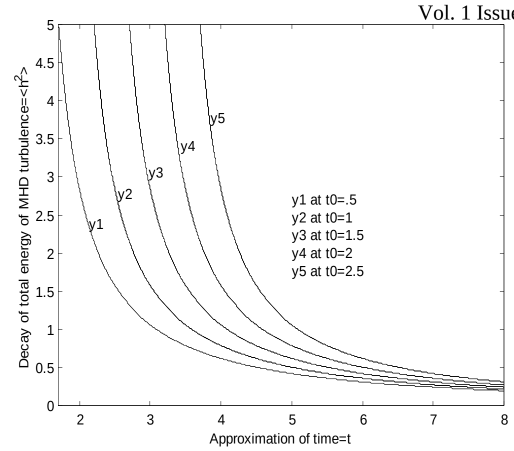

Fig. 6.3: Decay of energy of MHD turbulence for three-point correlation.

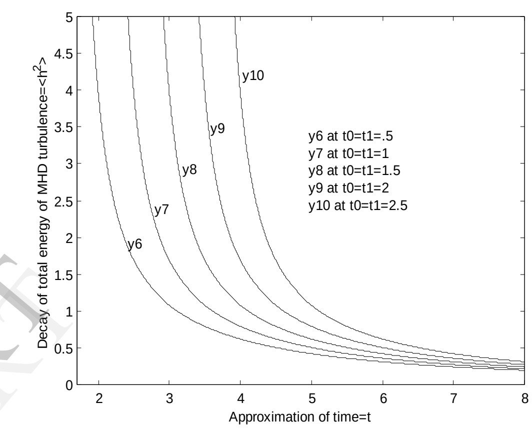

Fig. 6.4: Decay of energy of MHD turbulence for four-point correlation.

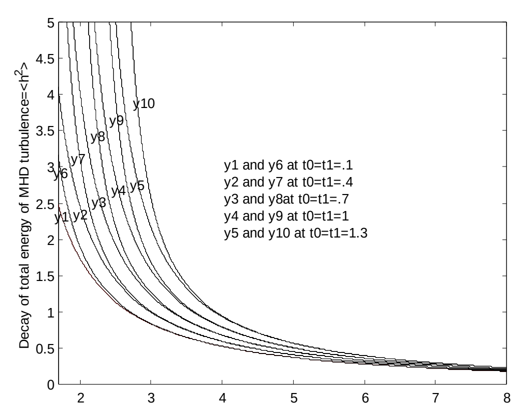

Fig. 6.5: Comparison between Figure 6.1 and Figure 6.2.

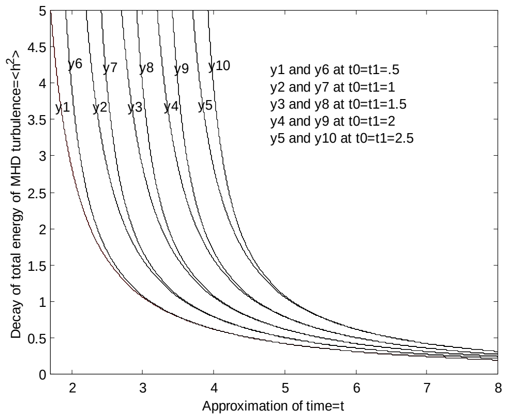

Fig.6.6: Comparison between Figure 6.3 and Figure 6.4.

Fig. 4.1 and Fig. 4.3 represent the energy decay of MHD turbulence for three-point correlations of equation (5.24). $y1$, $y2$, $y3$, $y4$ and $y5$ are solutions of equation (5.24) at t0=.1, .4, .7, 1 and 1.3 respectively; which indicated in the Figure 4.1 clearly. Similarly, in Figure 4.3; $y1$, $y2$, $y3$, $y4$ and $y5$ are represents solution curves of equation (5.24) at .5, 1, 1.5, 2 and 2.5 respectively, which indicated in Figure 4.2 and Figure 4.3. If the time is increases then the decay of energy is increases.

Fig. 4.2 and Fig. 4.4 represent the energy decay of MHD turbulence for four-point correlations of equation (5.23). $y6$, $y7$, $y8$, $y9$ and $y10$ are solutions of equation (5.23) at t0=t1=.1, .4, .7, 1 and 1.3 respectively; which indicated in the Figure 4.2 clearly. Similarly, in Figure 4.4; $y6$, $y7$, $y8$, $y9$ and $y10$ are represents solution curves of equation (5.23) at .5, 1, 1.5, 2 and 2.5 respectively, which indicated in Figure 4.4.

Fig. 4.5, represents the comparison between the Fig.4.1 and Fig.4.2 of three and four- point correlations of MHD turbulent flow at t0=.1, .4, .7, 1 , 1.3 and .5 , 1, 1.5, 2 , 2.5 respectively .

Fig. 4.6, represents the comparison between the Fig.4.2 and Fig. 4.4 of three and four- point correlations of MHD turbulent flow at t0=.1, .4, .7, 1 , 1.3 and .5 , 1, 1.5, 2 , 2.5 respectively .

In equation (5.23) the third and fourth term on the right hand side comes due to four point correlations. If we put C=0 and D=0 it will be in the form

$\overline{h^2} = A(t-t_0)^{-3/2} + B(t-t_0)^{-5}$, which is completely same with Sarker and Kshore [9] for the case of three -point correlation.

For large times second, third and fourth terms in equation (5.23) becomes negligible leaving only $A(t-t_0)^{-3/2}$ power decay law.

In equation (5.23), we shows that magnetic turbulent energy for four- point correlations systems decays more and more rapidly by exponential manner than the decays of three point correlation system.

If the quadruple and quintuple correlations were not neglected, the equation (5.23) appears that more terms in higher power of $(t-t_0) and (t-t_1)$ would be added to the equation (5.23).In this case, energy decays greater than the energy decays in equation (5.23) for four point correlation systems.

From Fig. 4.5 and Fig. 4.6, we see that, in four-point correlations system energy die out faster than the three- point correlations system in MHD turbulent flow.

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