Design and Failure analysis of Geodesic Dome of a Composite Pressure vessel

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Abstract

Composite pressure vessels are low weight structures mainly used in defence, aviation applications. A composite pressure vessel along with optimized dome ends avoids critical stresses that are incorporate with the structure when the structure is internally pressurized. Design of the Geodesic dome portion which leads to the optimization of the dome structure is calculated by using geodesic path equation. The thicknesses of laminate structure at various portions of the dome region and cylindrical region are calculated by using netting analysis. Carbon fibre is reinforcement material and epoxy resin is the matrix material selected for the present study. The operating and failure conditions of the Geodesic Dome portion are predicted by using classical laminate theory of composites. Finite element analysis of Geodesic dome end and cylindrical portion is done by using Ansys 12.0. The failure sequence of the fibres associated with different fibre orientations is identified. Mat lab code is developed to study the failure sequence of the fibres. Based on Tsai-Wu theory and maximum stress theory the operating and failure pressures of the composite structure are predicted.

Keywords: *Geodesic-dome profile, ANSYS, Tsai-Wutheory, composite pressure vessel.*

1. Introduction

The structures with parameters like high strength, high stiffness, with optimized mass are most effective and advantageous structures. These Composite structures are most widely used in aerospace and chemical engineering applications, composite rocket motor cases and CNG tanks. Failure prediction of the composite pressure vessel is very important. There are only two ASME standards are available for Glass fibre reinforced composite pressure vessels (water storage tanks).Composite pressure vessel fabricated by filament winding technique. Composite pressure vessel consists of filament wound laminated structure, along with polar fittings and thin non metallic low elastic modulus layer which does not share pressure load, prevents leakage of pressurized fluid, and efficiently transfers the loads to the composite structure. Metallic Polar fittings are provided at the dome region to avoid the failure at the ends of the dome surface (vicinity of the polar opening). In filament winding technique, filaments immersed in slightly heated resin, wound around collapsible foam mandrel with some tension, and then after completion of winding process, set the mandrel for curing in an air circulated oven for the specified time and temperature. By digging the foam from the cured structure we get the required composite pressure vessel. Lining process is carried out by using low modulus metallic or non metallic material, generally aluminium or polyurethane rubber layer can be used for lining. The tension of the fibres directly affects the volume fraction of the fibre and matrix material. Winding of the resin impregnated fibres around the mandrel with more tension increases volume fraction of the fibres and decreases the volume fraction of resin content. The design of the mandrel should be carried out in such a way that the structure should equally stressed at all regions when the pressure load is applied.

2. Design of the collapsible mandrel

The outer surface of the collapsible mandrel becomes inner surface of the composite pressure vessel after curing. Collapsible mandrel consists of cylindrical portion and two dome regions. Domes surface is highly stressed region under circumferential and longitudinal loads. Geodesic dome profile is most commonly used for pressure vessels. The material for the fabrication of the mandrel is foam or sand with polyvinyl alcohol binder (water soluble mandrel). The aluminium polar fittings are preassembled with collapsible mandrel before winding. Mandrel should not have voids or air gaps in the case of foam mandrel. The length of the cylindrical portion, the radius of the cylindrical portion, pole opening radius is taken as 1000mm, 206mm, 100mm for the present study. Fig .2 represents the collapsible mandrel along with Geodesic dome ends.

2.1. Design of Geodesic Dome Profile

The geodesic profile is the elliptical curve connecting the two points taking the consideration of shortest distance. We need to define Geodesic profile between pole opening radius and cylindrical portion radius [8],[3]. Generally friction is required to keep the path stationary. The fibre wound on the geodesic dome profile according to the claurit's principle [4] does not slip and it directly follows the Geodesic path, this type of winding is called Geodesic winding, which does not require any friction to keep the fibre stationary. Geodesic dome profile is created by using Geodesic path equation [8].

$$\bar{z} = -\sqrt{1-\bar{r}_0^2}\int_1^{\bar{\bar{r}}} \frac{\bar{r}^3 dr}{\sqrt{\bar{r}^2-\bar{r}_0^2-\bar{r}^6(1-\bar{r}_0^2)}} \quad (1)$$

z = Axial coordinate
r_0 = Polar opening radius

r = radial coordinate, R=radius of the cylindrical portion

$\bar{r} = \frac{r}{R}, \bar{z} = \frac{z}{R}, \bar{r_0} = \frac{r_0}{R}$

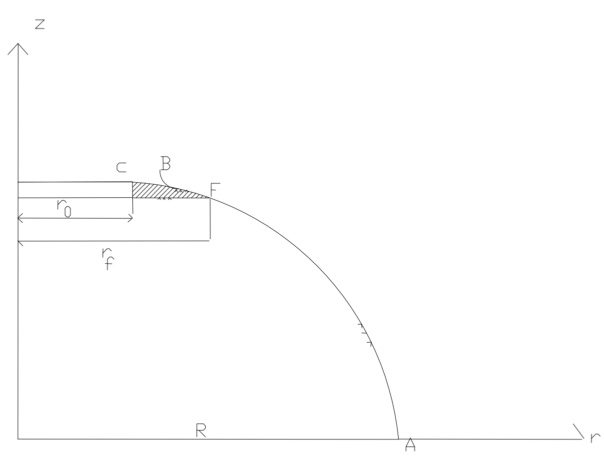

Figure 1. Geodesic dome profile pole opening

Geodesic curve which is obtained from the above Geodesic equation has the point of inflection at which the radius of curvature reverses its direction as shown in the Figure.1 along the line AFB. At a distance of 1.225 times of the pole opening radius the axial coordinate becomes infinity at that particular radius (i.e., point of inflection). In previous papers a tangent line drawn to connect the point of inflection and pole opening radius. Another geodesic profile equation is provided which can avoid the tangent line usage for connecting pole opening and inflection point. Second geodesic profile equation used for the calculation of dome coordinates. From the point of inflection the aluminium polar fitting is introduced to avoid the mode of failure at the vicinity of the pole opening [3], [1].

$$\bar{z} = -\overline{r_f}^2\sqrt{1-\bar{r}_0^2}\int_{\overline{r_f}}^{\bar{r}} \frac{\bar{r}\sqrt{\bar{r}^2-\bar{r}_0^2}\, dr}{\sqrt{(\overline{r_f}^2-\bar{r}_0^2)-\overline{r_f}^4((1-\bar{r}_0^2)\bar{r}^2(\bar{r}^2-\bar{r}_0^2)}} + \bar{z}_f \quad (2)$$

Where $\bar{z_f} = \bar{z}(\bar{r} = r_f)$

r_f=radius at point of inflection=1.225 times of pole opening.

The thickness of the metallic polar fitting can be calculated by using equation

t≥$3pr_0/4\tau_y$

τ_y =Yield stress of the material

t= thickness of the polar fitting

P=design pressure

2.3. Design of the laminate structure

The fibre material is carbon, the matrix material is epoxy resin selected for the present study. Circumferential layer and helical layers are required for constructing the laminate structure on cylindrical portion. Circumferential winding is not possible on the dome region of the mandrel. At the dome region slipping of the fibres occurs during winding. Doilies are group of layers in two mutually perpendicular directions are needed to cover the surface of the dome. The geodesic dome profile is calculated by using Geodesic path equations 1, 2, which are specified in the previous section. The below Tables. 1 , 2 represents the Geodesic dome coordinates for the required pole opening radius and cylindrical portion radius. Point of inflection (at which radius of curvature reverses its direction) is also considered. Laminate structure for the composite pressure vessels is clearly explained in the Table 3.

Table 1.Geodesic dome profile up to the point of inflection

z(mm)	r(mm)
0	206
14.612	205
21.05	204
25.93	203
30.074	202
33.67	201
36.91	200
50.42	195
59.428	190
68.432	185
75.63	180
82.83	175
88.24	170
108.05	145
126.05	122.5

Table 2. Geodesic dome profile from the point of inflection to the up to pole opening

z(mm)	r(mm)
126.4963	120
126.826	118
127.1306	116
127.3961	114
127.6668	112
127.8946	110
128.0958	108
128.2689	106
128.411	104
128.5189	102
128.5763	100

Previously the angle of doilies is taken as the winding angle of the dome portion at that particular radius, winding of the dollies is done by using filament winding machine. Now for the present study the dollies with 0° and 90° fibre layer orientations are considered. The process of covering the dome region doilies in the direction of 0° and 90° is carried out by manually. After completion of the circumferential winding with required thickness on arbitrary cylindrical mandrel, cut the layers according to the required dimensions using the template and cover the surface of the dome portion, by keeping the circumferential layers along 0° and 90° direction manually. Winding angle is calculated from the claurit's principle. Thickness of hoop, helical, doilies is taken as 0.2mm, 0.45mm, 0.5mm. Thickness calculations are done by using netting analysis [4].

3. Failure analysis of the composite structure

Classical laminate theory is used for analysis in which the behaviour of composite laminate structure is treated as an orthotropic material. Maximum stress, Tsai-Wu theories [9], [10] followed to predict the failure, Steps involved in the analysis:

1. Find the reduced stiffness matrix ($[Q_{xy}]_k$) from the orthotropic material properties of the carbon/epoxy structure.

2. Calculate reduced transformed stiffness matrix by using fibre angle orientation. Find coupling stiffness matrix, flexural stiffness matrix, and bending stiffness matrix (A, B, D matrices)

$$A = \sum_{k=1}^{n} [Q_{xy}]_k * [h_{k+1} - h_k]\,, B = \frac{1}{2}\sum_{k=1}^{n} [Q_{xy}]_k * [{h_{k+1}}^2 - {h_k}^2]$$

$D = \frac{1}{3}\sum_{k=1}^{n} [Q_{xy}]_k * [{h_{k+1}}^3 - {h_k}^3]$,k=number of layers

By using thickness coordinates from the midplane throughout the laminate h_k, h_{k+1} for each layer.

3. Calculate mechanical loading i.e., forces, (external moments [M] =0) of composite pressure vessel

$.N_x = pd/2t, N_y = pd/4t\ N_{xy} = 0$

Where P= applied pressure,

d=diameter at particular portion,

t=laminate thickness.

4. From the normal loading and moments find mid plane strains (ε_0) and curvatures (K)

$$[N] = [A]\varepsilon_0 + [B]K, [M] = [B]\varepsilon_0 + [D]K$$

5. Apply classical laminate theory to find local stresses, local strains, global stresses and global strains. Mat lab code is developed for the above algorithm to study the failure sequence of the laminate structure.

6. The failure of the laminates is predicted by using maximum stress theory and Tsai-wu theory.

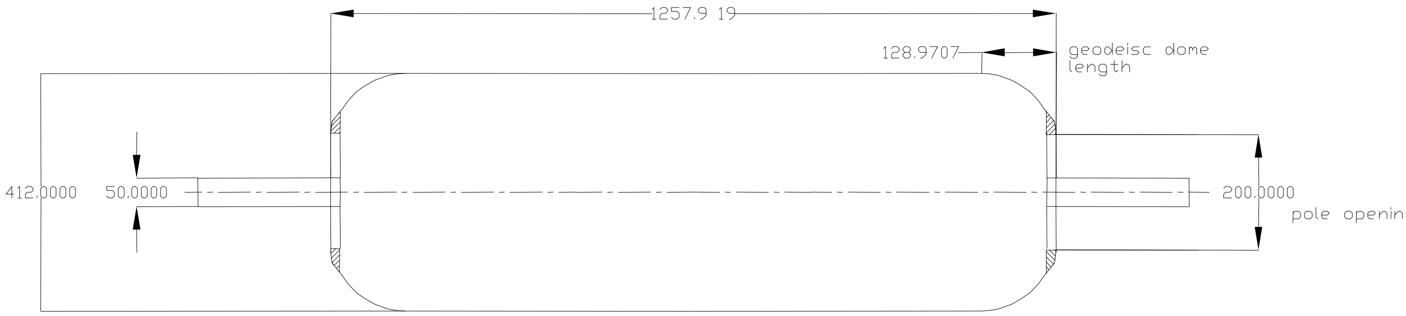

Figure 2. Collapsible mandrel with geodesic dome end (All dimensions are in mm)

Table 3. Winding layup details around the collapsible mandrel

Starting station	Ending station	radius	Winding angle	Sequence of winding pattern Around the mandrel
2.5263	20.523	122.5	54.7	$(54.7/54.7/54.7/doily/54.7/doily/54.7/54.7/doily/54.7/54.7/doily/54.7)_s$
20.523	40.3363	145	43.602	$(43.6/43.6/43.6/doily/43.6/doily/43.6/43.6/doily/43.6//doily/43.6/43.6)_s$
40.3363	52.9463	170	36.03	$(36.03/doily/36.03/doily/36.03/36.03/doily/36.03/36.03)_s$
52.9463	69.1483	180	33.748	$(33.7/doily/33.7/doily/33.7/33.7/doily/33.7/33.7)_s$
69.1483	91.663	190	31.7	[31.7/doily/31.7/doily/31.7/31.7/doily/31.7/31.7]s
91.663	113.9543	200	30	[30/doily/30/doily/30/30/doily/30]s
113.9543	128.5763	205	29.19	(29.1/doily/29.1/doily/29.1/29.1/doily/29.1)s
128.5763	1128.576	206	29.04	$(90/90/29.04/90/90/29.04/90/90/29.04/90/90/29.04/90/90/29.04/90)_s$
1128.576	1143.188	122.5	29.19	(29.1/doily/29.1/doily/29.1/29.1/doily/29.1)s
1143.188	1165.486	145	30	[30/doily/30/doily/30/30/doily/30]s
1165.486	1188.004	170	31.7	[31.7/doily/31.7/doily/31.7/31.7/doily/31.7/31.7]s
1188.004	1204.206	180	33.748	$(33.7/doily/33.7/doily/33.7/33.7/doily/33.7/33.7)_s$
1204.206	1216.816	190	36.03	$(36.03/doily/36.03/doily/36.03/36.03/doily/36.03/36.03)_s$
1216.816	1236.626	200	43.602	$(43.6/43.6/43.6/doily/43.6/doily/43.6/43.6/doily/43.6//doily/43.6/43.6)_s$
1236.626	1257.1526	205	54.7	$(54.7/54.7/54.7/doily/54.7/doily/54.7/54.7/doily/54.7/54.7/doily/54.7)_s$

3.1. Maximum stress theory

Ultimate strength of the carbon fibre epoxy layer is around 1260Mpa. Finding the strains and stresses in material coordinates which is illustrated in section-3 involves number of iterative calculations, stresses induced in the material and global coordinates are Using Mat lab program. The stress induced in the material coordinates should not exceed ultimate strength of the carbon/epoxy composite. The layer which exceeds the ultimate strength that layer fails first. (The stress induced in the layers should not exceed 1260Mpa).

3.2. Tsai-Wu theory

In this theory interaction between longitudinal, transverse, shear stress is considered. Using Tsai-Wu theory [9], [10] to find the strength ratio for all layers $a = f_{11}\sigma_{1k}^2 + f_{22}\sigma_{2k}^2 + f_{66}\tau_{6k}^2 + 2f_{12}\sigma_{1k}\sigma_{2k}$, $b = f_1\sigma_{1k} + f_1\sigma_{1k}$

Strength ratio$=\frac{-b\pm\sqrt{b^2+4a}}{2a}$, $f_1 = \frac{1}{(\sigma_1^T)_{ult}} - \frac{1}{(\sigma_1^C)_{ult}}$,

$f_2 = \frac{1}{(\sigma_2^T)_{ult}} - \frac{1}{(\sigma_2^C)_{ult}}$, $f_{66} = \frac{1}{(\tau_{12}^2)_{ult}}$

$$f_{11} = \frac{1}{(\sigma_1^T)_{ult}(\sigma_1^C)_{ult}}, f_{22} = \frac{1}{(\sigma_2^T)_{ult}(\sigma_2^C)_{ult}}$$

$$f_{12} = -\frac{1}{2}\sqrt{\frac{1}{(\sigma_1^T)_{ult}(\sigma_1^C)_{ult}(\sigma_2^T)_{ult}(\sigma_2^C)_{ult}}}$$

$(\sigma_1^t)_{ult}$=Ultimate longitudinal tensile strength

$(\sigma_2^t)_{ult}$= Ultimate transverse tensile strength

$(\tau_{12})_{ult}$ =Ultimate in-plane shear strength

σ_1, σ_2, τ_{12} are the local stresses induced in the lamina

Identify the minimum value of strength ratio of all the layers. Strength ratio gives the maximum value of the normal load. Strength ratio divided by the thickness of corresponding layer gives the allowable normal stress of the laminate.

4. Operating and failure conditions of cylindrical and Geodesic dome

Tsai –Wu theory is the best theory to predict the failure of composite structure. The present structure consists of epoxy impregnated carbon fibres which are oriented with different fibre angles. At the Cylindrical portion 22 hoop layers and 10 helical layers are required. The total thickness provided at the cylindrical 8.9mm. Symmetric laminate is selected for the present study. The helical winding angle is 29.04⁰ and hoop winding angle is 90⁰ both are constant throughout the cylindrical portion. Form the failure analysis (discussed in section-3), we can say that the operating pressure of the cylindrical portion is 25MPa and the ultimate failure pressure of the cylindrical portion is 32Mpa. Dome portion consists of different winding angles at different portions. The cross section of the dome consists of increase of thickness from the cylindrical portion to the polar opening. Slope of the contour is different at each point on the dome. This reason causes the more thickness of the composite laminates at the vicinity of the polar opening. The details of the winding angle at different portions of the dome are given in the table.3. The dome contour consists of different laminate structures with different winding angles, with different thicknesses. We need to find the laminate which is going to fail first under the given pressure. The dome portion the failure of first ply occurs at 21.5Mpa at radius 200mm. At this portion 10 helical, 6doilies, total thickness 10.5mm are provided. The Operating pressure of the dome portion is 20Mpa. The complete failure of the dome portion occurs at a pressure of 25MPa.

5. Finite element analysis of Geodesic dome portion

Finite element analysis of the Geodesic dome portion is carried out by considering the structure as orthotropic laminate structure. Shell 99 is selected as element type for the present study [8], [7], [2]. The layer sequence at the dome portion, along with orientation, thickness is completely specified in the Table 3. The below Table 4 represents the material properties of carbon epoxy laminate.

Table 4. Properties of Carbon/epoxy ($V_f = 60\%, V_m = 40\%$)

Properties of carbon/Epoxy	Values
E_x	139Gpa
$E_y = E_z$	9.8Gpa
$G_{xy} = G_{yz}$	4.818Gpa
G_{zx}	4.818Gpa
$\vartheta_{xy} = \vartheta_{yz}$	0.3
ϑ_{zx}	0.021

Where V_f= volume fraction of fibres.
V_m= volume fraction of matix.

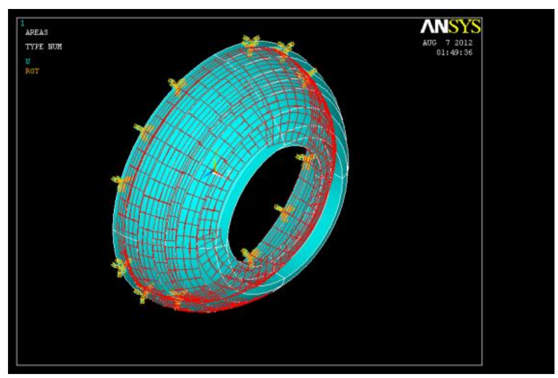

Figure 3. Finite element model of Geodesic dome portion

After specifying the layer configurations select the proper boundary conditions and apply the load in cyclic pattern. The failure of the Geodesic dome portions is predicted by using maximum stress theory, which is illustrated in section 3.1. Tsai Wu theory and maximum stress theory gives approximately equal results. From the maximum stress theory the burst pressure of the structure is identified as 22Mpa.

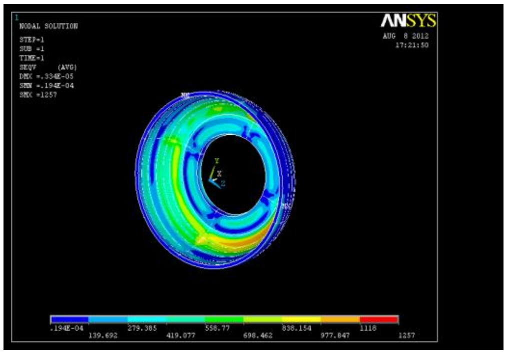

Figure 4. The stress distribution of the Geodesic dome at its failure pressure

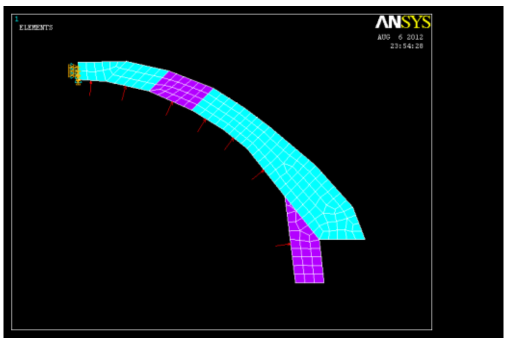

Figure 5. Finite element model of half section (axis-symmetric) of Geodesic dome portion

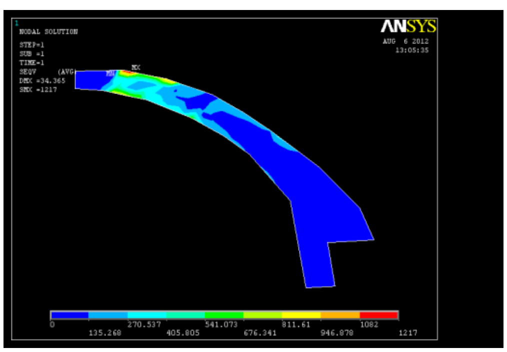

Figure 6. The stress distribution of the half section (axis-symmetric) of Geodesic dome at its failure pressure

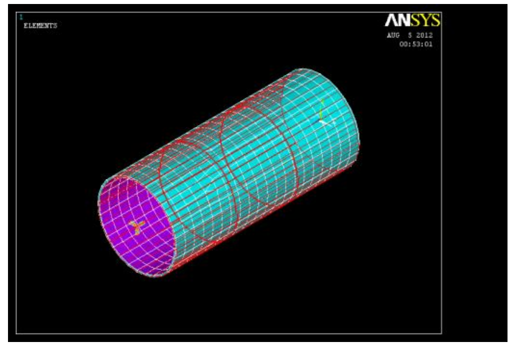

Figure 7. Finite element model of cylindrical portion of composite pressure vessel

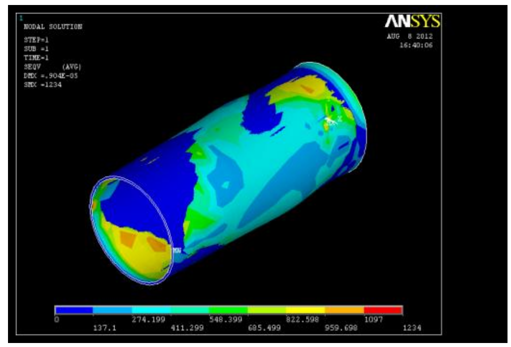

Figure 8. The stress distribution of cylindrical portion at failure pressure

All the above Figures represent finite element model and the stress distribution of the cylindrical and Geodesic dome portion at failure pressure.

Results and Discussions

The failure pressure of Geodesic dome portion is predicted by using finite element analysis software (Ansys 12.0), and classical laminate theory. The failure of the structure starts from the pressure of 21.5Mpa and the complete failure of the structure occurs at 25Mpa. The operating pressure of the structure should not exceed 21Mpa.Cylindrical portion consists of hoop layers. The layers with fibre orientation 90^o participate more actively in load bearing; the fibres which are oriented in the angle other than 90^o also participate in load bearing but lower than the hoop layers. This gives us a clear idea about predicting the failure of the layers. Increasing the pressure load beyond the limit causes the failure of hoop or circumferential layers first; this leads the total failure of the structure. The dome portion also consists of doilies oriented in 0^o, and 90^o, and helical layers. Here also hoop layers bear maximum load, and the next maximum load bearing layers are 0^o

layers. For increasing the load bearing capacity of the composite structure we must increase the circumferential layers at the cylindrical portion, and doilies at the dome portion. The failure sequence of the layers associated with different fibre orientation is as follows: the fibres with 90^o orientation fails first, next the failure of the 0^o layers takes place, lastly the failure of the helical layers takes place which leads the complete failure of the structure.

Table 5. Operating and failure pressures of composite structure

Pressure	Operating pressure	Failure pressure
Cylindrical portion	25Mpa	Failure Starts from 25Mpa, complete failure occurs at 32Mpa
Dome portion	21Mpa	Starts from 21.5Mpa and complete failure occurs at 25Mpa(at radius 200mm)
Composite structure	20Mpa	25Mpa

The above Table. 5 represent the operating and failure conditions of Geodesic dome as well as cylindrical portion.

The variation of the induced stresses in the corresponding layers is shown in the below Figures. 9, 10. The failure stresses induced in the layers which are oriented with different winding angles are calculated by using Mat lab code (which involves iterative calculations discussed in section-3) are represented in the following Figures.

From the graphs indicated in the following Figures. 9,10 we can clearly estimate the failure sequence of layers. The stress induce in the hoop layers is exceeding the ultimate strength of the layers (1260Mpa), from the maximum stress theory it is clear that the failure of the hoop layers will takes place first.

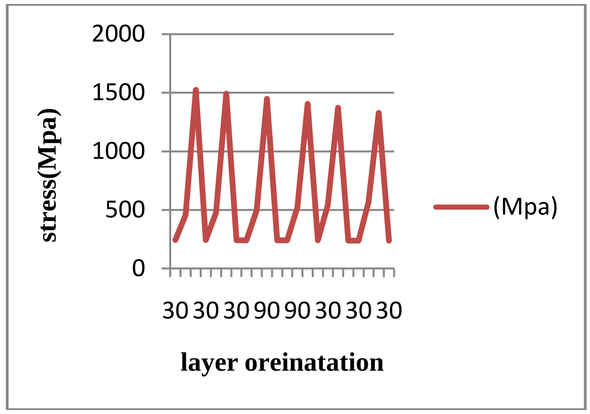

Figure 9. Failure stresses for each layer at dome portion

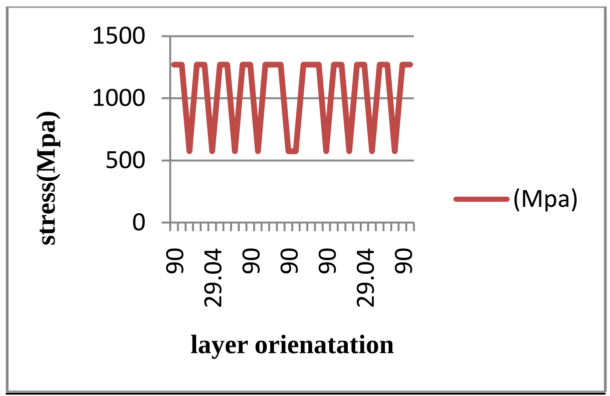

Figure 10. Failure stresses for each layer at cylindrical portion

References

[1] V. Varley Vasiliev, V. Evgeny Morozov, *"Mechanics and Analysis of Composite Materials*", 1st Edition, Elsevier Science Ltd., 2001.

[2] Nagesh, "Finite-element Analysis of Composite Pressure Vessels with Progressive Degradation", *Defence Science Journal*, Vol. 53, No. 1,pp. 75-86, 2003.

[3] Valery V. Vasiliev *"Composite Pressure Vessels: Analysis, Design, and Manufacturing "*.,2009 edition.

[4] M. Madhavi, K.V.J.Rao and K.Narayana Rao,"Design and Analysis of Filament Wound Composite Pressure Vessel with Integrated-end Domes". *Defense Science Journal*, Vol. 59, No. 1, January 2009, pp. 73-81.

[5] DOT CCFC standards.

[6] R.R. Chang, "Experimental and theoretical analyses of first-ply failure of laminated composite pressure vessels", *Composite Structures*,Vol. 49, pp. 237-243, 2000.

[7] Wang (Yingjun School of Science, Wuhan University of Technology), "Finite Element Modeling of Carbon Fiber Reinforced Polymer Pressure Vessel" *2010 International*

Conference on Educational and Network Technology (ICENT 2010).

[8] Mohammad Z. Kabir(Department of Civil Engineering, AmirKabir University of Technology, Tehran, Iran),"Finite element analysis of composite pressure vessels with a load sharing metallic liner" *Elsevier Science Ltd*.Composite Structures 49 (2000) 247-255.

[9] Isaac M., Daniel, Ori Ishai , *Engineering mechanics of Composite materials* Oxford Universiy Press(1994)

[10] Autar Kaw, *Mechanics of Composite Materials* second edition (2006).

[11] Vasiliev, V.V., Krikanov, A.A., & Razin, A.F. (2003). "New Generation of Filament -Wound Composite Pressure VesselsforCommercialApplications", *CompositeStructures, 62*, 449-459.