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Design of Artificial Neural Networks Controller for Stabilization of Ideal Juggler

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***Abstract*—An ideal juggler system which is nonlinear, stabilized around equilibrium point using controllers aided by artificial neural networks. Two control approaches are used. One is based on linear controller and another one is on nonlinear controller. Linear controller, controls the linearized plant around equilibrium, where as nonlinear controller considers the original nonlinear model of the plant. Both linear and nonlinear controllers are aided by neural networks.**

***Index Terms*—Artificial neural networks, nonlinear control, stabilization, approximation by neural networks.**

I. INTRODUCTION

THE theory of linear systems is well developed and linear controllers for linear systems are studied at vast. But in case of nonlinear plants the controllers are mostly plant specific. There are no general methods which can be applied to all class of systems. Neural networks are one such alternative for the control of nonlinear plants. In this work the nonlinear plant considered is ideal juggler. It will be stabilised around equilibrium point using neural controllers. For stabilizing the ideal juggler two controllers were developed and simulated. In the first approach nonlinear plant is linearised around the equilibrium and a linear controller is applied. In the second approach a neural network is made to approximate nonlinear control law of the plant and used as a controller.

II. CONTROL PROBLEM OF IDEAL JUGGLER

In this example we consider the task of juggling a ball using a flat board. We make the simplifying assumption that the ball moves in a two dimensional plane. We also assume that there are no losses in the system, that the ball follows a perfect ballistic curve while in the air, and that the collision with the board is elastic and always at the same height. The states of the system are given by the angle of the ball (ψ) and its horizontal location (ρ) just before impact. The direction of the ball is controlled by setting the angle of the board (α), as the control input.

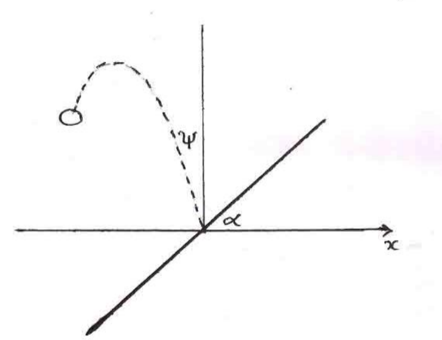

Fig.**1 Shows *the ideal juggler block diagram***

The state space equations of the system are:

$$\rho(k+1) = \rho(k) + \frac{2E}{mg}\sin\ 4\alpha(k) - 2\psi(k) \qquad (1)$$

$$\psi(k+1) = \psi(k) - 2\alpha(k)$$

k	count of impacts.
ρ	horizontal location of impact.
φ	angle at time of impact(with respect to vertical).
α	angle of board(with respect to horizontal), the control input.
E,m	enrgy, mass of the ball.

The origin (ρ=0, ψ =0), ball bounced vertically at center of board is an equilibrium state.

III. STABILIZATION OF JUGGLER

Consider the discrete time dynamical system described as

$x(k+1) = f[x(k),\ u(k)]$

$y(k) = h[x(k)]$

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The problem of controlling a plant, can be conveniently divided into the regulation and tracking problems. In the former, the main objective is to stabilize the plant around

a fixed operating point. In the latter, the aim is to make the output of the plant follow a specified signal asymptotically. While our ultimate goal is to determine the control input U based only on output measurements for both regulation and tracking, we will confine our attention in this work to the problem of regulation when the state of the system is accessible. This implies that our interest is only in the system described by the first part of (1), i.e., In the present work we discuss the stabilization of the nonlinear system represented by (1) around an equilibrium state.

$$\Sigma : \; x(k+1) = f(x(k), u(k))$$

The stabilization is done by two methods

(i) linear controller

(ii) nonlinear controller

The design of linear controller is based on the linearization of nonlinear system around the equilibrium point around which stabilization has to be done. After that a direct nonlinear controller is designed which produces control input to the plant based on the state feedback. Here the relation between state feedback and control input is nonlinear. The role of neural network in control is approximating plant and controller behaviour. The details of implementing linear and nonlinear controllers using neural network are discussed in following chapters.

A. Stabilization of Juggler using Linear Controller

Linearization of Nonlinear plant

A nonlinear system represented by equ (2) can be linearized around any equilibrium point (usually origin). The lineralized equation is given by

$$\Sigma_L : \; \delta x(k+1) = A\delta x(k) + b\delta u(k)$$

where $A = f_x|_{0,0}$ *and* $b = f_u|_{0,0}$ which are simply jacobians of *f* wrt to *x* and *u*

The lineralized equation of ideal juggler at the origin is

$$\begin{bmatrix} \delta\rho(k+1) \\ \delta\psi(k+1) \end{bmatrix} = \begin{pmatrix} 1 & \frac{-4E}{mg} \\ 0 & 1 \end{pmatrix} \begin{bmatrix} \delta\rho(k) \\ \delta\psi(k) \end{bmatrix} + \begin{bmatrix} \frac{8E}{mg} \\ -2 \end{bmatrix} \delta\alpha(k)$$

The controllability matrix M_C is given by

$$M_C = b \,|\, Ab = \begin{bmatrix} \frac{8E}{mg} & \frac{16E}{mg} \\ -2 & -2 \end{bmatrix}$$

Since *Mc* is full of rank, the system can be locally controlled from any initial point (ρ_i, ψ_i) to any other point in at most two steps.

The simplest scheme for stabilization is by the use of a linear controller. Let Σ_L be a linearization of (2) around an equilibrium point x= 0. If Σ_L is controllable, then linear theory tells us that there exists a linear feedback law $\boldsymbol{u}=\boldsymbol{K}^T\boldsymbol{x}$ that stabilizes Σ_L around the origin. Since Σ_L is the first order approximation of the original nonlinear system, one might expect that the same linear feedback law will locally make the origin an asymptotically stable point of (2)

Implementation of Linear Controller using Neural Network

Given the state space model of the nonlinear plant, the plant is approximated using feedforward neural network $NN_f(.)$. The *A, b* matrices of the liberalized plant are just the Jacobians of $NN_f(.)$ with respect to the states and the inputs. Once these are calculated, with a linear feedback law $\boldsymbol{u} = \boldsymbol{K}^T\boldsymbol{x}$, the linearized system is given by

$$\delta x(k+1) = A\delta x(k) + bK^T \delta u(kv)$$
$$=(A+bK^T)\delta x(k)$$

and it will be asymptotically stable if the eigen values of $A+bK^T$ lie inside the unit circle of the complex plane. With a feedback law chosen in this manner, the above theorem assures that the nonlinear system will also be locally asymptotically stable.

Though the linear controller will stabilize the nonlinear system around the origin, the range over which the system will be stable depends upon the system and may be small for nonlinear systems. Thus one hopes that by employing an appropriate nonlinear controller, the range over which the system is stabilized can be increased. The following sections address the issue of nonlinear controllers, and the linear controller is used as a benchmark for the evaluation of the performance of more sophisticated controllers.

B. Stabilization of Juggler by Nonlinear Feedback

Concept of Nonlinear State Feedback Controller

Let $\sum$ be the nonlinear dynamical system (2) and Σ_L it's linearization around the origin. If the linearized system is controllable then there exists a neighborhood $V_x \subset X$ around the origin and a continuous feedback law $u(k) = g[x(k)]$ that will make *V* n-step stable.

Now, assume the system was started at *x(0)* = x1. Since x1, the sequence of inputs *u(k)*= g k(x1) will drive it to the origin in *n* steps. On the other hand, the original input sequence *u(k)*= gk+1(xo) will drive it to the origin in *n* - 1

steps. The origin, however, is an equilibrium state (with zero input the system will remain at the origin). Thus the input sequence (gl(xo),g 2(xo),. . .gn-1(x0),0) will also drive x1 to the origin in ***n*** steps. But for any x the input sequence that drives it to the origin in *n* steps is unique and thus we get that go(x1) must be equal to gl(x0).
The same reasoning, applied to each of the ***xi,*** will lead to go(xi) = gi(xo*)*. Hence, for any x E *Vx*, the system

x(k + 1) = f[x,go(x)]

will converge to the origin in at most *n* steps. The equivalent result for a linear systems *x(k + 1) = Ax(k) + bu(k)* is that using state feedback the $u=K^Tx$ is combined matrix $\bar{A}$ = *A* + bk^T is made nilpotent. For a two-dimensional canonical system

$$x(k+1)=\begin{pmatrix}1 & 0\\ a_1 & a_2\end{pmatrix}x(k)+\begin{pmatrix}0\\1\end{pmatrix}u(k)$$

Choosing $k^T = a_1 \quad a_2$ the state feedback becomes $u(k)=-a_1x_1(k)-a_2x_2(k)$ and that will bring the system to the origin in at most two steps. A controller that stabilizes a system around a point in finite time is called a *dead beat controller.*

Again the juggler's equations are given by

$$\rho(k+1)=\rho(k)+\frac{2E}{mg}\sin\ 4\alpha(k)-2\psi(k) \qquad (2)$$
$$\psi(k+1)=\psi(k)-2\alpha(k)$$

From above theorem it follows that the system can be stabilized around the origin using a nonlinear feedback control. In fact, this will be accomplished by the control law.

$$\alpha(k)=g[\rho(k),\psi(k)]=\frac{1}{4}\left[2\psi(k)-\sin^{-1}\left(\frac{mg}{2E}\rho(k)\right)\right]$$

Implementing Nonlinear Controller using Neural Network

In this implementation the nonlinear feedback law *u=g(x)* is also approximated by neural network NN_g. Hence nonlinear controller requires two neural networks , one approximating plant and the another one which approximates nonlinear control law. For the above said method to be applied, the rank condition needs to be checked. This is done by determining the Jacobian of NN_f with respect to the inputs at the equilibrium point. Let $\hat{A}$ and $\hat{b}$ be defined as

$$\hat{A}=\frac{\partial NN_f\ \ x,u}{\partial x}\,|\,0,0' \quad \hat{b}=\frac{\partial NN_f\ \ x,u}{\partial u}\,|\,0,0 \quad (3)$$

Using the matrices *A* and *b,* the rank of the model's controllability matrix *Mc,* is checked.

Let *Mc* be of full rank. Let *S* denote the region of interest in which we wish to stabilize the system. Our goal is to train a neural network NN_g **,** as a controller of (2) that will make S finitely stable with respect to the origin. The results developed earlier establish that there exists a control law *u = g(x)* for which the following is true. (i) There exists an open set *V* containing the origin such that for all w⊃ *V,* F(z) = 0.(ii) There exists a larger open set *W 2 V* such that for all x E *W, F(z)* is a contraction mapping.
Based on these results, the performance of a controller can be evaluated only in intervals of *n* steps. We assume that a control law can be determined so that *W* covers *S.* Though our ultimate goal is to stabilize the actual system, the training of the controller is done using the model, and thus we can assume arbitrary initial conditions. The latter are selected using a random uniform distribution over S. Let

$$NN_{f,g}(x)=NN_f\left[x,N_g(x)\right] \qquad (4)$$

Once an initial point xo is chosen, $x_n=NN^n_{f,g}(x_0)$ is calculated by running the controlled model *n* steps. Since it is only for x E *V* (which is unknown) that the system can be brought to zero in *n* steps, the training error for the controller must be chosen as follows

IV. Results

The proposed control algorithms were simulated using MATLAB software. The figures 2,3,4 show the stabilization of states when linear controller is used and the figures 5,6,7 show the stabilization of states of ideal juggler towards origin for both the cases All the states were stabilized at equilibrium which is origin. The initial state is chosen at random near the origin

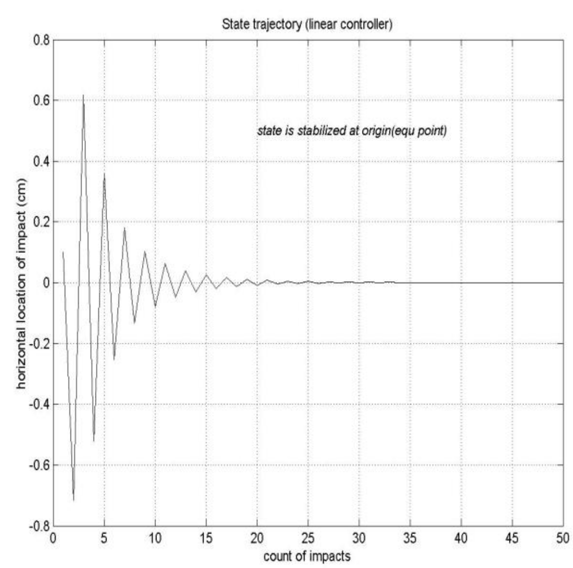

Fig.2 shows the stabilization of states when linear controller(k vs ρ).

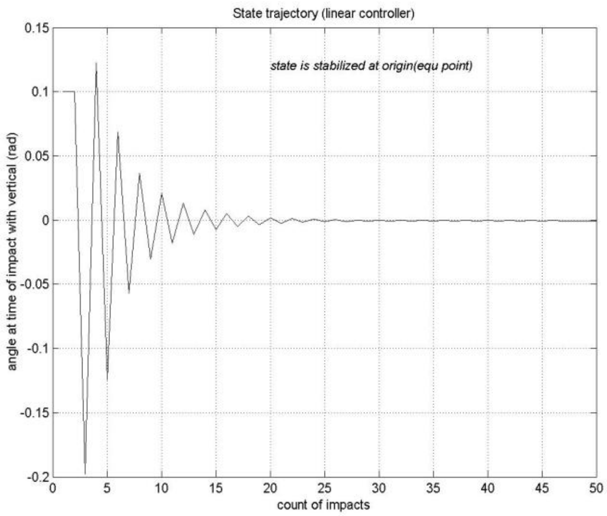

Fig.3 shows the stabilization of states when linear controller(k vs φ).

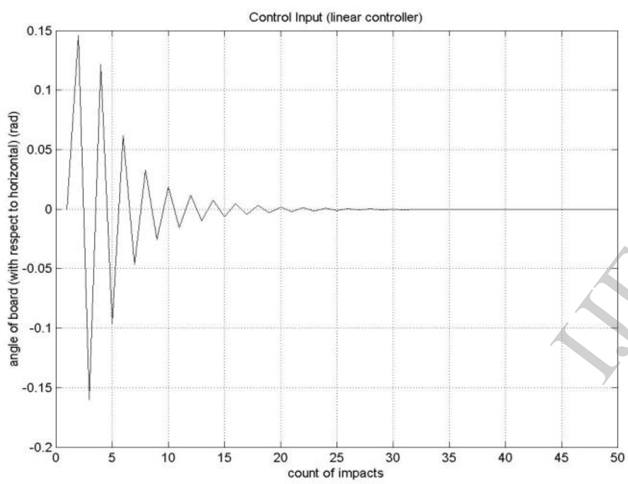

Fig 4 shows the stabilization of states when linear controller(k vs α).

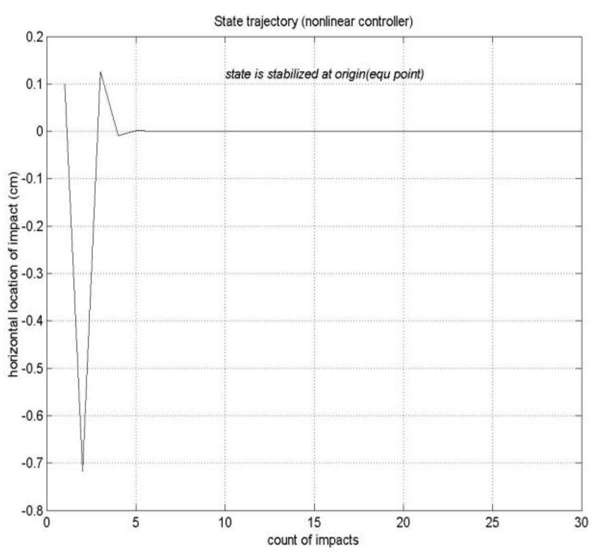

Fig.5 shows the stabilization of states of ideal juggler towards origin (k vs ρ).

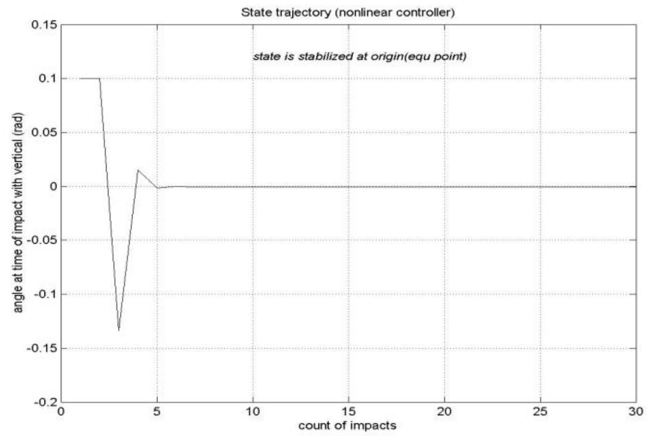

Fig.6 shows the stabilization of states of ideal juggler towards origin (k vs φ).

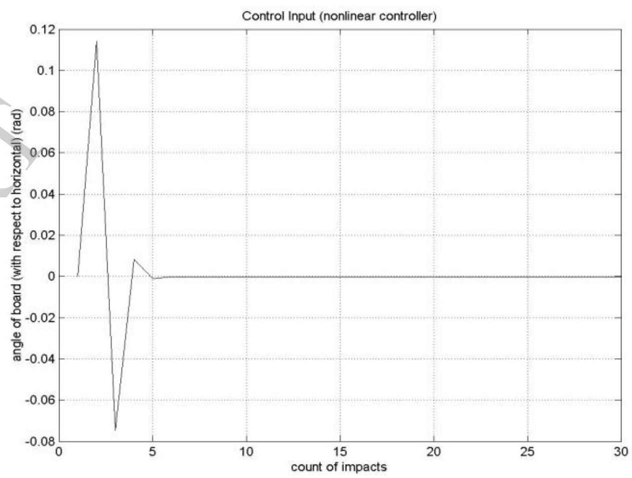

Fig.7 shows the stabilization of states of ideal juggler towards origin (k vs α).

V. CONCLUSION

The nonlinear system can be controlled with the aid of neural networks directly and indirectly by linearising the nonlinear point around equilibrium point. The universal approximation capability of neural networks can be used in aiding the controllers

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