

# Design of Cascade Control System Based on Sustained Oscillation

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## Abstract:

The aim of this paper is to eliminate the load or static disturbance in the response. Initially, the inner loop and outer loop controllers of cascade system are designed using Zeigler Nichols tuning method. Relay is connected to the outer loop controller without affecting the performance of the cascade control system. The new controller parameters are found by Simple IMC tuning method from sustained oscillation. The simultaneous action of new controllers results in complete elimination of disturbance.

**Keywords:** disturbance, IMC, relay, Tuning.

## 1. Introduction

Cascade control systems are mainly used for eliminating disturbance. The controllers of the cascade system can be designed in many ways. Simply, Zeigler Nichols method is used[5]. Inner controller should be faster than outer controller. So conventionally, proportional controller is used for inner control, proportional integral or proportional integral and derivative controller is used for outer control. Conventional cascade control system is ineffective, because loop interaction is neglected and also controllers are tuned sequentially. Two point method[5] is used for outer loop controller design. So it takes more time. Hence we go for sustained oscillation method. In this method relay is connected to the master controller. From the output of inner loop and outer loop, some simple calculations are done. Using these calculations, controllers are designed by Simple or Skogestad IMC (SIMC) tuning method[3]. This method can be used for any cascade control systems such as continuous stirred tank reactor or drum boiler. Here the controllers are tuned simultaneously. It is a single step process. The disturbance may be water level fluctuations or valve position changes.

These changes are eliminated by proposed controllers.

## 2. System description:

### 2.1 Cylindrical tank system:

The liquid which is pumped fills the process tank according to the percentage of opening of the inlet control valve. The tank has two openings at the top, one to permit water inflow and the other for level switch. At the bottom end two drain paths are placed to facilitate water outflow through the manual hand valve and solenoid valve.



Figure 1: Experimental Setup of a cylindrical Tank level system

The desired level 'h' is maintained by manipulating the inlet flow rate 'q1' to the system. Thus 'h' is the controlled variable and 'q1' is the manipulated variable.

### 2.2 Block diagram:

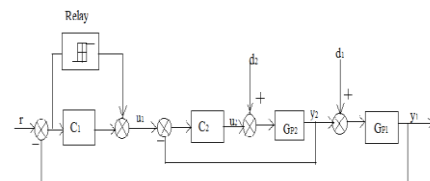


Figure 2: Block diagram of proposed method

In the proposed method, controllers are designed from sustained oscillation[2]. For getting these oscillations, a relay is connected to the inner controller. The disturbance can be eliminated effectively by SIMC tuning method.

**2.3 Process model**

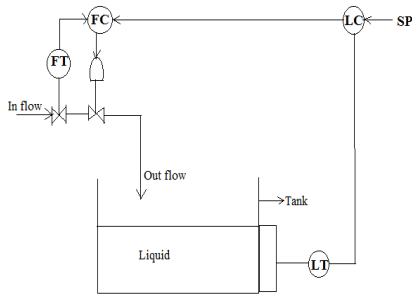


Figure 3: Process model of proposed system

Where

- FT :Flow transmitter.
- FC :Flow controller.
- LT :Level transmitter.
- LC :Level controller
- SP :Set point

The figure 3 shows process diagram of cascade control system consisting of two controllers in which one controller's output drives the set point of another controller. The controller driving the set point is called the primary, outer or master controller. The controller receiving the set point is called the secondary, inner or slave controller. Level controller driving the set point of a flow controller to keep the level at its set point. The flow controller, in turn, drives a control valve to match the flow with the set point, the level controller is requesting.

**3. Mathematical modeling**

**3.1 Level process**

The cylindrical tank is the process considered which is given in figure 4.

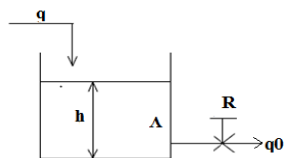


Figure 4: cylindrical tank

- $\rho$  :density of liquid in the tank Kg/cm<sup>3</sup>
- $\rho_1$  :density of liquid in the inlet stream Kg/cm<sup>3</sup>
- $\rho_2$  :density of liquid in the inlet stream Kg/cm<sup>3</sup>
- $V$  :the total volume of the cylindrical tank cm<sup>3</sup>
- $q_0$  :volumetric flow rate of inlet stream LPH
- $q$  :volumetric flow rate of outlet stream LPH
- $h$  :Height of the cone at steady state cm

Using the law of conservation of mass,

$$\left[ \begin{matrix} \text{accumulation of} \\ \text{total mass} \\ \text{time} \end{matrix} \right] = \left[ \begin{matrix} \text{input of} \\ \text{total mass} \\ \text{time} \end{matrix} \right] - \left[ \begin{matrix} \text{output of} \\ \text{total mass} \\ \text{time} \end{matrix} \right]$$

$$\frac{d(\rho V(t))}{dt} = \rho_1 q(t) - \rho_2 q_0(t)$$

Assume that the room temperature as well as the density of liquid is constant,  $\rho = \rho_1 = \rho_2$ .

The volume of cylinder  $A \cdot h$ .

Where,  $q_0 = \frac{h}{R}$

The transfer function model for level is

$$\frac{Q(s)}{H(s)} = \frac{R}{1 + \tau S}$$

Where,  $\tau = AR$

The transfer function obtained from the open loop response of level is  $\frac{3.21}{1.91s + 1}$ .

**3.2 flow process**

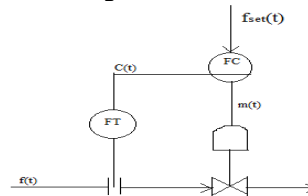


Figure 5: flow process

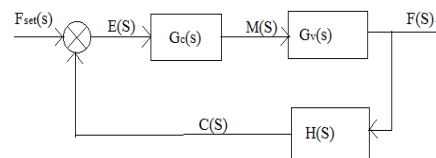


Figure 6: block diagram of flow process

- $F_{set}(S)$  : Flow input(set point)
- $F(S)$  : Flow output
- $G_c(s)$  :Transfer function of the controller
- $G_v(s)$  : Transfer function of the valve
- $H(s)$  :Transfer function of the transmitter
- $E(s)$  :Error signal

From the figure 6, loop transfer function will be,

$$\frac{F(s)}{F_{set}(s)} = \frac{G_v(s)}{1 + k_t G_v(s) G_c(s)}$$

$$\text{Where, } G_v(s) = \frac{F(s)}{M(s)} = \frac{k_v}{\tau_v s + 1}$$

$$G_c(s) = k_c$$

The transfer function model for flow is

$$\frac{F(s)}{F_{set}(s)} = \frac{k}{\tau s + 1}$$

The transfer function obtained from the open

$$\text{loop response of flow is } \frac{0.575}{180s + 1} e^{-42s}.$$

Specifications of cylindrical tank:

Height	: 80 cm
Volume	: 25.7 litres
Diameter	: 25 cm
Material	: Stainless Steel

#### 4. Conventional cascade controller design

For conventional cascade control system, usually P and PI controllers are used. So firstly, open loop test should be done for both level and flow process. This is actually for controller design. From the open loop flow response, find the tuning parameters of the P controller. Here Zeigler Nichols tuning method is used for controller design.

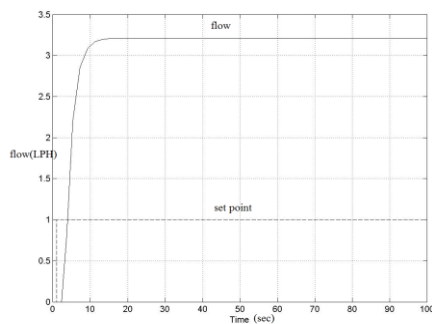


Figure 7: open loop flow response

From the figure 7,

$$K_p = 3.21$$

$$0.283 \text{ of } K_p = 0.9339$$

$$0.6325 \text{ of } K_p = 2.087$$

$$t_1 = 4 \text{ sec}$$

$$t_2 = 6 \text{ sec}$$

$$t_d = t_2 - T = 3 \text{ sec}$$

Time constant,

$$T = [t_2 - t_1] * 1.5 = 3 \text{ sec}$$

P mode;

$$K_c = T / [t_d * K_p] = 3 / [3 * 3.3] = 0.303$$

From the open loop level response, find the tuning parameters of the PI controller. Here Zeigler Nichols tuning method is used for controller design[5].

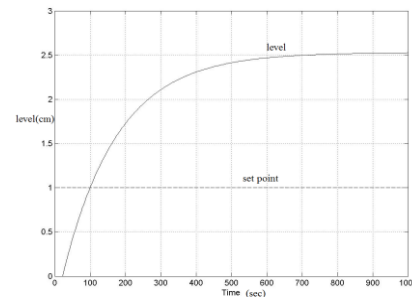


Figure 8: open loop level response

From the figure 8 we can find,

$$K_p = 0.283$$

$$0.283 \text{ of } K_p = 0.080089$$

$$0.6325 \text{ of } K_p = 0.1790$$

$$t_1 = 102 \text{ sec}$$

$$t_2 = 222 \text{ sec}$$

$$t_d = t_2 - T = 42 \text{ sec}$$

Time constant,

$$T = [t_2 - t_1] * 1.5 = 180 \text{ sec}$$

PI mode;

$$K_c = 0.9T / [t_d * K_p] = 13.6294$$

$$T_i = 3.33 * t_d = 139.986$$

$$K_i = K_c / T_i = 0.09736$$

Using this value, we will stabilize the process, and also find the controller parameters by using relay[3]

**5. SIMC based cascade controller tuning[1]:**

The table 1 shows the tuning rule for cascade controller. Initially we are using P controller for flow process and PI controller for level process by Zeigler Nichols tuning method. After stabilizing the process, relay based tuning method is used[3].

PI controller		PID controller		
$K_{p2}$	$T_{i2}$	$K_{p1}$	$T_{i1}$	$T_{d1}$
$\frac{0.5\tau_2}{K_2\theta_2}$	$\tau_2$	$\frac{0.5\tau_1}{K_1(\theta_1+\theta_2)}$	$\tau_1$	$\theta_2$

Table 1: Tuning rule for cascade controller

Where the controller parameters[4] are given by,

$$k_1 = \frac{-\tau_1 y_1(t_2)}{h_{y2}}$$

$$k_2 = \frac{1}{k_{p2}} \left[ \frac{h_{y2}}{h - h_{y2}} \right]$$

$$\tau_2 = \frac{2a}{b + \sqrt{b^2 - 4ac}}$$

$$\tau_1 = - \frac{t_2 - t_p}{\ln \left[ \frac{k_1 h_{y2}}{y_1(t_p) + k_1 h_{y2}} \right]}$$

Connecting relay to the master controller such as level controller, we obtained the response as shown in the below figure 8.

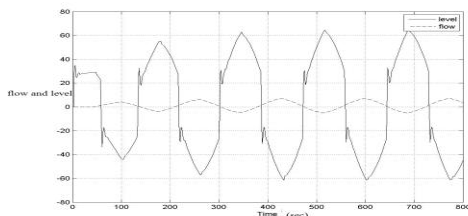


Figure 9: relay based cascade control system response

From the relay based cascade control system[1] response, the parameters can be calculated as,

$$\begin{aligned} h &= \pm 40 \\ t_0 &= 42 \\ t_1 &= 60 \\ t_p &= 100 \\ t_2 &= 135 \\ \theta_1 &= t_p - t_1 = 40 \\ \theta_2 &= t_1 - t_0 = 18 \end{aligned}$$

$$\begin{aligned} y_2(t_1) &= 28 \\ y_1(t_p) &= 4.25 \end{aligned}$$

$$\begin{aligned} y_1'(t_2) &= -0.12 \\ y_2(t_1)' &= -25 \end{aligned}$$

PI Controller parameters are,

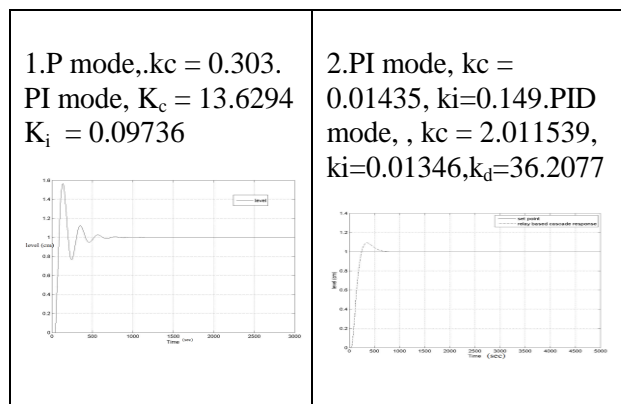
$$\begin{aligned} k_{p2} &= 0.01435 \\ T_{i2} &= 3.7790 \\ K_{i2} &= 0.0037973 \end{aligned}$$

PID controller parameters are,

$$\begin{aligned} k_{p1} &= 2.011539 \\ T_{i1} &= 149.36 \\ k_{i1} &= 0.01346 \\ T_{d1} &= 18 \\ k_{d1} &= 36.2077 \end{aligned}$$

Where  $K_{p2}$  and  $K_{i2}$  are the proportional gain and integral gain of flow controller respectively. The proportional, integral and derivative gains of PID are  $K_{p1}, K_{i1}, K_{d1}$  respectively.

**6. Results and discussion:**



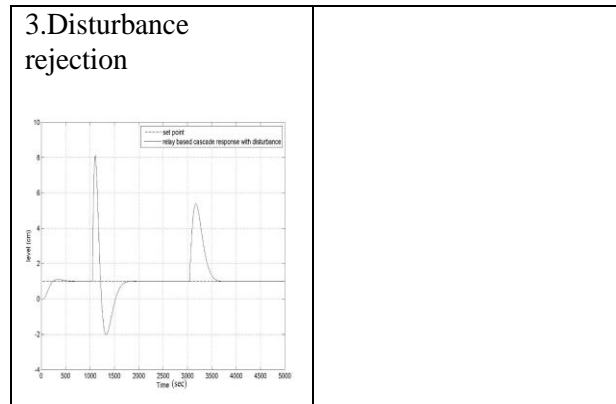


Figure 10: Responses of cascade systems

The conventional P and PI controllers for cascade system are designed using Zeigler Nichol's tuning method. The conventional P controller parameter is  $k_c = 0.303$  and PI controller parameters are,  $K_c = 13.6294$ ,  $K_i = 0.09736$ . The relay based cascade control system is designed by SIMC tuning method and its PI parameters are,  $k_c = 0.01435$ ,  $k_i = 0.149$  and PID controller parameters are  $k_c = 2.011539$ ,  $k_i = 0.01346$ ,  $k_d = 36.2077$ . The relay based cascade control system can eliminate the disturbance present in the inner loop and outer loop.

### 6.1.1 Explanation:

The conventional cascade control tuning method is a two step process and it requires more time. The response of the conventional cascade control system is sluggish in nature. And also, the tuning method is sequential. But, the relay based cascade control system is single step process. The tuning method for relay based system is simultaneous in nature. Its response have less overshoot and less undershoot.

## 7. Conclusion and Future Work:

The proposed method is used for tuning the controllers simultaneously. The tuning method of conventional cascade control system is done sequentially. Sequential tuning method is a time consuming task. Some simple calculations are done using the relay based response. The main advantage of this project is eliminating the effect of load disturbance in the inner and outer loop of

the cascade control system. Compared to conventional cascade control system, relay based method is more effective and not time consuming. Future work is the real time implementation of cascade control system based on sustained oscillation for on line identification using LabVIEW.

## 8. References:

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