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Design of FRP (Fiber Reinforced Polymer) For Bending Moment, According To European Normatives, For Reinforcement of Flexural Reinforced Concrete Elements

Igli Kondi
M.Sc. Civil Engineer
Polytechnic University of Tirana, Faculty of Civil Engineering, Tirana, Albania

Julian Kasharaj
M.Sc. Civil Engineer
Polytechnic University of Tirana, Faculty of Civil Engineering, Tirana, Albania

Elvis Capo
Civil Engineer
Polytechnic University of Tirana, Faculty of Civil Engineering, Tirana, Albania

Abstract

Reinforcement is the process of strengthening existing structural members in order to make them, or all the structure, resistant to a new system of action that was not taken in consideration during the design period. This article is focused on FRP method of reinforcement. As we know, nearly all the members on an engineering object are subject of flexural condition. Taking in consideration the importance of bending moment in the structure, this article bring a design procedure of FRP for bending moment, as a reinforcement method for a RC member. This design procedure is according the European Normatives and all the approaches are made using the ultimate limit state design.

Keywords-*reinforcement, FRP, bending moment, Eurocode 2*

1. Introduction

In order to design FRP for bending moment, at first some assumption about how the elements react under flexural condition needs to be made. To fully understand the behaviour of a RC member reinforced with FRP the ways of failure of this member must be determined. Along this article, a stress-strain relationship of RC member under the bending moment action is given. With the use of equilibrium equations written for the element cross section the design formula for FRP design are founded. At the end of the article, to create a better idea of FRP design for bending moment, a solved case of a beam under flexural condition is provided.

2. Basic Assumption

Some of the basic assumptions taken in consideration during design procedure are:

- concrete works only in compression zone
- plain section before deformation remain plain even after deformation
- Hook's law take place
- FRP are placed in tensile zone
- FRP are perfectly attached to concrete
- The material used for attaching the FRP to concrete doesn't affect the material behaviour
- FRP behaviour is the same as reinforcing bar
- Existing tensile steel bars area could or could not be taken in consideration
- Deformation from shear may be not taken in consideration.
- The effect from previous loading before reinforcement should be taken in consideration.

3. Types of failure

Types of failure of a RC member reinforced with FRP under bending moment can be categorized in two groups

3.1. Failure because of destruction of one of the components

This type of failure comes from the crush of concrete in compressed zone (compressive stresses exceed the compressive strength), rebar in tensile zone breaks (tensile stresses on rebar exceed the tensile strength) or FRP breaks (tensile stresses on FRP exceed the tensile strength). Those types are called the 'classic' type of failure.

3.2. Premature types of failure

Premature failure is also called “peeling-of” and “concrete-ripping”. In this case there is no components destruction but the failure comes from the separation of FRP from concrete surface. Failure because the separation of FRP from concrete or separation of a piece of concrete together with FRP from the element must be studied carefully because a perfect connection between FRP and concrete could make possible common work of FRP and concrete. The disconnection might not happen in the entire length of element, but only in a certain area. In this case the beam may not fail, but is certain that its strength is decreased. When the disconnection happens in entire length of the element, we could say that the reinforcement has failed. FRP doesn’t support any action and the element without reinforcement fails. This type of failure is called “peeling-of” and in this case the failure is sudden, without visible signs, so the failure is not ductile but fragile. FRP disconnection from concrete may happen because the contact surface isn’t ready for the implementation of FRP, poor ability of adhesive used and especially low adhesive resistance in high temperatures. A premature failure that happens because of the loss of links that leads to disconnection of concrete form FRP and longitudinal reinforcing is called “concrete ripping”. This happens because the tensile and shear strength of adhesive used is higher than concrete. In this case concrete cracking begins from FRP and progresses with 45° angle inclination till the longitudinal tensile rebar. Another type of failure happens when FRP components fiber and resin split from each other. This type of failure is particular for high strength concrete. In conclusion we can say that those types of failures leads to the failure of the flexural element (beam) for loads much smaller compared with the classical types of failure.

4. Ultimate state design analyse of classic types of failure.

Which are the classic ways of failure?

- Concrete fail in compression zone. Reinforcing steel has not reached the yield strength.
- Reinforcing steel fail in tensile zone. In this case the reinforcing area is not enough to resist the tensile stresses from bending moment. This is the most typically case of usage of FRP for reinforcement of the flexural RC members from bending moment.
- Simultaneous fail of both concrete and reinforcing steel.

To avoid the destruction of concrete compressive zone, the FRP area must be smaller than a certain value A_{fmax}. EC 2 recommends show that maximum height (x) of the compression zone must be less than 0.45d for concrete with $f_{ck} \leq 35 N/mm^2$ and less than 0.35d for concrete with $f_{ck} > 35 N/mm^2$. This restriction makes the cross section to show a ductile behaviour and reinforcement steel has reached the yield strength. So the section could rotate significantly. With increasing of concrete strength its ductility decreases. That why for concrete strength $f_{ck} > 35 N/mm^2$ the maximum height of compressive zone is less than 0.35d. Figure 1 show stresses distribution of a cross section for a flexural element with rectangular section, maximal height of compression zone is accepted x=0.45d.

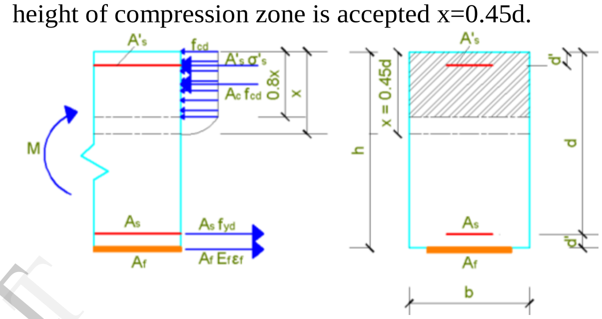

Fig 1.

A_s – rebar area in tensile zone
A'_s – rebar area in compressive zone
A_f – FRP cross section area
f_{cd} – concrete design compressive strength
M – bending moment from external load
$\sigma'_s = E'_s \cdot \varepsilon'_s$ – stress on A'_s
E'_s – modulus of elasticity of A'_s ($E'_s = E_s$)
E_s – modulus of elasticity of A_s
ε'_s – relative deformation of A'_s
$A_c = 0.8 \cdot x \cdot b$ – concrete compressive zone
x – compression zone height
f_{yd} – design tensile strength of reinforcement
E_f – FRP modulus of elasticity
ε_f – relative deformation of FRP
b – cross section width
h – cross section height
d – effective depth of cross section
d’ – concrete cover

Figure 2 shows cross section strains condition (see line nr. 3’) corresponding to Figure 1 stress condition.

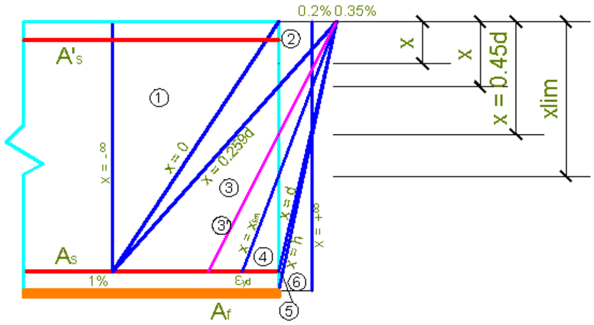

Fig.2.

Writing the equilibrium equation for forces on "x" direction:

$$-A'_s \cdot \sigma'_s - A_c \cdot f_{cd} + A_s \cdot f_{yd} + A_f \cdot E_f \cdot \varepsilon_f = 0 \quad (1)$$

On equation (1) are known A'_s, A_s, f_{cd}, $f_{yd} = f'_{yd}$, b, h, $E'_s = E_s$, d, d' and also we know the FRP type used so E_f is known.

$$\sigma'_s = E'_s \cdot \varepsilon'_s \quad (2)$$

From triangle identities of deformed shape of the cross section:

$$\frac{x}{0.35\%} = \frac{x-d'}{\varepsilon'_s} \Rightarrow \varepsilon'_s = \frac{x-d'}{x} 0.35\% \quad (3)$$

On equation (3) x = 0.45d and ε_c = 0.35% is the ultimate concrete deformation. Solving equation (3) for ε'_s we can calculate σ'_s with the help of equation (2). If $\varepsilon'_s \geq \varepsilon'_{yd}$ than $\sigma'_s = f'_{yd} = f_{yd}$ ($\varepsilon'_{yd} = f'_{yd} / E_s$). Using again the triangle identites for the deformed shape of cross section:

$$\frac{x}{0.35\%} = \frac{h-x}{\varepsilon_f} \Rightarrow \varepsilon_f = \frac{h-x}{x} 0.35\% \quad (4)$$

In this case we can write:

$$A_f = (A'_s \cdot f'_{yd} + A_c \cdot f_{cd} - A_s \cdot f_{yd}) / (E_f \cdot \varepsilon_f) = A_{fmax} \quad (5)$$

With the help of equation (5) we can calculate the FRP area for maximal height of compressed area accepted 0.45d or 0.35d according to EC 2 recommendations. If FRP area is greater than the area calculated with equation (5), is the risk of element failure because the crush of compressed zone. Let suppose having a beam with b = 30cm and h = 50cm, d' = 3.5cm, d = 46.5cm so x = 0.45d = 0.45·46.5 = 20.92cm. Concrete C25/30 with f_{cd} = 141.7daN/cm^2 and S500 steel with f_{yd} = 4348daN/cm^2, E_s = 2100000daN/cm^2. A'_s = 3Φ16 = 3·2.01 = 6.03cm^2. A_s = 4Φ20 = 4·3.14 = 12.56cm^2. E_f = 2350000daN/cm^2.

$$\varepsilon'_s = \frac{x-d'}{x} 0.35\% = \frac{20.92-3.5}{20.92} 0.35\% = 0.291\%$$

And $\varepsilon'_{yd} = f'_{yd}/E_s$ = 4348/2100000 = 0.207%. Because ε'_s = 0.291% > ε'_{yd} = 0.207% than $\sigma'_s = f'_{yd} = f_{yd}$ = 4348daN/cm^2 and also A_c = 0.8·x·b = 0.8·20.92·30 = 502.08cm^2 so:

$$\varepsilon_f = \frac{h-x}{x} 0.35\% = \frac{50-20.92}{20.92} 0.35\% = 0.486\%$$

Finally: $A_{fmax} = (A'_s \cdot \sigma'_s + A_c \cdot f_{cd} - A_s \cdot f_{yd}) / (E_f \cdot \varepsilon_f)$ = (6.03·4348 + 502.08·141.7 – 12.56·4348) / (2350000·0.00486) = 3.74cm^2

Failure type with the breaking of steel reinforcement is more acceptable because is more ductile. If FRP area is smaller from what is needed the element will fail. According to Eurocodes, the ultimate deformation for reinforcing steel is accepted 1%, but this value is very conventional because, in reality, reinforcing steel deformations are grate than 1%. Most FRP have deformation less than 1% and according to this, we can suppose, if the element fail from tensile stress, the failure comes from FRP destruction (breaking). To design FRP from bending moment we use figure 3. Solving the equilibrium equation of forces on horizontal axes:

$$-0.8 \cdot x \cdot b \cdot f_{cd} - A'_s \cdot f_{yd} + A_s \cdot f_{yd} + A_f \cdot E_f \cdot \varepsilon_f = 0 \quad (6)$$

This equation is true if $\varepsilon'_s \geq \varepsilon'_{yd}$ and $\varepsilon_s \geq \varepsilon_{yd}$. Only if this condition is fulfilled we can write say that $\sigma_s = \sigma'_s = f'_{yd} = f_{yd}$. So:

$$\varepsilon'_s = \varepsilon_c \frac{x-d'}{x} \geq \frac{f'_{yd}}{E_s} = \varepsilon'_{yd} = \varepsilon_{yd} \quad (7)$$

And:

$$\varepsilon_f = \varepsilon_c \frac{h-x}{x} - \varepsilon_0 \quad (8)$$

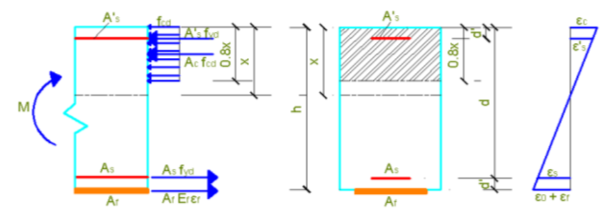

Fig. 3.

The design should take in consideration that during the reinforcement procedure the element's extreme tensile fiber have an initially deformation ε_0. FRP deformation is caused from the additional loading. This is the reason because on equation (7) from ε_f is subtracted ε_0. For a better understanding, see figure 4.

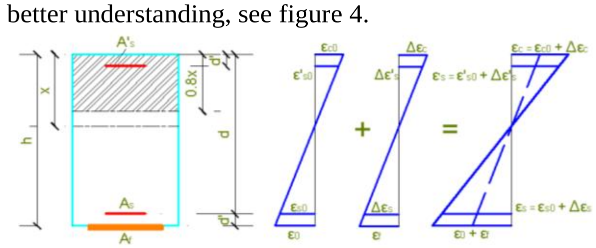

Fig. 4.

The ultimate design bending moment is calculated with the help of the second equilibrium equation which is a sum of moments about the axes passing on centre of gravity of concrete compressed zone:

$$M_{Rd} = A_s f_{yd}(d-0.4x) + A_f E_f \varepsilon_f (h-0.4x) + A'_s E_s \varepsilon'_s (0.4x-d') \quad (9)$$

Supposing that $\varepsilon'_s \geq \varepsilon'_{yd}$:

$$M_{Rd} = A_s f_{yd}(d-0.4x) + A_f E_f \varepsilon_f (h-0.4x) + A'_s f_{yd} (0.4x-d') \quad (10)$$

And also:

$$\varepsilon_f = \varepsilon_c \frac{h-x}{x} - \varepsilon_0 \leq \varepsilon_{f,lim} \quad (11)$$

$\varepsilon_{f,lim}$ must not exceed 50% of initially FRP deformation (ε_{fd}) and also must not exceed 5 time the yield deformation of reinforcing steel (ε_{yd}).

5. Solved case

A flexural RC beam is studied. Bending moment form external loading M_{Ed} = 15000daNm; rectangular cross section with width b=30cm and height h=50 cm, concrete cover d'=3.5cm, effective depth of cross section d=h-d'=50-3.5=46.5cm; C25/30 concrete with f_{cd}=141.7daN/cm^2; S500 reinforcing steel with f_{yd}=4348daN/cm^2, E_s = 2100000daN/cm^2, $\varepsilon_{yd} = f_{yd}/E_s$ = 4348/2100000 = 0.207%; FRP modulus of elasticity E_f = 2350000daN/cm^2, ultimate design deformation ε_{fd} = 1.8%. Calculate the FRP area for reinforcing the element for a new external loads moment M_{Ed} = 20000daNm. Initially let calculate the reinforcing steel area needed to resist a bending moment of M_{Ed} = 15000daNm:

$\mu = M_{Ed}/(b \cdot d^2 \cdot f_{cd}) = 1500000/(30 \cdot 46.5^2 \cdot 141.7) = 0.163$

On equation:

$$\mu = 0.8 \cdot \frac{x}{d}(1-0.4 \cdot \frac{x}{d}) \quad (12)$$

if x/d=0.45 is replaces than μ_{max} is calculated:

$$\mu_{max} = 0.8 \cdot 0.45 \cdot (1-0.4 \cdot 0.45) = 0.295 \quad (13)$$

Because $\mu = 0.163 < \mu_{max} = 0.295$ there is no need for reinforcing on compressed zone, so $A'_s = 0$. However 2 diameter 16 mm bars are used as reinforcing on the compressed zone.

$$\xi = 1.25 - 1.25 \cdot (1-2\mu)^{1/2} = 1.25 - 1.25 \cdot (1-2 \cdot 0.163)^{1/2} = 0.223 \quad (14)$$

So:

$$A_s = 0.8 \cdot \xi \cdot d \cdot b \cdot f_{cd}/f_{yd} = 0.8 \cdot 0.223 \cdot 46.5 \cdot 30 \cdot 141.7/4348 = 8.11\text{cm}^2$$

4 diameter 16mm bars are used with an area of A_s =4·2.01 = 8.04cm^2 (-0.86%).

From $\xi = x/d = 0.223$ we can find the compressed zone height: $x = \xi \cdot d = 0.223 \cdot 46.5 = 10.36$cm

According to EC 2, if a 0.35% deformation of the compressed zone and 0.35% deformation of reinforcing steel are accepted than the cross section is balanced or equilibrated. Graphically this is described on figure 5(the pink line):

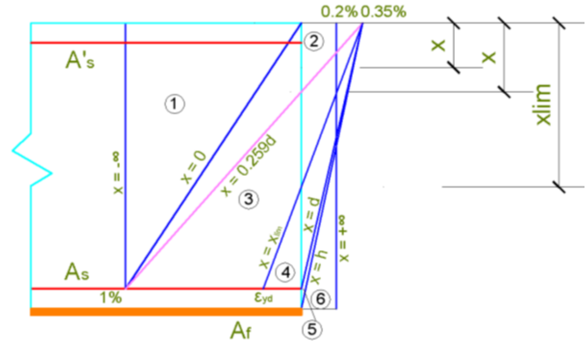

Fig. 5.

The compressed zone height: x=0.259d=0.259·46.5 = 12.04cm. Since x = 10.36cm < 0.259d = 12.04cm then we can say that we are in the second case (see figure 5). In a more detailed way, adapted to our case, the deformed shape and stresses distribution are given on figure 6 and figure 7.

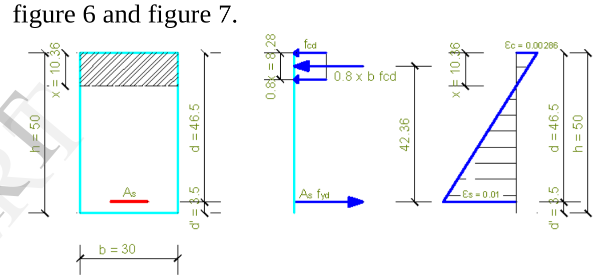

Fig. 6.

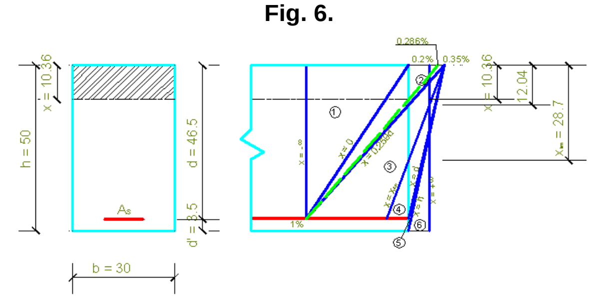

Fig. 7.

Some of the values given on figure 6 and figure 7 can be found as following. So, to calculate the concrete deformation ε_c according to the deformed shape of the section we have:

$$\frac{x}{\varepsilon_c} = \frac{d-x}{\varepsilon_s} \quad (14)$$

Solving equation (14) for ε_c:

$$\varepsilon_c = \frac{x \cdot \varepsilon_s}{d-x} = \frac{10.36 \cdot 0.01}{46.5 - 10.36} = 0.00286 = 0.286\%$$

Because ε_c = 0.286% > 0.2% than $\sigma_c = f_{cd}$. Relative deformation of bottom extreme fiber, which is in the same time the initially deformation ε_0 is:

$$\frac{d-x}{\varepsilon_s}=\frac{h-x}{\varepsilon_0} \qquad (15)$$

Solving equation (15) for ε_0:

$$\varepsilon_0=\frac{(h-x)\varepsilon_s}{d-x}=\frac{(50-10.36)*0.01}{46.5-10.36}=0.0109=1.09\%$$

Before the application of FRP, the bottom part of the cross section will be relived so a part of the deformation will disappear, this deformation decrease is accepted 30% of the initially deformation. In this case the bottom extreme fiber of the cross section, also ε_0, the effective deformation is calculated:

ε_0 (effective) = $0.7\cdot\varepsilon_0 = 0.7\cdot1.09\% = 0.763\%$

Let suppose that for different reason the bending moment increase to a new value of M_{Ed} = 20000daNm. For this new condition we need to calculate the FRP are in order that the element must not fail. To solve this problem we refer to figure 3. Equation (6) and (9) or (10) are used. On equation (6) x, A_f, and ε_f are unknown. With the help of equation (11) we can calculate FRP deformation (ε_f) function of x. So, now, there are to unknowns, x and A_f. On equation (10), if $M_{Ed} = M_{Rd}$, there are also two unknowns, x and A_f. Solving those two equations with two unknown variable we found:

x = 13.97cm dhe $A_f = 3.83cm^2$

In reality, we use another method. A certain value of FRP area is accepted and the values of x, ε_f and M_{Rd} are calculated. If $M_{Rd} > M_{Ed}$ the problem is solved, but if $M_{Rd} < M_{Ed}$ than a bigger value of FRP area is accepted and the calculations are repeated. Equations (6) and (9) are true only if $\varepsilon_s \geq \varepsilon_{yd}$. To calculate ε_s equation (15) is used.

$\varepsilon_s = 0.0035\cdot(46.5\text{-}13.97)/13.97 = 0.814\% \geq \varepsilon_{yd} = 0.207\%$

Checking if FRP ultimate deformation passes the equation (11) condition:

$$\varepsilon_{flim} = 0.5\cdot\varepsilon_{fd} = 0.5\cdot1.8\% = 0.9\%$$
$$\varepsilon_{flim} = 5\cdot\varepsilon_{yd} = 5\cdot0.207\% = 1.035\%$$

ε_{flim} = 0.9% is accepted. From equation (11) we calculate $\varepsilon_f = 0.139\% < \varepsilon_{flim} = 0.9\%$, the condition is fulfilled.

6. Conclusion

This article provided a design approach of FRP as a reinforcing material for flexural RC elements. All the design process was done by following some steps given along the material. The focus of this material was to give a better understanding of FRP behavior as a reinforcing material and to give e clearly stress-strain relationship of the reinforced element. All the design formulas are used from Eurocode 2 to calculate the design area of FRP in order that the reinforced element must resist a new load case. At the end of the article, a solved numerical case was given. This solved case gives a practical way of calculating the necessary FRP area for reinforcing.

7. References

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