ISSN: 2278-0181
ACMEE - 2016 Conference Proceedings

Design of IMC based PID Controller for Coupled Tank System

Alok Prakash
Electrical and Instrumentation department SLIET, Longowal, Sangrur, Punjab India-148106

Ashutosh Prasad Yadav
Electrical and Instrumentation department SLIET, Longowal, Sangrur, Punjab India-148106

Raj Kumar
Electrical and Instrumentation department SLIET, Longowal, Sangrur, Punjab India-148106

***Abstract*—The performance of the Proportional Integral Derivative (PID) controllers commonly used in process control industries depends upon the controller tuning parameters. In this paper an Internal Model Control (IMC) based PID controller is proposed to estimate controller tuning parameters in terms of single parameter, known as closed-loop time constant which provides improved performance and robustness to control system. An IMC based controller is designed and presented here for a coupled tank level control system which is of non-interacting type. The transfer function of the system is obtained from the equipment specifications. The obtained transfer function is approximated into first order plus delay time (FOPDT) model for the estimation of the IMC-PID controller tuning parameters in terms closed-loop time constant. The process is simulated in MATLAB/Simulink to record the closed-loop performance with IMC-PID based tuning parameters. The result are compared with the Ziegler-Nichols, Cohen-Coon and Tyreus-Luyben tuning methods in terms of time response characteristics and various performance Indices like Integral of Absolute Error (IAE), Integral Squared Error (ISE) and Integral Time Absolute Error (ITAE). The robustness is checked by incorporating uncertainties in the process. The results indicate PID controller tuned with IMC has better performance and robustness as compared to other tuning techniques.**

Keywords*— *Proportional Integral Derivative, Internal Model Control (IMC), First Order plus Delay Time Model, Closed-loop time constant, tuning, robustness.

I. INTRODUCTION

PID Controllers are extensively employed in process control industries because of their relatively simple structure and design. Tuning technique is adopted for determining the proportional, integral and derivative constants of these controllers which depend upon the dynamic response of the plants. Ziegler-Nichols and Cohen-Coon [1-3] tuning methods are the most popular methods used in process control to determine the parameters of a PID controller. Although these methods are very old, they are still widely used because of their capability to achieve desired optimal performance for specific inputs with less tolerance to plant variations. PID controller tuned with these methods shows less robust results. A controller is said to be robust if it is insensitive to small changes in process or to inaccuracies in process model. Robustness can be defined as amount of Uncertainty in process parameters or inaccuracy in Process model that can be tolerated by controller before the closed-loop system becomes unstable [4-5]. In reality a, model is never perfect, so controllers must be designed to be robust (to remain stable even when the true plant characteristics are different from the model). Internal model control (IMC) based PID controller has gained attention because of its robustness and single tuning parameter selection [6-7].

Maintaining the level at a desired state is an important and common task in all process industries. IMC based PID controller is developed in this paper to control the liquid level in the coupled tank system. Among the other tuning methods, IMC based PID controllers tuning methods has gained widespread acceptance in the process industries because of its easy in design and simple in understand, robustness and fast in real time applications.

II. INTERNAL MODEL CONTROL

A. Internal Model Control Strategy

Internal Model Control (IMC) has been presented by Garcia and Morari [6] which is developed upon Internal Model principle to combine the process model and external signal dynamics. The IMC controller is a model based procedure, where a process model is embedded in the controller, and is considered to be robust. Mathematically, robust means that the controller must perform to specification, not just for one model but also for a set of models [4]. The IMC controller design philosophy adheres to this robustness by considering all process model errors as bounded and stable. IMC Theory states that a perfect control can be achieved only if the control system encapsulates, either implicitly or explicitly, some representation of the process to be controlled.

The IMC basic structure is shown in Fig.3 is characterized by a controller $Gc(s)$, actual process or plant $Gp(s)$ and predictive model of the plant $Gp^*(s)$. $d(s)$ is an unknown disturbance affecting the system. The manipulated input U(s) is introduced to both the process and its model. The process output is $Y(s)$. d*(s) is the difference between the output of the actual process $Gp(s)$ and process model $Gp^*(s)$which is the result of model mismatch and the disturbances; this is used by the internal model controller.

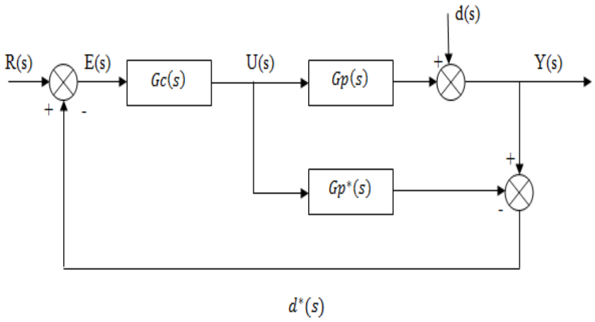

Fig.3 IMC basic structure

The design procedure of IMC involves factorization of the plant model $Gp^*(s)$ as invertible $Gp-^*(s)$ and non-invertible $Gp+^*(s)$ parts as shown in Eq. (1) by simple factorization or all pass factorization. Ideal IMC controller $Gc^*(s)$ is the inverse of the invertible portion $Gp-^*(s)$ of the process model $Gp^*(s)$.

$$Gp^*(s) = Gp+^*(s)Gp-^*(s) \quad (1)$$

$$Gc^*(s) = [Gp-^*(s)]^{-1} \quad (2)$$

$Gc^*(s)$ will be stable, but may not be proper.A low pass filter f(s) of the form of Eq. (3) is added to $Gc^*(s)$ for making it proper, which also attenuates the effects of process model mismatch, which usually occurs at high frequency and provides good set point tracking.

$$f(s) = \frac{1}{(\lambda s+1)^n} \quad (3)$$

$$Gc(s) = Gc^*(s)f(s) \quad (4)$$

Value of n is chosen to make $Gc^*(s)$ proper or semiproper. λ is filter time constant or closed-loop time constant or filter tuning parameter whose value is adjusted to vary the speed of response of the closed-loop system. Gc(s) in Eq. (4) is the final form of the IMC Controller.

B. IMC based PID Controller

Although the Internal Model Control (IMC) procedure is simple but it cannot be implemented practically since most industries still uses the PID controller. So the IMC structure can be modified and rearranged to the form of a standard feedback control diagram or Conventional PID structure shown in Fig.4.

$G_{PID}(s)$ is standard feedback controller which is a function of plant model $Gp^*(s)$ and IMC Controller Gc(s) shown in Eq. (5) which can be obtained by rearrangement of IMC basic structure Fig.3 to Feedback control structure Fig.4

$$G_{PID}(s) = \frac{Gc(s)}{1 - Gc(s)\,Gp^*(s)} \quad (5)$$

$$G_I(s) = Kp[\frac{\tau_i\tau_d s^2 + \tau_i s + 1}{\tau_i s}] \quad (6)$$

In IMC based PID design procedure Gc(s) is made semi proper or even improper to give the resulting PID controller derivative action. A first or second order pade approximation is used if a process model has a time delay. The standard PID controller in Eq. (5) is compared to ideal PID Controller Eq. (6) to find out PID parameters (Kc,τ_i,τ_d) in terms of closed-loop time constant λ whose value is adjusted to give IMC-PID tuning.

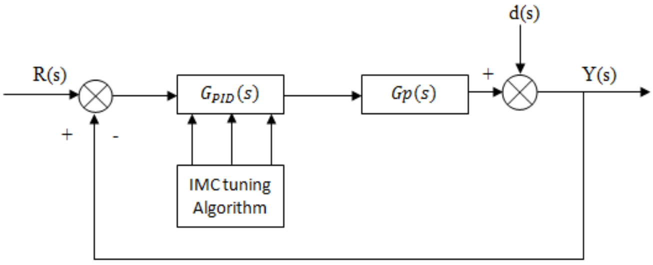

Fig.4 Feedback control structure

III. MATHEMATICAL MODELLING AND CONTROLLER DESIGN

A. Obtaining Transfer Function of the Process

The control objective in a coupled tank system is that a desired level of the liquid in tank is to be maintained when there is an inflow and outflow of water out of the tank respectively. The coupled tank system [8-9] is a multi-input multi output system (MIMO) consisting of two independent single-input single-output systems (SISO) with control voltage as input and water level as the output.

Consider the process consisting of two non-interacting liquid tanks in the Fig.1 here Load Changes in first tank affects the second tank but not the vice-versa. Q_i is the volumetric flow rate into Tank1, Q is the volumetric flow rate from Tank 1 to Tank 2 and Q_o is the volumetric flow rate out of Tank 2.Height of liquid level in Tank1 is H_1 and in Tank 2 is H_2.Both tanks are having same cross-sectional area A.Two ball valves V_1 and V_2 having Hydraulic resistances R_1 and R_2 are connected at the outlet of each tanks.V_i is the control input voltage to pump.

Assuming linear resistance to flow, transfer function of the coupled tank system through mathematical modeling is

$$G(s) = \frac{H_2(s)}{Q_i(s)} = \frac{R_2}{(\tau_1 s+1)(\tau_2 s+1)} \quad (7)$$

where $\tau_1 = AR_1$ and $\tau_2 = AR_2$ are the time constants of Tank 1 and Tank 2 related to operating levels in the tank

Flow rate of the pump is related as:

$Q_i(s) = \eta Vi(s)$; η is pump constant relating to control voltage

Hence, overall transfer function of the process becomes

$$Gp(s) = \frac{H_2(s)}{Vi(s)} = \frac{\eta R_2}{(\tau_1 s+1)(\tau_2 s+1)} \quad (8)$$

Here H2 is controlled variable and Vi is manipulated variable

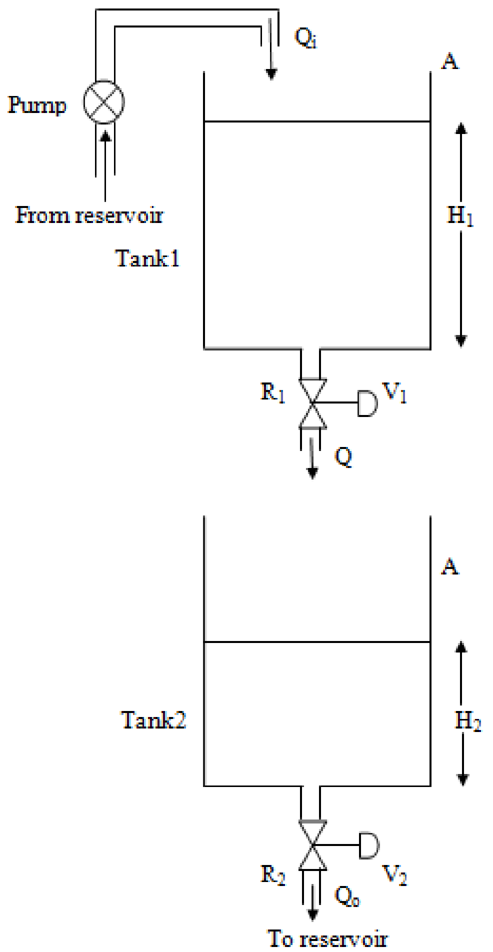

Fig.1 Two tank non-interacting process

Therefore, Obtained Transfer function of coupled two tank non-interacting level process using coupled tank parameters from Table 1 is

$$Gp^{*}(s) = \frac{2.9646}{(11.2509s+1)^2} \quad (9)$$

$$Gp^{*}(s) = \frac{2.9646}{126.5827s^2 + 22.5018s + 1} \quad (10)$$

Table 1 parameters of coupled tank system

Parameter	Description	Value	Unit
A	Cross-sectional area of tanks	138.9	cm^2
R_1	Hydraulic resistance of ball valve 1	0.081	sec/ cm^2
R_2	Hydraulic resistance of ball valve 2	0.081	sec/ cm^2
η	Pump constant related to flow rate into tank	36.6	cm^3/v.sec

B. FOPDT Approximation of process model

Industrial processes are of higher order so finding a real value of it is very difficult. The transfer functions of plants that can be approximately modelled by some definite transfer function. Sundaresan and Krishnaswamy [10] have proposed a simple method for fitting the dynamic response of higher order systems in terms of first order plus time delay transfer functions. The obtained second order transfer function of the coupled tank system is approximated into a FOPDT transfer function using the same method as:

The method is based on times, t_1 and t_2, which can be estimated from a step response curve (Fig.2), corresponding to the 35.3% and 85.3% response times, respectively.

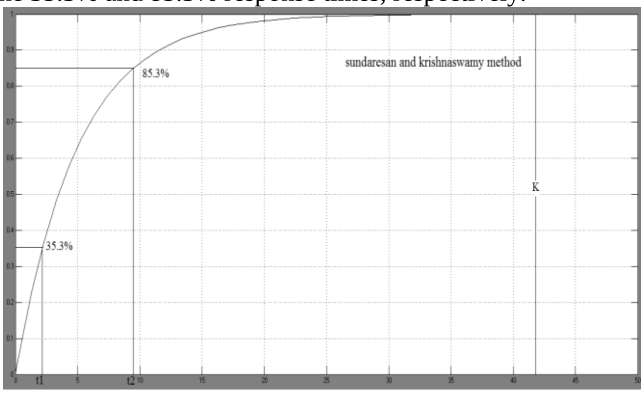

Fig.2 FOPDT approximation curve

The time delay and time constant are then estimated from the following equations:

$$\theta = 1.3t_1 - .29t_1 \quad (11)$$
$$\tau = .67(t_2 - t_1) \quad (12)$$

The FOPDT Transfer function is given by:

$$\frac{K}{(\tau s + 1)} e^{-\theta s} \quad (13)$$

FOPDT model of Coupled Tank System is represented as:

$$Gp^{*}(s) \approx \frac{2.9646}{16.22s + 1} e^{-7.1s} \quad (14)$$

C. IMC based PID Controller Design

FOPDT transfer function of Coupled Tank System obtained in Eq. (8) is:

$$Gp^{*}(s) = \frac{2.9646}{16.22s + 1} e^{-7.1s} \quad (15)$$

Process Model $Gp^{*}(s)$ after first order pade approximation [6] for time delay is:

$$Gp^{*}(s) = \frac{2.9646(1 - 1.35s)}{(16.22s + 1)(1 + 1.35s)} \quad (16)$$

Performing Simple factorization:
Invertible part

$$Gp -^{*} (s) = \frac{2.9646}{(16.22s + 1)(1 + 3.5s)} \quad (17)$$

Non-invertible part

$$Gp +^{*} (s) = (1 - 3.5s) \quad (18)$$

Adding Filter with n=1 to make our Controller Improper in order to obtain an ideal PID Controller

ISSN: 2278-0181
ACMEE - 2016 Conference Proceedings

$$Gc(s) = \frac{(16.22s + 1)(1 + 3.5s)}{2.9646}\left(\frac{1}{1 + \lambda s}\right) \qquad (19)$$

Using Eq. (13) and (19)

$$G_{PID}(s) = \frac{(16.22s + 1)(1 + 3.5s)}{2.9646(\lambda + 3.5)s} \qquad (20)$$

Again, by comparing Eq. (19) with ideal PID Controller Eq. (20), PID parameters are obtained in terms of closed-loop time constant λ which has been easily adjusted to tune the controller.

$$Kp = \frac{6.6397}{\lambda + 3.5} \qquad (21)$$

$$\tau_i = 19.72sec \qquad (22)$$

$$\tau_d = 2.9199sec \qquad (23)$$

Thus we can easily do IMC-PID tuning by adjusting λ.Rivera et al. [7] recommend that $\lambda > 0.8\theta$ because of the model uncertainty due to the pade approximation.

IV. SIMULATION RESULTS

Simulation results are presented to illustrate the effectiveness of IMC based PID Controller for coupled tank liquid level control system.

A. *Simulation Results for different values of 'λ'*

Simulation was performed for IMC-PID tuning for different values of 'λ'.Simulation response in Fig.5 shows the behavior of response with increase in value of 'λ'.Results in Table 2 indicates the variation of Gain and Phase Margin with 'λ' as Gain and Phase Margins are related to Robustness of the Controller.

Table 2 PID parameters for different values of λ

λ	K_p	$K_i = K_p/\tau_i$	$K_d = K_p\tau_d$	Gain Margin	Phase Margin
0.9θ	0.6713	0.03404	1.901	2.7862	69.5402
θ	0.6263	0.03175	1.8287	2.9864	70.9729
1.1θ	0.5870	0.02977	1.7139	3.1865	72.1926
1.2θ	0.5523	0.02801	1.6126	3.3866	73.2730
1.3θ	0.5215	0.02644	1.5227	3.5866	74.2363

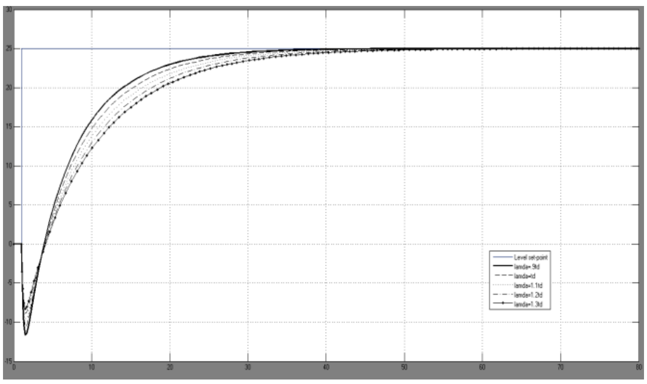

Fig.5 simulation Response for Step input for various λ values

B. *Simulation Results for Performance*

The Controller performance is measured by calculating performance indices like ISE, IAE and ITAE and determining the time response characteristics like rise time(t_r), settling time(t_s) and peak overshoot(M_p) through closed-loop simulation in MATLAB/Simulink. Performance results for IMC-PID tuning were compared with ziegler-nichols, cohen-coon and tyreus-luyben tuning methods to see its effectiveness. The Simulations performed for step changes in set-point and in the disturbance at t=100sec for different tuning methods. Simulation responses in Fig 9 and Fig. 10 shows set-point tracking and disturbance rejection capability of IMC-PID tuning in comparison with other tuning methods used.

Table 3 Performance results for different tuning methods

Specifications	IMC-PID	Ziegler-Nichols	Cohen-Coon	Tyreus-Luyben
Rise Time(sec)	17.9511	8.9575	10.3185	22.7988
Settling Time(sec)	38.8085	73.6954	51.5864	115.8182
Peak Overshoot (%)	2.2617	30.1450	16.1750	0
IAE	158.3	182.1	135.3	233.1
ISE	1430	1166	1029	1346
ITAE	1424	3382	1505	7138

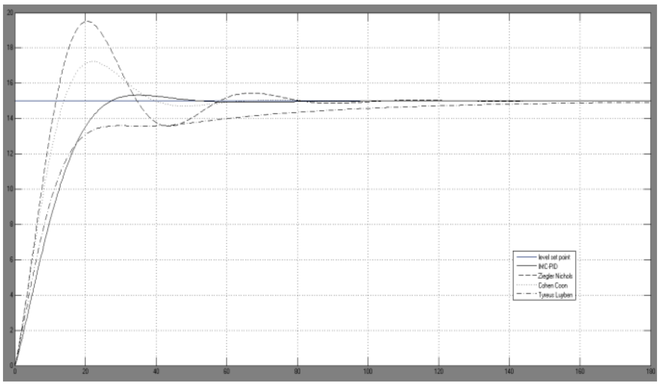

Fig.6 simulation response for step input for different tuning methods

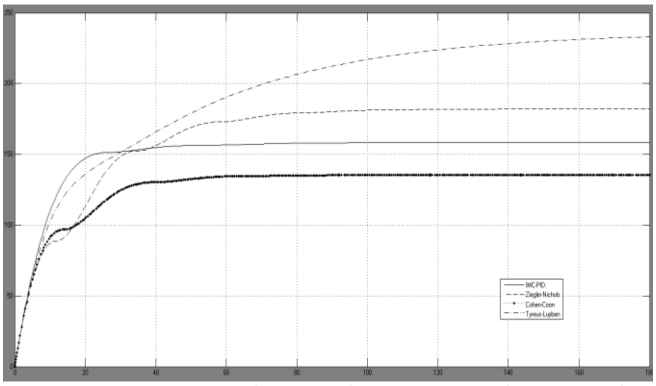

Fig.7 simulation response of Integral of Absolute value of error (IAE) for different tuning methods

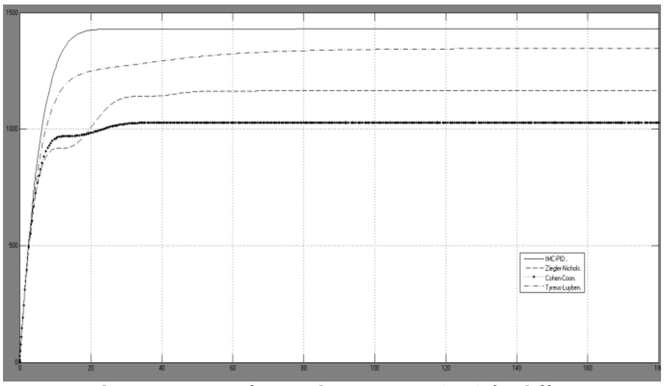

Fig.8 simulation response of Integral square error (ISE) for different tuning methods

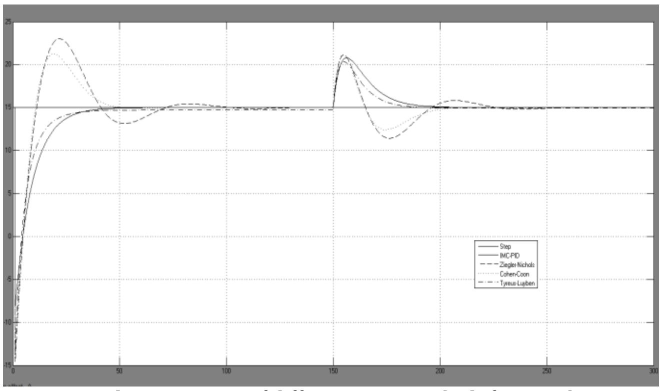

Fig.9 Simulation response of different tuning methods for step change in disturbance

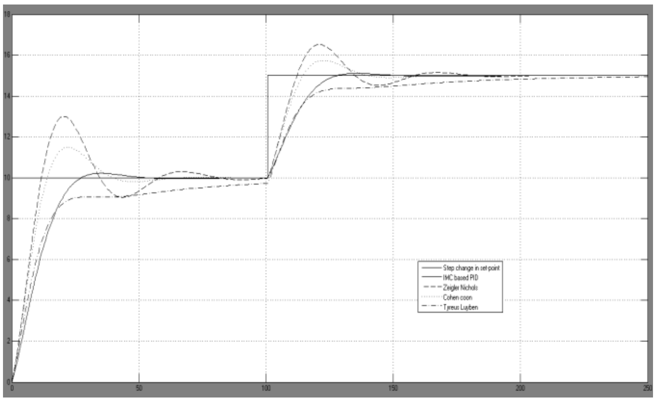

Fig.10 Simulation response of different tuning methods for step change in set-point

C. Simulation Results for Robustness testing

The robustness testing of IMC tuned PID Controller was evaluated by incorporating uncertainty in the actual process by a factor of 20% and 25% in gain(K), delay time(θ) and time constant(τ).Results in Table 4-9 and simulation responses Fig.11-16 were presented to show the robustness of IMC tuned PID Controller in comparison with other tuning techniques.

Table 4 results with 20% change in gain(K)

Specifications	**20% change in gain(K)**			
	IMC-PID	**Ziegler-Nichols**	**Cohen-Coon**	**Tyreus-Luyben**
Rise time(sec)	10.4603	4.4962	4.4384	6.7058
Settling time(sec)	18.8504	20.555	18.6782	65.9047
Peak Overshoot (%)	0	40.6757	16.0683	0
IAE	124	150	109.1	194.9
ISE	1630	2220	1891	1730
ITAE	597.2	1021	541	5435

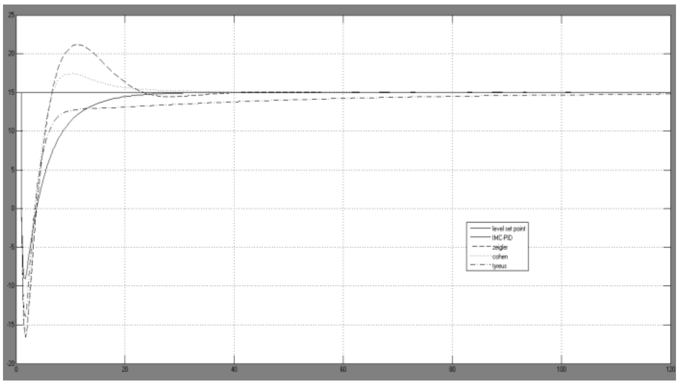

Fig.11 simulation response for step input for different tuning methods for 20% change in gain (K)

Table 5 Performance results with 25% change in gain(K)

Specifications	**25% change in gain(K)**			
	IMC-PID	**Ziegler-Nichols**	**Cohen-Coon**	**Tyreus-Luyben**
Rise time(sec)	9.7258	4.2	4.0947	5.9553
Settling time(sec)	17.5648	19.6499	17.4233	61.2076
Peak Overshoot (%)	0	41.5147	17.0456	0
IAE	119.1	147.3	108.1	187.1
ISE	1617	2303	1970	1745
ITAE	655.6	1080	621.5	5317

Table 6 Performance results with 20% change in delay time(θ)

Specifications	**20% change in delay time$(\boldsymbol{\theta})$**			
	IMC-PID	**Ziegler-Nichols**	**Cohen-Coon**	**Tyreus-Luyben**
Rise time(sec)	12.6976	5.9046	6.0346	9.3253
Settling time(sec)	19.3810	50.9195	22.9978	84.0347
Peak Overshoot (%)	0.5638	63.8238	28.7914	0
IAE	150.2	249.5	152.7	233.3
ISE	2021	3130	2318	2069
ITAE	941.3	3376	1086	6924

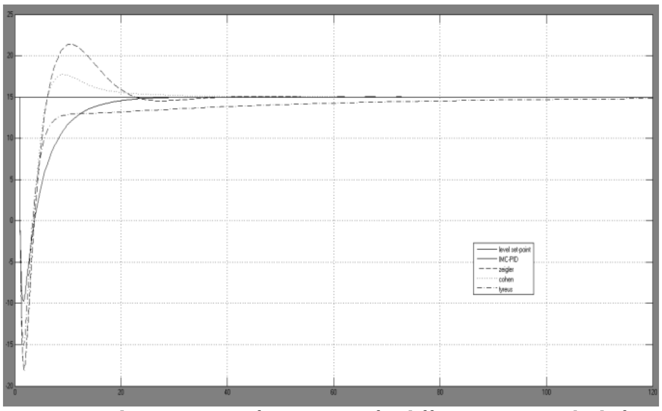

Fig.12 simulation response for step input for different tuning methods for 25% change in gain (K)

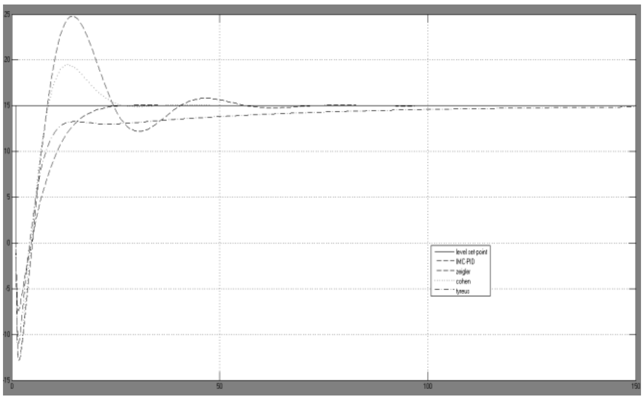

Fig.13 simulation response for step input for different tuning methods for 20% change in delay time (θ)

Table 7 Performance results with 25% change in delay time(θ)

Specifications	25% change in delay time(θ)			
	IMC-PID	**Ziegler-Nichols**	**Cohen-Coon**	**Tyreus-Luyben**
Rise time(sec)	12.5172	5.9184	6.0447	9.1274
Settling time(sec)	18.4789	52.1761	22.9321	83.5019
Peak Overshoot (%)	1.2969	71.1971	33.1745	0
IAE	153.2	280.3	164.4	233.4
ISE	2097	3514	2495	2186
ITAE	978.8	4211	1238	6840

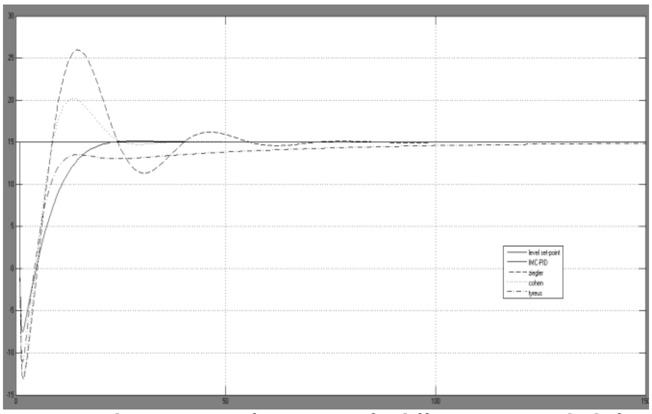

Fig.14 simulation response for step input for different tuning methods for 25% change in delay time (θ)

Table 8 Performance results with 20% change in time constant(τ)

Specifications	20% change in Time Constant(τ)			
	IMC-PID	**Ziegler-Nichols**	**Cohen-Coon**	**Tyreus-Luyben**
Rise time(sec)	16.1031	6.9982	7.5528	15.9628
Settling time(sec)	24.1089	45.5994	32.5747	86.0580
Peak Overshoot (%)	2.3230	41.1355	15.7591	0
IAE	170.1	199.1	140.2	233.8
ISE	1853	2090	1713	1809
ITAE	1735	2677	1231	6604

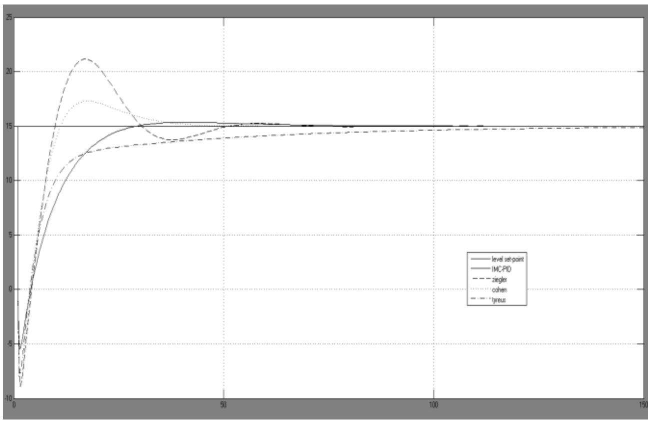

Fig.15 simulation response for step input for different tuning methods for 20% change in time Constant (τ)

Table 9 Performance results with 25% change in time constant (τ)

Specifications	25% change in Time Constant(τ)			
	IMC-PID	**Ziegler-Nichols**	**Cohen-Coon**	**Tyreus-Luyben**
Rise time(sec)	16.5442	7.2542	7.8865	16.7595
Settling time(sec)	44.8506	47.2821	34.1925	85.5761
Peak Overshoot (%)	3.00	41.9782	16.4586	0
IAE	176.7	207.8	145.6	233.9
ISE	1886	2118	1725	1825
ITAE	1943	2976	1339	6448

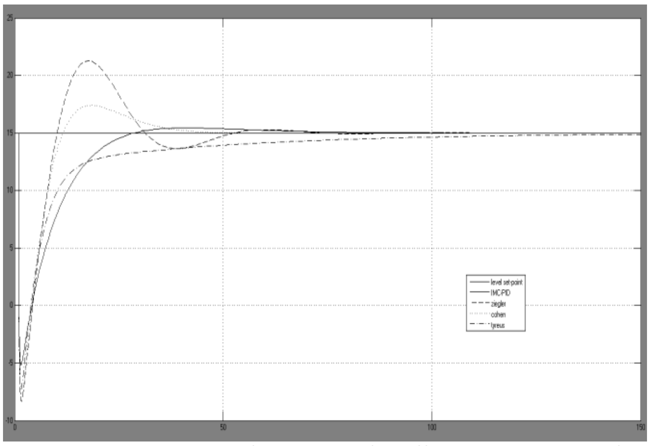

Fig.16 simulation response for step input for different tuning methods for 25% change in time Constant (τ)

V. REAL TIME RESULTS

Real time closed-loop responses obtained for IMC-PID tuning and Ziegler-Nichols tuning for Coupled-Tank System are shown in Fig.17-19.Step changes in set point is made at t=350sec for IMC-PID tuning to see its set-point tracking capability.

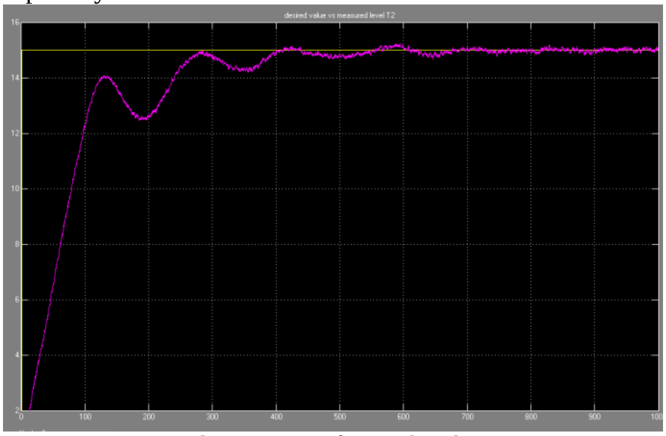

Fig.17 Real time response for IMC based PID tuning

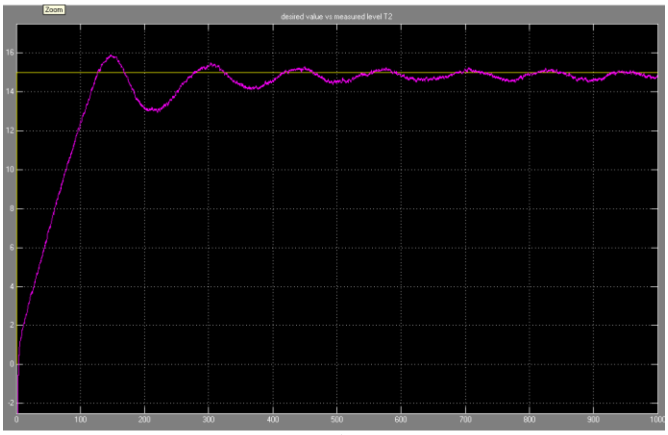

Fig.18 Real time response for Ziegler-Nichols tuning

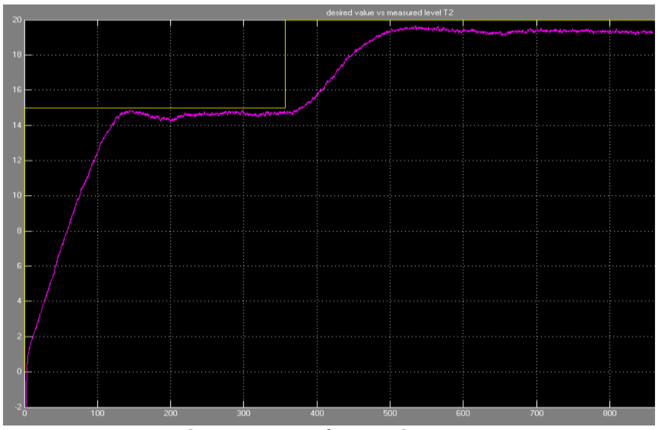

Fig.19 Real time response for step changes in set-point

VI. CONCLUSION

Simulation Results were presented to illustrate the IMC based PID tuning and to demonstrate its effectiveness, we considered Coupled Tank System for Liquid level Control. The four tuning methods IMC-PID, Ziegler-Nichols, Cohen-Coon and Tyreus-Luyben are considered for PID Controller and are comparatively analyzed based on performance and robustness. From Table 3 it is evident that IMC based PID tuning provides better time response characteristics i.e. optimum settling time and reduced overshoot as compared to other tuning methods. Table 3 and Fig.6-8 also shows that IMC based PID tuning exhibits minimum Integral error criteria's i.e. ISE, IAE and ITAE compared to other tuning methods. Simulation responses in Fig.9 and Fig.10 shows that IMC-PID tuning has better set-point tracking and disturbance rejection capability than other tuning methods. The Robustness of IMC tuned PID Controller was tested by incorporating uncertainty in the actual process by a factor of 20% and 25% in gain(K), delay time(θ) and time constant(τ).Results in Table 4-9 and simulation responses in Fig.11-16 indicates that IMC based PID tuning shows robust performance in Comparison with other tuning techniques. It is evident from the robustness analysis that Gain Margin is related to the amount of gain uncertainty that can be tolerated, and the Phase Margin is related to the amount of delay time uncertainty that can be tolerated. Therefore we can say that Gain and Phase Margin indicates the Robustness of the Controller. The result can be found from Table 3 that as we increase the value of λ the Gain and Phase Margin values increases which indicates Robustness increases. Decreasing the value of λ makes the closed-loop response fast whereas increasing its value makes the closed-loop system more robust. Hence, the IMC based PID controller tuning has the advantage of using only a single tuning parameter (λ) whose value is adjusted to achieve a clear trade-off between the closed loop performance and robustness.

REFERENCES

[1] G. H. Cohen and G. A. Coon, "Theoretical investigation of Retarded Control," Transactions of American Society of Mechanical Engineers,Vol. 75, pp. 827-834, 1953.

[2] K. J. Astrom, "Automatic tuning of PID regulators," Instrument Society of America ,1988.

[3] J. G. Ziegler and N. B. Nichols, "Optimum settings for automatic controllers," Transactions of American Society of Mechanical Engineers, Vol. 64, pp. 759-768, 1942.

[4] M. Morari and E. Zafiriou, Robust process control, Englewood Cliffs, Prentice-Hall, NJ, 1989.

[5] B. Wayne Bequette, Process Control Modeling, Design and simulation,Prentice-Hall of India,2004.

[6] C. E. Garcia, and M. Morari, "Internal Model Control - Unifying Review and Some New Results," Industrial Engineering Chemical Process Design and Development, Vol. 21,1982.

[7] D. E. Rivera, M. Morari, S. Skogestad, "Internal model control. 4. PID controller Design," Industrial Engineering Proceedings Design. Dev., 25, p.p. 252-265,1986.

[8] "Coupled tank system control experiment manual," Feedback Instruments Ltd.

[9] Elizabeth Rani T and Samson Issac J, "Modeling and Design Aspects of PI Controller for Coupled Tank Process," International Journal of Computer Applications(0975-8887), pp. 10-14, 2013.

[10] Sundaresen, K. R., Krishnaswamy, R. R., "Estimation of Time delay, time constant parameters in Time, Frequency and Laplace Domains," Journal of Chemical Engineering, 56, pp. 257-262, 1978.

[11] A.Iyswariya, N.Nimitha, Aarthi K.J Veronica, R.Selvarasu, "Design of PI Controller using First Order Plus Time Delay Model for Process Control," International Journal of Advanced Research in Electronics and Communication Engineering, Volume 4, Issue 3,pp. 687-691, March 2015.

ISSN: 2278-0181
ACMEE - 2016 Conference Proceedings

[12] P. V. Gopi Krishna Rao, M. V. Subramanyam, K. Satyaprasad, "Model based Tuning of PID Controller,"Journal of Control & Instrumentation, Vol. 4, Issue 1, ISSN: 2229-6972,2013.
[13] P. Aravind, M. Valluvan, B. Sangeetha, "Model based Controller design for level Process," The International Journal of Computer Applications(0975-8887), Vol. 88 - No. 18, February 2014.
[14] B. D. Tyreus and W. L. Luyben, "Tuning PI Controllers for integrator/deadtime processes," Ind. Eng. Chem. Res., pp. 2628-2631,1992.
[15] K. J. Astrom and T. Hagglund, "Automatic tuning of simple regulator with specifications on phase and amplitude margins," Automatica, Vol. 20, pp. 645-651, 1984
[16] S. Skogestad, "Modeling, Identification and Control," Vol. 25, No. 2, pp. 85-120, 2004
[17] Scott A. Geddes ,Thesis, "Internal Model Control (IMC) of a Fruit Drying System," University of Southern Queensland,2006.
[18] George Stephanopoulos, Chemical Process Control, Prentice Hall, NJ, 1984.