Design of Optimal Controller for A Non-Linear Batch Process

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***Abstract*— This paper provides a theoretical framework for modelling and simulation for optimal control design of a nonlinear dynamic system. In this paper we have considered a Batch reaction as nonlinear dynamics. During the mixing of the chemical reactants, a sudden unpredictable amount of heat is released causing exothermic reaction. This may affect the product quality and may damage the system. An optimal control technique such as Linear Quadratic Regulator (LQR) and Proportional Integral Control(PID) method are used for control of the temperature of the chemical process and hence maintain the adequate conditions for the process to take place. The non-linear system states are fed to the LQR which is designed using linear state space model. The analysis of the simulation results revealed that LQR and two PID controllers together can give better performance than a simple LQR controller**

Key words*— *Batch process , LQR, PID, Optimal control, Non-linear dynamic system.

I. INTRODUCTION

In an exothermic reactor, a large amount of heat liberated during the mixing of reactants can cause thermal runaway[1] if the generated heat exceeds the cooling capacity of the reactor tank. This may affect the product quality and pose safety problem to the plant. Hence it is necessary to have a precise temperature control[2] in such reactors. Here the control problem consists of obtaining the model of the reactor, and using this model to determine the control laws or strategies to achieve the desired system response and performance.

The Proportional-Integral-Derivative (PID) control is used to give efficient solution to various real-world control problems[3]. The transient and steady- state responses are taken care of with three-terms (i.e. P, I, and D). To make the performance of the system optimal LQR(Linear Quadratic Regulator) optimization is used.

As the input flow rates of the reactants increases the tank temperature as well as level also increases. Here we use LQR to control the temperature and later we use LQR +two PID controllers[6] for controlling temperature as well as level.

The organisation of the paper is as follows. Section II discusses about the mathematical modelling of the mixing tank which includes linearization of the system equation and modelling of the reactor. Section III presents the design of controller

II. MATHEMATICAL MODELLING

A. Linearization of the System

Jacobian Linearization method is used to linearize the non linear system, about a specific operating point, called an equilibrium point.

Consider a non-linear differential equation

$$\dot{x}(t) = f(x(t), u(t)) \tag{1}$$

where f is a function mapping $R^n \times R^m \rightarrow R^n$. A Point $\hat{x} \in R^n$ s called an equilibrium point if there is a specific $\in R^m$ such that $f(\hat{x}, \hat{u}) = O_n$.

Defining deviation variables to measure the difference

$$\delta_x(t) = x(t) - \hat{x}. \tag{2}$$

$$\delta_u(t) = u(t) - \hat{u}. \tag{3}$$

The Relation between $x(t)$ and $u(t)$ are given by the differential equation

$$\dot{x}(t) = f(x(t), u(t)) \tag{4}$$

Substituting the deviation variables in (4), we get

$$\dot{\delta_x}(t) = f(\hat{x} + \partial_x(t), \hat{u} + \partial_u(t)) \tag{5}$$

Using Taylor series expansion in equation (5)

$$\dot{\partial}_x(t) \approx \frac{\partial f}{\partial x}\delta_x(t) + \frac{\partial f}{\partial u}\delta_u(t). \tag{6}$$

The higher order terms are neglected.

The above differential equation holds good for the deviation variables as long as the deviation variables are small. It is a linear, time-invariant[9], differential equation, since the derivatives of δx are linear combinations of the δx variables and the deviation inputs, δu. The matrices

$$A = \frac{\partial f}{\partial x} \; at \; x = \hat{x} \; and \; u = \hat{u} \;, \tag{7}$$

$$B = \frac{\partial f}{\partial u} \; at \; x = \; \hat{x} \; \; and \; u = \hat{u} \in \boldsymbol{R}^{n \times n} \tag{8}$$

are constant matrices. The Linear system can now be defined as

$$\dot{\delta_x}(t) = A\delta_x(t) + B\delta_u(t)). \tag{9}$$

This is the Jacobian Linearization of the nonlinear system about the equilibrium point $(\hat{x}, \hat{u})$. For small values of δ_x *and* δ_u the linear equation approximately governs the exact relationship between the deviation variables δ_x *and* δ_u .

B. Chemical Reactor

Consider a mixing tank[7], with constant supply temperatures T_C and T_H and input flow rates qc(t) and $q_H(t)$. The equations for the tank are:

$$\dot{h}(t) = \frac{1}{A}\left(q_c(t) + q_h(t) - C_D A_0\sqrt{2gh(t)}\right) \qquad (10)$$

$$\dot{T}_T(t) = \frac{1}{Ah(t)}\left(q_c(t)[T_C - T_T] + q_h(t)[T_H - T_T]\right) \quad (11)$$

where A is the area of the tank, T_T is the temperature of the product inside the tank.

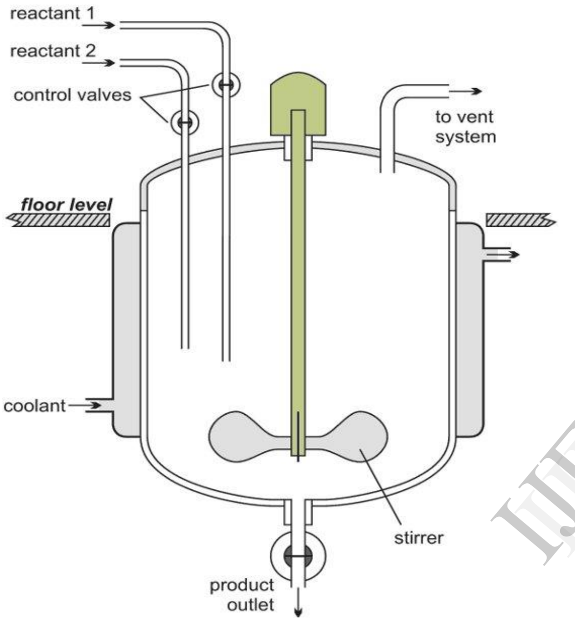

Fig.1 Batch Process - Mixing Tank

Let the state vector *X* and input vector *U* be defined as:

$$X(t) = \begin{bmatrix} h(t) \\ T_T(t) \end{bmatrix} ,U(t) = \begin{bmatrix} q_c(t) \\ q_H(t) \end{bmatrix} \qquad (12)$$

$$f_1(x,u) = \frac{1}{x_1 A}\left(u_1 + u_2 - C_D A_0\sqrt{2gx_1}\right) \qquad (13)$$

$$f_2(x,u) = \frac{1}{A}\left(u_1(t)[T_C - x_2] + u_2(t)[T_H - x_2]\right) \quad (14)$$

Where $u_1 = q_c(t), u_2 = q_H(t)$ and $x_1 = h(t), x_2 = T_T(t)$

For any height $\hat{h} > 0$ and any tank temperature $\widehat{T_T}$, satisfying $T_c < \widehat{T_H} < T_H$ should be a possible equilibrium point. With $\hat{h}$ and $\widehat{T_T}$ chosen, the equation $f(\hat{x}, \hat{u})=0$ can be written as

$$\begin{bmatrix} 1 & 1 \\ T_c - \widehat{x_2} & T_H - \widehat{x_2} \end{bmatrix} \begin{bmatrix} \widehat{u_1} \\ \widehat{u_2} \end{bmatrix} = \begin{bmatrix} C_D A_0\sqrt{2g\widehat{x_1}} \\ 0 \end{bmatrix} \qquad (15)$$

The 2×2 matrix is invertible if T_C not equal to T_H. Hence as long as T_C not equal to T_H, there is a unique equilibrium input for any choice of $\hat{x_2}$. It is given by :

$$\widehat{u_1} = \frac{C_D A_0\sqrt{2g\,\widehat{x_1}}(T_H - \widehat{x_2})}{T_H - T_c} \qquad (16)$$

$$\widehat{u_2} = \frac{C_D A_0\sqrt{2g\,\widehat{x_1}}(-T_C + \widehat{x_2})}{T_H - T_c} \qquad (17)$$

As u_i represents flow rates into the tank, they are non-negative real values due to physical restrictions. This implies that $\hat{x} \geq 0$ and $T_C \leq \widehat{T_T} \leq T_H$.The differential equation for T_T , the tank temperature, implies that it is inversely proportional to the height of the tank . *Hence*, the differential equationof a model is valid while h(t) > 0, so we further restrict $\widehat{x_1} > 0$. Under those restrictions, the state $\widehat{x}$, is indeed an equilibrium point.

The necessary partial derivatives are given by :

$$\begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} \end{bmatrix} = \begin{bmatrix} -\frac{gC_D A_0}{A\sqrt{2gx_1}} & 0 \\ \frac{u_1[T_C - x_2] + u_2[T_H - x_2]}{x_1^2 A_T} & \frac{-(u_1 + u_2)}{x_1 A_T} \end{bmatrix}$$

$$\begin{bmatrix} \frac{\partial f_1}{\partial u_1} & \frac{\partial f_1}{\partial u_2} \\ \frac{\partial f_2}{\partial u_1} & \frac{\partial f_2}{\partial u_2} \end{bmatrix} = \begin{bmatrix} \frac{1}{A} & \frac{1}{A} \\ \frac{[T_C - x_2]}{x_1 A_T} & \frac{[T_H - x_2]}{x_1 A_T} \end{bmatrix}$$

In order to linearize the given system it is required that the matrices of partial derivatives be evaluated at the equilibrium points.

III. CONTROLLER DESIGN

Optimal control is used to minimize the performance index. A control law is synthesized using optimal control technique, which results in best possible behaviour of the system .Linear quadratic regulator (LQR) is one of the optimal control techniques, which takes into account the states of the dynamical system and control input to make the optimal control decisions. The control law is given by

$$U = -KX \qquad (18)$$

where, X is the states of the system and K is feedback gain matrix[8] and it is derived from minimization of the cost function

$$J = \int (X^T QX + U^T PU)dt \qquad (19)$$

where, Q and R are positive semi-definite and positive definite symmetric constant matrices respectively. The LQR gain vector K is given by

$$K = R^{-1}B^T P \qquad (20)$$

where, P is the solution of the Algebraic Ricatti Equation[10] - (21)

$$A^T P + PA - PBR^{-1}B^T P + Q = 0 \qquad (21)$$

In the optimal control of mixing tank total temperature of the tank have been considered available for measurement which are directly fed to the LQR. The LQR is designed using the linear state-space model of the system. The optimal control value of LQR is given as a negative feedback along with the PID controller. The tuning of the PID controller and PID+LQR controller is done by Zeigler Nicholas method[11].

IV. SIMULATION & RESULT

The MATLAB-SIMULINK models for the control of temperature and height of the mixing tank have been developed. The typical parameters of the reactor is selected as $T_C = 10^{\circ}$, $T_H = 90^{\circ}$, $A_T = 3m^2$, $A_0 = 0.05$m, constant $C_D = 0.7$. After linearization the system matrices used to design LQR are computed as below:

$$A=\begin{bmatrix} -0.0258 & 0 \\ 0 & -0.0517 \end{bmatrix}$$

$$B=\begin{bmatrix} 0.333 & 0.333 \\ -21.67 & 5 \end{bmatrix}$$

$$C=\begin{bmatrix} 0 & 1 \end{bmatrix}$$

With the choice of

$$Q=\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$$

$$R=\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

we obtain LQR gain vector as following:

$$K=\begin{bmatrix} 0,9079 & -0.2295 \\ 0.2338 & 0.9495 \end{bmatrix}$$

the temperature response with LQR is shown in fig. 4 .

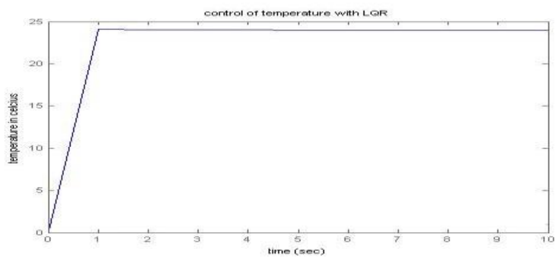

Fig.4 Control of temperature with LQR

SIMULINK model of the system using two PID controllers having parameters,

PIDControl schemes	Height	temperature
K_P	1.9438	1.45028
K_I	3.16	7.146180
K_D	-0.0049493	-0.145766

Table 1. PID controller parameters

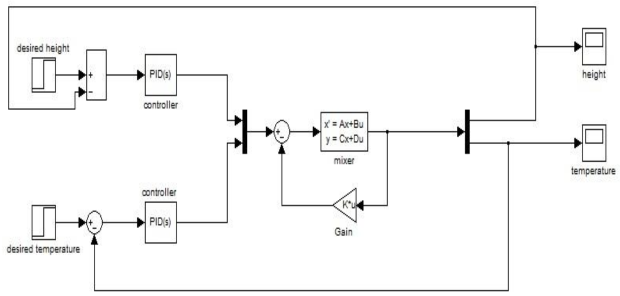

Fig.5 Block diagram of Two PID With LQR

The response of above model is as shown in fig. 6.

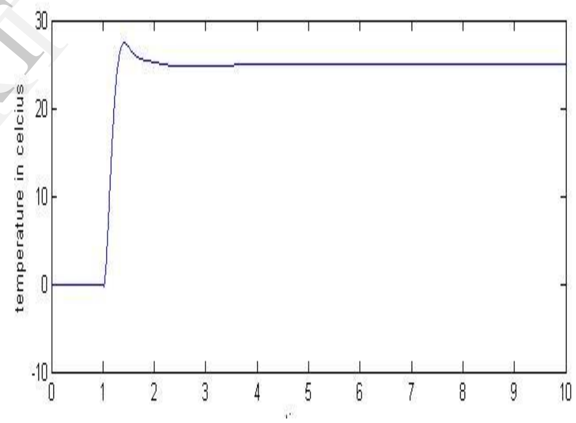

Fig.6 Response of temperature with two PID and LQR

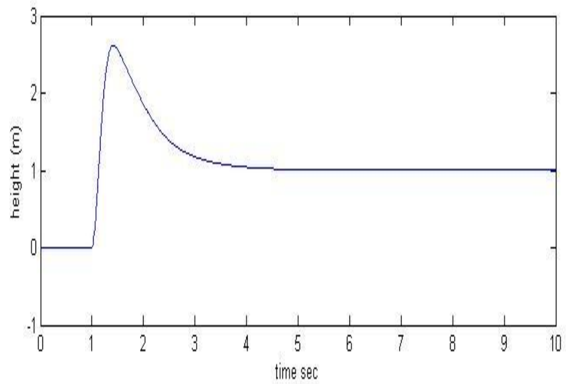

Fig.7 Response of height with two PID and LQR

TIME DOMAIN SPECIFICATION	2PID+LQR	LQR
RISE TIME	6.3053	21.6764
SETTLING TIME	17.7636	108.5134
OVERSHOOT	0	9.6023
UNDERSHOOT	0	0.8522
PEAK	27.4006	12.5025
PEAK TIME	57	63

Table 2. Time Domain Specification Comparison

It is observed that the product temperature reaches the setpoint without overshoot and offset while using two PID and LQR.

The setpoint tracking and disturbance rejection capability of the controller is verified by using the fig 7

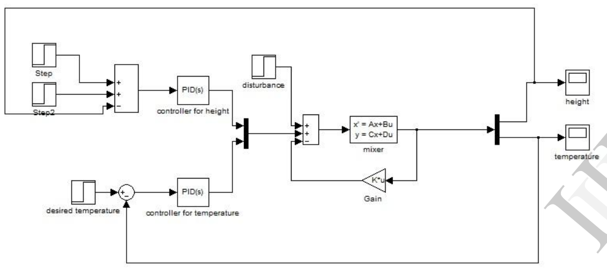

Fig 7. Block diagram of setpoint tracking and disturbance rejection of the system

Its response is as shown in fig 8.

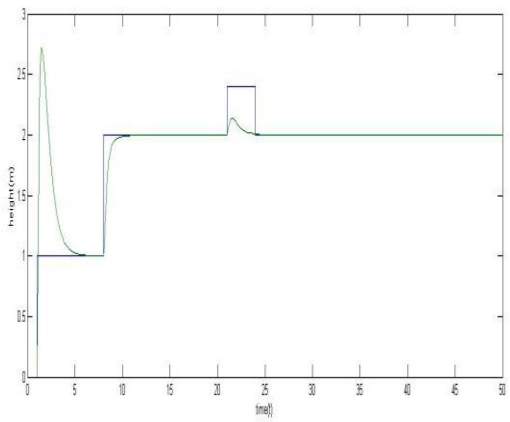

Fig 8. Response for setpoint tracking and disturbance rejection

V. CONCLUSION

PID with LQR controller, is used to control the effective temperature of a batch reactor. In order to compare the results initially system with only LQR is implemented and later on system with two PIDs with LQR is implemented. The MATLAB-SIMULINK models have been developed for the simulation of both control schemes. The simulation results justify that performance of two PID+LQR control scheme is better than LQR control scheme. Also it is verified that the system tracks the setpoint and rejects the disturbance in an effective manner. The performance investigation of this approach with fuzzy controller may be done as a future scope of this work.

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