Design of Set-Point Filter and PID Controller in Double Feedback Loop for Conical Tank

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Abstract:
The aim of this paper is to eliminate the peak overshoot in the response of a conical tank. To design a set-point filter, the following values are required: the peak overshoot values and the corresponding peak time. The controller of the inner loop in the double feedback is tuned by Zeigler Nichol's method, the outer loop is tuned by internal model controller (IMC) based PID tuning rule. This method needs a tuning parameter which is obtained from the set-point filter. The time constant of the set-point filter is used to tune the internal model controller based PID tuning rule. The simultaneous usage of the set-point filter and the double feedback results in complete elimination of peak over shoot.
Keywords: ***peak*** *overshoot, PID, IMC, Set-point filter, Tuning.*

1. Introduction:

The objective of every industry is to measure and control its parameters. In most of the chemical industries, controlling the process parameters such as level, pressure, flow, temperature etc is very essential. For example, controlling the level of the dangerous liquids such as acids in Chemical Industries.PID controllers are the simplest and perfect controllers available today. Various tuning methods can be employed for designing the controller parameters. These parameters are obtained by designing the filter. The time constant of the filter is obtained by performing simple calculations. It requires only the peak overshoot value and the corresponding peak time. The conical tank is commonly used in process industries because its shape provides better drainage of viscous liquids. The nonlinear shape of the conical tank makes the level control a typical problem. The non linearity and varying area of cross section of a conical tank is a use the filter in the circuit, the amplitude of the oscillation can be reduced. The proposed method will give us the k_p, k_i, and k_d values by using the time constant of the filter[5]. In double feedback loop the controller used can be P, PI, or PID. Among these the PID controller is the most effective one.

2. System description:

2.1 Conical tank system:

The conical tank level process which is a nonlinear process whose parameters vary with respect to the process variable is considered. At a fixed outlet flow rate the system is controlled and maintained at the desired level. The time constant and gain are the important variables which vary as a function of level in the chosen process.

Figure 1: Experimental Setup of a Conical Tank level system

The desired level 'h' is maintained by manipulating the inlet flow rate 'q1' to the system. Thus 'h' is the controlled variable and 'q1' is the manipulated variable.

2.2 Block diagram:

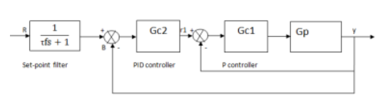


Figure 2: Block diagram of proposed method

The double-feedback loop provides stability and better performance. The stability is obtained from the inner loop. The outer loop and set-point filter can be used for good set-point tracking and reducing peak overshoot.

2.3 Process model

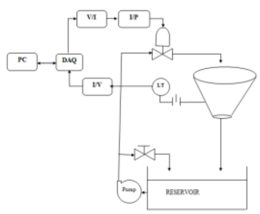


Figure 3: Process model of proposed system

Where

LT :Level transmitter.
I/V :Current to voltage converter.
V/I :Voltage to current converter.
I/P :Current to pressure converter.
DAQ :data acquisition card.

3. Mathematical modeling:

The conical tank is the process considered which is given in figure 4.

Figure 4: conical tank

ρ :density of liquid in the tank Kg/cm^3
ρ1 :density of liquid in the inlet stream Kg/cm^3
ρ2 :density of liquid in the inlet stream Kg/cm^3
V :the total volume of the conical tank cm^3
qI :volumetric flow rate of inlet stream LPH
q :volumetric flow rate of outlet stream LPH
R :Maximum radius of the cone cm
r :Radius of the cone at steady state cm
H :Maximum height of the cone cm
h :Height of the cone at steady state cm

Using the law of conservation of mass,

$$\left[\frac{\text{accumulation of total mass}}{\text{time}}\right] = \left[\frac{\text{input of total mass}}{\text{time}}\right] - \left[\frac{\text{output of total mass}}{\text{time}}\right]$$

$$\frac{d\left(\rho V(t)\right)}{dt} = \rho_1 q(t) - \rho_2 q_0(t)$$

Assume that the room temperature as well as the density of liquid is constant, ρ=ρ1=ρ2.

The volume of cone $V = \frac{1}{3}\pi r^2 h$

Where, $r = \frac{R}{H}h$

$$\frac{H(S)}{Q(S)} = \frac{m_t}{(\tau s+1)}$$

Where, time constant $\tau = m_t \alpha h s^2$ and process gain $m_t = \frac{2\sqrt{h}}{c}$

Specifications of conical tank:

Height	: 80 cm
Volume	: 33.5 litres
Bottom Diameter	: 7.62 cm
Top Diameter	: 36.62 cm
Angle	: 10deg
Material	: Stainless Steel

The transfer function of the conical tank system obtained is

$$\frac{H(s)}{Q(s)} = Gp = \frac{0.9363e^{-20s}}{86.982s+1}$$

4. Set point filter design[3]:

Numerous methods are available to design a set-point filter which needs extensive calculations. Moreover, the existing techniques need information about the process parameters, controller setting values and are laborious. But the proposed method is simple and requires only the peak overshoot value and the peak time of the system response regardless of the type and order of the system with arbitrary PID parameters.

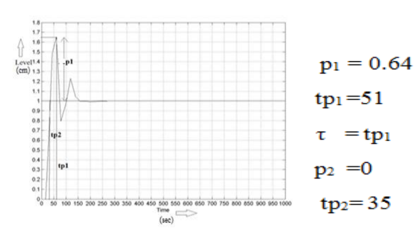


Figure 5: Peak over shoot response of PID controller

P_1 : Closed loop Peak overshoot
tp_1 : Corresponding Peak time
P_2 : desired overshoot of closed loop
tp_2 : Corresponding Peak time

Gain k = $p_1/0.6321$

The process transfer function is $Gp = \frac{0.9363e^{-20s}}{86.982s+1}$

The approximated transfer function is

$$\frac{Y(s)}{U(s)} = \frac{k}{\tau s+1} e^{-ds}$$

τ : time constant
d : time Delay
τf : time constant of set-point filter

Introducing Set Point filter

$$\frac{Y(s)}{1/s} = \frac{k}{(\tau s+1)} e^{-ds} * \frac{1}{\tau fs+1}$$

$$Y(s) = \frac{k}{s(\tau s+1)} e^{-ds} * \frac{1}{\tau fs+1}$$

Apply partial fraction

$$\frac{k}{s(\tau s+1)} e^{-ds} * \frac{1}{\tau fs+1} = \frac{A}{s} + \frac{B}{\tau s+1} + \frac{C}{\tau fs+1}$$

A = K, $B = -\tau^2 \frac{k}{\tau - \tau f} e^{d/\tau}$ and

$$c = -\tau f^2 \frac{k}{\tau - \tau f} e^{d/\tau f}$$

Inverse Laplace Transform

$Y(t) = K + (k/\tau - \tau f)\,)[\,\tau f^* e^{d-1/\tau f} - \tau^{*\, d-1/\tau}]$

At t = tp_2 +d then Y(t) = p_2 , take -d+1 = -t',
If Y(t) = p_2 then t' becomes tp_2

$p_2 = K + (k/\tau - \tau f) * [\ -\tau^* e^{-tp2/\tau}\ ]$

Since $e^{-tp2/\tau f} << e^{-tp2/\tau}$

$\tau f = \tau^*(K - p_2 - K^* e^{-tp2/\tau}) / (K - p_2) = 23.0482$

5. IMC based PID controller tuning[2]:

Gc1 is a proportional controller, therefore Gc1=kc1.Gc2 is tuned by this tuning rule. The transfer function of the conical tank is given as

$$Gp = \frac{0.9363e^{-20s}}{86.982s+1}$$

Let Gp1 is the inner loop transfer function.

$$Gp = \frac{G_{c1}G_p}{1+G_{c1}G_p}$$

$$Gp1 = \frac{5.45(fs+1)e^{-20s}}{86.982s+1+5.45*e^{-20s}}$$

Where K = kc1*kp =5.816*0.9363 =5.45

G_D is the desired closed loop transfer function of the block diagram.

$$\frac{Y}{B} = G_D = \frac{\frac{G_{C2}G_{C2}G_P}{1+G_{C1}G_P}}{1+\frac{G_{C2}G_{C1}G_P}{1+G_{C1}G_P}}$$

$$G_{C2} = \frac{G_D(1+G_{C1}G_P)}{G_{C1}G_P(1-G_D)} = \frac{G_D}{G_{P1}(1-G_D)}$$

Let G_D is given by $G_D = \frac{e^{-ds}}{(xs+1)(ys+1)}$

Where x= τ_f (23.0482) and y=0 for the first order system. Whereas x=y=τ_f for second order system.

Here the time constant of set-point filter is used as the tuning parameter to tune kc, ki, and kd values.

$$G_D = \frac{e^{-20s}}{(23.0482s+1)}$$

Outer controller Gc2 is given by

$$G_{c2} = \frac{15.96s+0.1835+e^{-20s}}{1+23.0482s-e^{-20s}}$$

Infinite series is given by,

$$e^{-Ds} = 1 - \frac{Ds}{1!} + \frac{(Ds)^2}{2!} - ...., \quad -\infty < \mathrm{Ds} < \infty$$

$$G_{c2} = \frac{15.96s + 0.1835 + 1 - 20s + 200s^2}{1 + 23.0482s - 1 + 20s - 200s^2}$$

$$= \frac{1}{s} * \left[\frac{1.1835 - 4.04 + 200s^2}{43.0482 - 200s} \right]$$

Gc_2 can be written as $Gc_2 = \Phi(s)/s$

$$\phi(s) = \left[\frac{1.1835 - 4.04s + 200s^2}{43.0482 - 200s} \right]$$

According to Laurent series[4]

$$\phi(s) = -\infty .... + \phi(0) + \frac{\phi'(0)}{1!} + \frac{\phi''(0)}{2!} + ...\infty$$

To find out kc, ki, and kd. $\Phi(0)=0.02749, \Phi'(0)=0.03388, \Phi''(0)=9.606746$

$$Gc_2 = \frac{1}{s} * \left[0.02749 + \frac{0.03388s}{1!} + \frac{9.606746s^2}{2!} \right]$$

$$= 0.03388 \left[1 + \frac{0.811393152}{s} + 141.7761s \right]$$

The standard form of PID controller is given by

$$Gc_2 = kc \left[1 + \frac{1}{\tau is} + \tau ds \right]$$

Kc =0.03388

τi = 1.23245

τd = 141.7761

kd = kc* τd

= 4.8034

$$ki = \frac{kc}{\tau i}$$

=0.02748

6. Results and discussion:

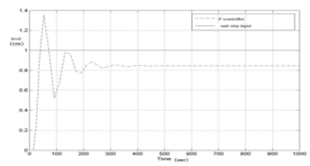
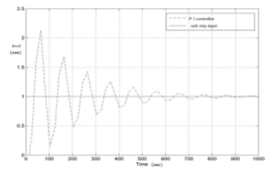
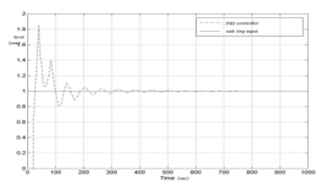
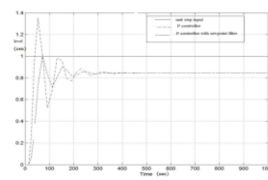
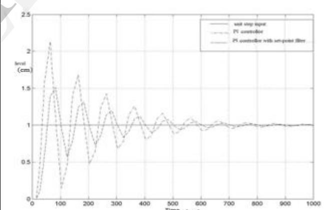
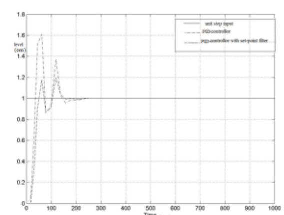
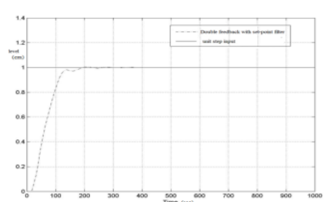
1.kc = 5.86 	2. kc = 5.86, ki=0.149 
3.kc=5.86,ki=0.149, kd=56.706 	4.kc=5.816, τf=23.0482 
5. kc= 5.816, ki= 0.149, τf = 23.0482 	6. kc= 5.816, ki= 0.149, kd = 56.706, τf = 23.0482 
7. kc=0.03388,kd=4.8034, ki = 0.02748 	

Figure 6: Responses of closed loop systems

The P, PI, PID controllers are designed using Zeigler Nichol's tuning method[6]. In proportional controller the output is proportional to the error signal. The response has offset error. The PI controller response is sluggish and the output is proportional to error signal as well as the integral of error signal. Even though PID controller is a robust controller, its response has peak overshoot. The set-point filter is designed and the filter time constant obtained is τf = 23.0482. The IMC based PID[1] tuning rule is used to tune the parameters kc=0.03388, kd = 4.8034, ki =0.02748. The tuning parameter considered is the filter time constant τf = 23.0482.

6.1 Comparison:

	P	PI	PID	Set-point filter	Double feedback loop
Peak overshoot	0.4	1.1	0.85	0.64	0
Settling time	300	1000	400	250	240

Table 1: comparison of performance measures

6.1.1 Explanation:

The peak overshoot of the response is zero by using double feedback loop with set-point filter. The proportional controller is having rapid response. The proportional integral controller provides large settling time compared to other controllers. The usage of set-point filter along with single closed loop will give more fast settling. The best settling time is obtained from the double feedback loop with set-point filter.

7. Conclusion and Future Work:

Here the proposed method is a simple first order set-point filter and a new IMC based PID tuning rule for open loop low order processes. The tuning parameter τ that is used for tuning the PID controller is used as filter time constant. According to design objective, the peak overshoot as well as the settling time is reduced. Future work is real time implementation of PID controllers in double feedback loops for conical tank with set-point filters using LabVIEW. The controller can be used for disturbance rejection and performance under the presence of measurement noise.

8. References:

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