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Design of Sliding Mode Controller via Novel Model Order Reduction Technique

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***Abstract*—This paper presents a mixed method for finding stable reduced order models using error minimization technique and modified pole clustering technique for single input-single output (SISO) large scale systems. The denominator polynomial of the reduced order model is determined by forming the clusters of the poles of original system and the cluster centers are identified by using inverse distance measure criterion and the numerator polynomial is obtained by the error minimization technique. For this reduced order model, sliding mode controller is designed. The performance of the system for both PID controller as well as SMC is compared. This reduced system with SMC is stable even when the original high order system is unstable.**

Keywords—Modified pole clustering, Order reduction, Error minimization technique, Stability, Transfer function, Sliding mode controller.

I. INTRODUCTION

Model order reduction (MOR) is a technique for reducing the computational complexity of mathematical models in numerical simulations. Many modern mathematical models of real-life processes pose challenges when used in numerical simulations, due to complexity and large size (dimension). Model Order Reduction aims to lower the computational complexity of such problems, for example, in simulations of large-scale control systems. By a reduction of the model's associated state space dimension or degrees of freedom, an approximation to the original model is computed. This reduced order model can then be evaluated with lower accuracy but in significantly less time.

A model is often too complicated to be used in real life problems. It is an un-debated conclusion that, the development of mathematical model of physical system made it feasible to analyze and design. So the procedures based on the physical considerations or mathematical models are used to achieve simpler models than the original one. Whenever a physical system is represented by a mathematical model it may be a transfer function of very high order. Available methods for analysis and design may become cumbersome when applied to a system of higher order [1-4].

The modeling of complex dynamic system is one of the most important subject in engineering .A High order system are not well suited for analysis and design. It becomes necessary to reduce the order of these high-order systems. Model order reduction technique plays an important role in various engineering applications, especially in large scale control systems. Many reductiontechniques for linear continuous systems such as Padé approximations, Routh approximations and time moment matching based methods have been proposed [5-9].

In the proposed method, the denominator polynomial of the reduced order model is obtained by using modified pole clustering technique while the coefficients of the numerator are obtained by error minimization technique [10-12].

II. PROBLEM STATEMENT

Let the transfer function of high order original system of the order 'n' be

$$G_n(s) = \frac{N(s)}{D(s)} = \frac{a_0+a_1s+a_2s^2+\cdots+a_{m-1}s^{m-1}+a_m s^m}{b_0+b_1s+b_2s^2+\cdots+b_{m-1}s^{m-1}+b_m s^m} \qquad (1)$$

Where $a_i; 0 \le i \le m$ and $b_i; 0 \le i \le n$ are known scalar constants.

Let the transfer function of the reduced model of the order 'k' be

$$R_k(s) = \frac{N_k(s)}{D_k(s)} = \frac{d_0+d_1s+d_2s^2\ldots+d_{q-1}s^{q-1}+d_q s^q}{e_0+e_1s+e_2s^2\ldots+e_{r-1}s^{r-1}+e_r s^r}$$
(2)

Where $d_i; 0 \le i \le q$ and $e_i; 0 \le i \le r$ are unknown scalar constants.

The objective of this paper is to realize the k^{th}-order reduced model in the form of (2) from the original system (1).

III. COMPUTATION OF ROM

The reduction procedure for getting the k^{th}-order reduced models consists of the following two steps:

Step-1: Determination of the denominator polynomial using modified pole clustering technique [9]:

The following rules are to be followed for clustering the poles of the original system given in frequency domain:

- The number of cluster centers to be calculated is equal to the order of the reduced system.
- The poles are distributed in to the cluster center for the calculation such that none of the repeated poles present in the same cluster center.

- Minimum number of poles distributed per each cluster center is at least one. There is no limitation for the maximum number poles per cluster center.

The procedural steps for realizing the denominator polynomial by using the modified pole clustering are:

Let there be r real poles in i^{th}cluster are $p_1, p_2, \ldots p_r$ where $|p_1| < |p_2| \ldots < |p_r|$and then modified cluster center p_{ei}can be obtained by using the algorithm of modified pole clustering suggested in this paper.

Let $m$pair of complex conjugate poles in the j^{th}cluster be $[(\alpha_1 \pm j\beta_1), (\alpha_2 \pm j\beta_2), \ldots\ldots\ldots (\alpha_m \pm j\beta_m)]$ where

$$|\alpha_1| < |\alpha_2| \ldots < |\alpha_m|.$$

Now using the same algorithm separately for real and imaginary parts of the complex conjugate poles, the modified cluster center is obtained, which is written as

$$\emptyset_{ej} = A_{ej} \pm jB_{ej},$$

where $\dot{\emptyset}_{ej} = A_{ej} + jB_{ej}$ and $\ddot{\emptyset}_{ej} = A_{ej} - jB_{ej}$.

For finding the modified cluster center, the below algorithm has to be followed:

Step-1: Let r real poles in a cluster be

$$|p_1| < |p_2| \ldots < |p_r|.$$

Step-2: Set $j= 1$.

Step-3: Find pole cluster center

$$c_j = \left[\sum_{i=1}^{r}\left(\frac{-1}{|p_i|}\right) \div r\right]^{-1}$$

*Step-4:*Set $j = j+1$.

Step-5: Now find a modified cluster center from

$$c_j = \left[\left(\frac{-1}{|p_1|} + \frac{-1}{|c_{j-1}|}\right) \div 2\right]^{-1}$$

Step-6:Is r = j? , if No, then go to step-4 otherwise go to step7.

Step-7: Take a modified cluster center of the k^{th}–cluster as

$$p_{ek} = c_j$$

After calculating the modified cluster centers, one of the following cases may occur:

Case-1: If all modified cluster centers are real, then denominator polynomial of the k^{th}-order reduced model can be obtained as

$$D_k(s) = (s - p_{e1})(s - p_{e2})\ldots\ldots\ldots(s - p_{ek}) \quad (3)$$

Where $p_{e1,}p_{e2}, \ldots\ldots p_{ek}$ are $1^{st}, 2^{nd}, \ldots\ldots k^{th}$modified cluster center respectively.

Case-2: If all modified cluster centers are complex conjugate then the k^{th}-order denominator polynomial is taken as

$$D_k(s) = (s - \dot{\emptyset}_{e1})(s - \ddot{\emptyset}_{e1})\ldots(s - \dot{\emptyset}_{ek/2})(s - \ddot{\emptyset}_{ek/2}) \quad (4)$$

Case-3: If some cluster centers are real and some are complex conjugate. For example, (k - 2) cluster centers are real and one pair of cluster center is complex conjugate, then k^{th}- order denominator can be obtained as

$$D_k(s) = (s - p_{e1})(s - p_{e2})\ldots(s - p_{e(k-2)})(s - \dot{\emptyset}_{e1})(s - \ddot{\emptyset}_{e1}) \quad (5)$$

Hence, the denominator polynomial $D_k(s)$ is obtained as

$$D_k(s)=e_0+e_1s+\ldots\ldots\ldots+e_rs^r \quad (6)$$

Step-2: Determination of the numerator of the reduced model by Error minimization technique:

The obtained reduced order denominator polynomial is used here to obtain the values of reduced order numerator. The given higher order transfer function and the general form of reduced order transfer function are equated.

$$\frac{a_0+a_1s+a_2s^2\ldots+a_{m-1}s^{m-1}+a_ms^m}{b_0+b_1s+b_2s^2\ldots+b_{n-1}s^{n-1}+b_ns^n} = \frac{d_0+d_1s+d_2s^2\ldots+d_{q-1}s^{q-1}+d_qs^q}{e_0+e_1s+e_2s^2\ldots+e_{r-1}s^{r-1}+e_rs^r} \quad (7)$$

On cross multiplying the above equation and comparing the same powers of ‘s’ on both sides, we get the following equations:

$$\begin{aligned} a_0e_0 &= b_0d_0 \\ a_0e_1 + a_1e_0 &= b_0d_1 + b_1d_0 \\ a_0e_2 + a_1e_1 + a_2e_0 &= b_0d_2 + b_1d_1 + b_2d_0 \\ &\cdot \\ &\cdot \\ &\cdot \\ a_me_r &= b_nd_q \end{aligned} \quad (8)$$

On solving the above equations, we can find the unknown numerator coefficients $d_0, d_1, \ldots d_q$. Hence the reduced order numerator polynomial can be written as:

$$N_r(s) = d_0 + d_1s + d_2s^2\ldots+d_{q-1}s^{q-1} + d_qs^q \quad (9)$$

IV. SLIDING MODE CONTROL

In control theory, sliding mode control, or SMC, is a control method that alters the dynamics of a system by application of discontinuous control signal that forces the system to "slide" along a cross-section of the system's normal behavior. The state-feedback control law is not a continuous function of time. Instead, it can switch from one continuous structure to another based on the current position in the state space. Hence, sliding mode control is a variable structure control method. The multiple control structures are designed so that trajectories always move toward an adjacent region with a different control structure, and so the ultimate trajectory will not exist entirely within one control structure. Instead, it will slide along the boundaries of the control structures. The motion of the system as it slides along these boundaries is called a sliding mode and the geometrical locus consisting of the boundaries is called the sliding (hyper) surface.

A. Linear Control Law

The system is given by the equation

$\dot{x}(t) = Ax(t) + Bu(t)$
(10)

The sliding surface can be represented by

s(x,t)= Sx(t),where S∈ $R^{m\times n}$

Also, $S\,\dot{x}(t) = SAx(t) + SBu(t)$
(11)

Once the sliding surface is reached, then Sx(t)=0

Therefore, from (7), the linear control laws can be obtained as
$u_L = -(SB)^{-1}SAx(t)$
(12)

which is equivalent to linear control law equation

$k = (SB)^{-1}SA$
(13)

designed such that the states would remain on sliding surface. SBmust be non-singular. Here S is a design parameter and B has a rank of m.

B. Reachability of the sliding surface

The second order system is given as

$\ddot{x}(t) + f(x,t) = u(t)$
(14)

Sliding mode control for this system may be selected as

$u(t) = -k_1x(t) - k_2\dot{x}(t) - \rho sign(\dot{s}(t))$
(15)

The linear part of this control input is state feedback law, which is in agreement with the structure of above equation and the discontinuous part is to be arranged so that the system is insensitive to disturbances.

V. PID Controller

The PID controller is the most common form of feedback. It was an essential element of early governors and it became the standard tool when process control emerged in the 1940s. In process control today, more than95% of the control loops are of PID type, most loops are actually PI control. PID controllers are today found in all areas where control is used.

The PID algorithm is described by:

$u(t) = K\left(e(t) + \frac{1}{T_i}\int_0^t e(\tau)d\tau + T_d\frac{de(t)}{dt}\right)$
(16)

wherey is the measured process variable, *r* the reference variable, *u* is the control signal and *e* is the control error ($e = y_{sp} - y$). The reference variable is often called the set point. The control signal is thus a sum of three terms: the P term (which is proportional to the error), the I term (which is proportional to the integral of the error), and the D term (which is proportional to the derivative of the error). The controller parameters are proportional gain *K*, integral time *Ti*, and derivative time *Td*.

VI. Numerical Examples

The proposed method is explained by considering two numerical examples, taken from the literature. The goodness of the proposed method is measured by observing the step responses of the original and reduced model using MATLAB. The step response of the reduced model should be nearly same as that of original higher order system. [13, 14].

Example-1: Consider an sixth-order system [9]

$$G(s) = \frac{3s^5 + 108s^4 + 1221s^3 + 5792s^2 + 11860s + 8400}{s^6 + 41s^5 + 571s^4 + 3491s^3 + 10060s^2 + 13100s + 6000}$$

The poles are: -1, -2, -3, -5, -10,-20.

Let the 2nd -order reduced model is required to be realized, for this purpose only two real clusters are required.

Let the first and second cluster consists the poles (-1, -2, -3) and (-5, -10, -20) respectively.

The modified cluster centers are computed as

P_{e1}=-1.1076

P_{e2}=-5.5813

Using equation (3), the denominator polynomial $D_2(s)$ is obtained as

$D_2(s)$=(s+1.1076)(s+5.5813)

=s^2+6.6889s+6.1818

Using the equations (8) and (9), the following coefficients are calculated:

d_o= 8.6545

d_1= 2.6881

Therefore, finally 2nd –order reduced model is obtained as

$$R_2(s) = \frac{2.6881s + 8.6545}{s^2 + 6.6889s + 6.1818}$$

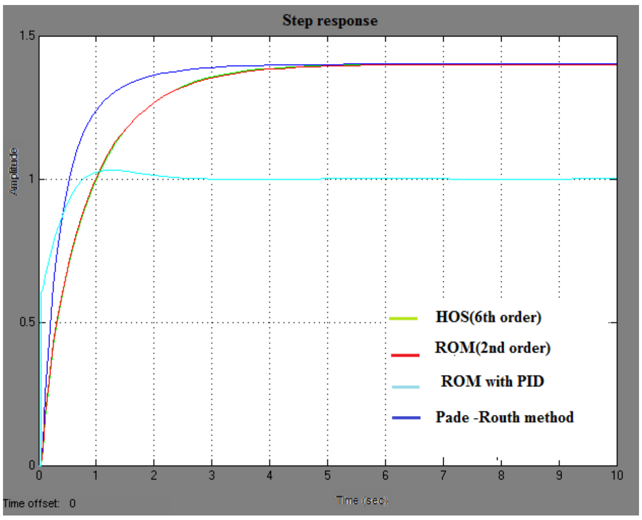

Fig. 1. Comparison of the step responses for the given example 1

TABLE I. COMPARISON WITH OTHER EXISTING METHODS

Method	Reduced model	ISE
Proposed method	$\frac{2.6881s + 8.6545}{s^2 + 6.6889s + 6.1818}$	0.00019
Pade-Routh method	$\frac{0.992s + 1.4017}{0.243s^2 + 1.1434s + 1.0006}$	0.97

VII. EXTENSION FOR DESIGNING OF SLIDING MODE CONTROLLER

*Example 2:*Consider a fourth order system described by the transfer function [3]

$$G(s) = \frac{72 + 54s + 12s^2 + S^3}{100 + 180s + 97s^2 + 18s^3 + s^4}$$

First we have to find the reduced denominator polynomial by using modified pole clustering technique:

Using step-1, two cluster centres from the real poles -1,-2,-5 and -10 can be formed. The first cluster contains two poles i.e -1 and -2. The second cluster contains remaining two poles i.e -5 and -10.

By using the modified cluster center algorithm steps, we get

P_{e1}=-1.1416

P_{e2}=-5.7142

Therefore, by using equation (3) we get

$D_2(s)$ = (s+1.1416) (s+5.7142)

$=s^2$+6.8558s+6.5233

Next for the reduced order numerator coefficients in error minimization technique, using the equations (8) and (9), the following coefficients are calculated:

d_o=4.69

d_1=0.0045

Therefore, finally 2nd –order reduced model is obtained as

$$R_2(s) = \frac{4.69 + 0.0045s}{6.5233 + 6.8558s + s^2}$$

For this 2nd order model, a sliding mode controller is designed with a control input which is calculated by the given procedural steps from equations (10)-(15) and is obtained as

$$U_l = -171.395x_1 - 163.08x_2 - sign(5x_1 + x_2)$$

This control input is given to the reduced model and a Simulink block is designed for the sliding mode controller.

Step input is given to the original higher order system, reduced order model, ROM with sliding mode controller as well as PID controller and their responses are shown in the figure 2.

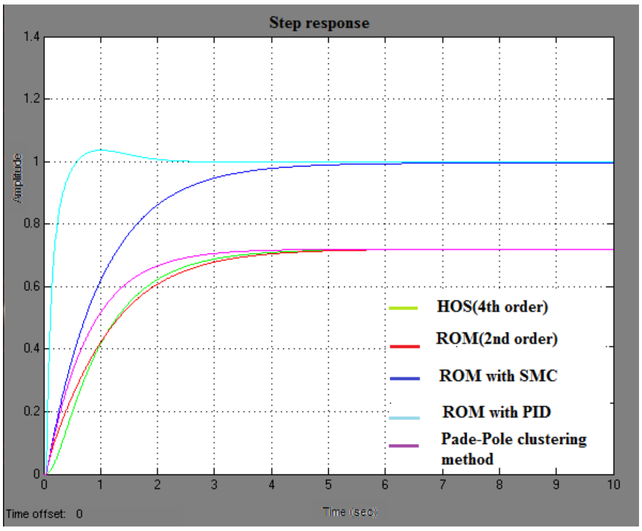

Fig. 2. Comparison of the step responses for the given example 2

TABLE II. COMPARISON WITH OTHER EXISTING METHODS

Method	**Reduced model**	**ISE**
Proposed method	$\frac{4.69 + 0.0045s}{6.5233 + 6.8558s + s^2}$	0.000025
Pade-Pole clustering method	$\frac{6.3920 + 0.9523s}{8.8778 + 7.9999s + s^2}$	0.002025

VIII. CONCLUSION

In this proposed order reduction method for the linear single-input-single-output high order systems, the determination of denominator polynomial of the reduced model is done by using the modified cluster method while the numerator coefficients are computed by error minimization technique. The proposed algorithm has been explained with a numerical example. The step responses of the original and reduced system of second order are shown in the Figure 1 and 2. This method is simple, proficient and takes little computational time. This method secures the stability and avoids steady-state error between the step responses of the original and reduced models. The results of SMC for second order system show the stabilized performances and robustness of SMC.

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