### Design of the Steering System of an SELU Mini Baja Car

Alfred Showers and Ho-Hoon Lee Department of Computer Science and Industrial Technology Southeastern Louisiana University, Hammond, LA 70402

### **Abstract**

In this paper a steering system is designed for a SELU Mini Baja car, which adopts a rack-and-pinion steering mechanism. The theoretical modeling of the system as well as the derivations of optimal parameter values are presented here. First, the steering angles of the front wheels are derived based on the geometry of the steering system. Second, linear equations representing the axial lines of the front wheels are derived based on the steering angles of the front wheels. Then the Ackermann steering errors are computed on the axial line of the rear wheels using the axial lines of the front wheels. Finally, the optimum values of the parameters of the steering system are obtained via computer programming such that the obtained values of the parameters minimize the Ackermann steering error on the axial line of the rear wheels.

### 1. Introduction

The steering systems play an important role in maneuvering vehicles, but they also have to provide good ergonomics. Substantial research and development have been performed for the design and optimization of the steering system of a vehicle [1-8].

There are two main types of steering systems for modern cars and light trucks: the conventional linkage steering system and the rack-and-pinion system. The conventional system was the only type used until the 1970s. In small cars, it has been replaced by the rack-and-pinion steering, but the conventional system is still used for many light trucks. The advantages of the rack and pinion steering system is that it is simple to construct, economical to manufacture, and compact and easy to operate [9].

The steering mechanism of a car must follow the Ackermann principle to reduce the skidding and wear of the tires. This principle states that all the axial lines of the wheels should meet at a point, i.e. the instant centre of turning.

Real steering mechanisms are complex and spatial in nature. However, the real rack and pinion steering linkage may be modeled as a planar linkage for the investigation of the Ackermann conditions [5]. Therefore, in this paper, the steering mechanism of a vehicle is considered as a planar linkage [3,5].

The objective of this project at Southeastern LA University (SELU) was to design a steering system for an SELU Mini Baja car, based on a rack-and-pinion steering mechanism. For this vehicle, the steering mechanism is built using a rack-and-pinion assembly and a steering knuckle assembly, which are available for commercial go-carts. As part of the design process, the optimum values of the steering parameters were computed. The derivation of these parameters was based on geometric modeling and the use of numerical methods. The value of the modeling and the derivation of optimal parameter values is of broader value than just the purposes of our project and can be directly applied to different size vehicles.

In this paper, the optimum distance from the front wheel axle to the steering rack axis is obtained such that the steering system satisfies the Ackermann principle with minimum steering error on the axial line of the rear wheel. First, the steering angles of the front wheels are derived based on the geometry of the steering system. Then linear equations representing the axial lines of both front wheels are obtained based on the steering angles, the distance between the left and right wheels, and the distance between the front and rear axles. Finally, the steering error, based on the Ackermann principle, is computed as a function of the displacement of the steering rack and the distance from the front axle to the steering rack axis.

The remainder of this paper is organized as follows. In Section 2, the model of a rack-and-pinion steering mechanism, the Ackermann principle, and the design objective are described. In Section 3, the steering angles of the front wheels are derived and the linear equations for the axial lines of the front wheels are obtained. In Section 4, the optimum distance from the front axle to the steering rack axis is computed so that the steering error is minimized. Finally, in Section5, conclusions are drawn from this study.

## 2. Ackermann principle of steering design2.1. Ackermann condition

Figure 1 shows the Ackermann principle, in which  $\theta_l$  and  $\theta_r$  denote the left and right steering angles, respectively;  $l_w$  and  $l_a$  denote the distance between the left and right wheels and the distance from the front axle to the rear axle respectively.

The Ackermann principle requires that the axial lines of all the wheels should meet at the same point which represents the centre of turning (point O in Fig. 1). A steering linkage must be designed based on the Ackermann principle to ensure pure rolling and to minimize the skidding and thus wear of the tires.

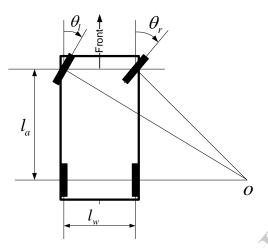


Fig. 1 Ackermann principle

### 2.2. Ackermann steering design

The Ackermann principle states that the extended lines from the steering arms meet in the middle of the rear axle as shown in Fig. 2.

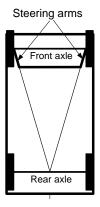


Fig. 2 Ackermann condition

The steering system of the SELU mini Baja car will be constructed by using a commercial steering knuckle assembly with a fixed angle of steering arm.

As a result of using an off-the-shelf assembly, the conventional linkage steering mechanism does not satisfy the Ackermann condition as shown in Fig. 2.

As a possible solution, a rack-and-pinion steering mechanism, shown in Fig. 3, is used in this steering design, in which the distance from the front axle to the rack axis is considered a design parameter.

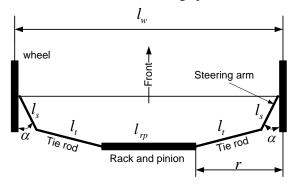


Fig. 3 A rack and pinion steering system

In the figure, s denotes the distance from the front wheel axle to the steering rack axis; r is the distance from the wheel to the rack;  $\alpha$  denotes the angle of steering arms;  $l_s$ ,  $l_t$ , and  $l_{rp}$  denote the lengths of the steering arms, tie-rods, and the rack, respectively.

### 2.3. Design objective

The design objective in this work is to find an optimum value of the distance between the front axle and the rack axis so that the optimum value results in minimum steering error on the line of rear axle; the steering error is defined in Section 4.

# 3. Design of the Steering System 3.1. Derivation of steering angles

As described above, the steering mechanism is built using a rack-and-pinion assembly and a steering knuckle assembly, which are designed for commercial go-carts. Therefore, the parameters  $l_{rp}$ ,  $l_s$ , and  $\alpha$  are constant. Then the parameters r and  $l_t$  are computed as

$$r = \frac{l_w - l_{rp}}{2} \tag{1}$$

$$l_t = \sqrt{(s - l_s \cos \alpha)^2 + (r - l_s \sin \alpha)^2}$$
 (2)

Figure 4 shows the rack and pinion steering system with the rack displaced by u, in which  $\theta_l$  and  $\theta_r$  represent the left and right steering angles, respectively;  $\phi_l$  and  $\phi_r$  represent the left and right tie rod angles, respectively.

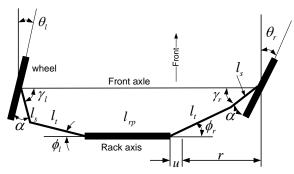


Fig. 4 Steering system having rack displacement

From Fig. 4, the following geometric equations are obtained for the right wheel:

$$\theta_r = \frac{\pi}{2} - \gamma_r - \alpha,\tag{3}$$

$$l_s \cos \gamma_r + l_t \cos \emptyset_r = r + u, \tag{4}$$

$$l_s \sin \gamma_r + l_t \sin \phi_r = s. \tag{5}$$

Similarly, for the left wheel,

$$\theta_l = \gamma_l + \alpha - \frac{\pi}{2},\tag{6}$$

$$l_s \cos \gamma_l + l_t \cos \phi_l = r - u, \tag{7}$$

$$l_s \sin \gamma_l + l_t \sin \phi_l = s. \tag{8}$$

For the right steering angle, Eqs. (4) and (5) can be rewritten as

$$l_t \cos \phi_r = r + u - l_s \cos \gamma_r, \tag{9}$$

$$l_t \sin \phi_r = s - l_s \sin \gamma_r \,. \tag{10}$$

By squaring and adding Eqs. (9) and (10),  $\emptyset_r$  is removed:

$$P\cos\gamma_r + O\sin\gamma_r + R = 0, \tag{11}$$

where

$$P = 2l_s(u+r), (12)$$

$$Q = 2l_s s, (13)$$

$$R = l_t^2 - l_s^2 - r^2 - s^2 - u^2 - 2ru.$$
 (14)

Dividing Eq. (11) by  $\sqrt{P^2 + Q^2}$  yields

$$\frac{1}{\sqrt{P^2 + O^2}} [P\cos \gamma_r + Q\sin \gamma_r + R] = 0, \tag{15}$$

which is equivalent to

$$\sin \delta_r \cos \gamma_r + \cos \delta_r \sin \gamma_r = -\frac{R}{\sqrt{P^2 + O^2}},$$
 (16)

where  $\delta_r = \tan^{-1}(P/Q)$ .

Eq. (16) is equivalent to

$$\sin(\delta_r + \gamma_r) = -\frac{R}{\sqrt{P^2 + Q^2}},\tag{17}$$

from which it follows that

$$\gamma_r = -\sin^{-1}\frac{R}{\sqrt{P^2 + Q^2}} - \delta_r.$$
 (18)

Likewise, for the left steering angle,  $\gamma_l$  can be obtained as

$$\gamma_{l} = -\sin^{-1} \frac{R_{l}}{\sqrt{P_{l}^{2} + Q_{l}^{2}}} - \delta_{l}, \tag{19}$$

where  $\delta_l = \tan^{-1}(P_l/Q_l)$ ,

$$P_l = 2l_s(r - u), \tag{20}$$

$$Q_l = 2l_s s, (21)$$

$$R_{l} = l_{t}^{2} - l_{s}^{2} - r^{2} - s^{2} - u^{2} + 2ru.$$
 (22)

Then, from Eqs. (3) and (6), the steering angles  $\theta_r$  and  $\theta_l$  are computed as

$$\theta_r = \frac{\pi}{2} + \sin^{-1} \frac{R}{\sqrt{R^2 + \Omega^2}} + \delta_r - \alpha,$$
 (23)

$$\theta_l = -\sin^{-1} \frac{R_l}{\sqrt{p_l^2 + Q_l^2}} - \delta_l + \alpha - \frac{\pi}{2}.$$
 (24)

### 3.2. Computation of steering error

Figure 5 shows the coordinate system that defines the steering error on the axial line of the rear wheels, based on the Ackermann principle.

The slope of left and right axial lines of the front wheels are  $-\tan\theta_l$  and  $-\tan\theta_r$ , respectively, as can be seen in Fig. 5. Then the equations of the axial lines of the left and right front wheels are obtained as

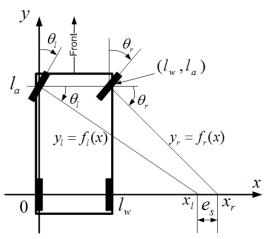


Fig. 5 Definition of Ackermann steering error

$$y_l = -\tan\theta_l \ x + l_a \tag{25}$$

$$y_r = -\tan\theta_r \, x + (l_a + l_w \, \tan\theta_r) \tag{26}$$

At y = 0, i.e., on the axial line of the rear wheels,

$$\chi_l = \frac{l_a}{\tan \theta_l} \tag{27}$$

$$\chi_r = \frac{l_a + l_w \tan \theta_r}{\tan \theta_r} \tag{28}$$

Then the steering error is defined as

$$e_s = x_l - x_r = \frac{l_a}{\tan \theta_l} - \frac{l_a + l_w \tan \theta_r}{\tan \theta_r}$$
 (29)

The objective of this project is to minimize  $e_s$  by finding the optimum value of s, the distance from the front axle to the steering rack axis.

### 4. Computing the optimal value of s

The optimum parameters are computed so that the steering error (29) is minimized. In the computation of the optimum value of s, the following parameters are used:  $l_a = 79$  in;  $l_w = 49$  in;  $l_{rp} = 14$  in;  $l_s = 3.5$  in;  $\alpha = 16.6^{\circ}$ .

The steering error  $e_s$  is computed numerically via C programming, the flow chart of which is shown in Fig. 6. First, for the fixed value of u = 2 in, which is the maximum displacement of the rack, the steering angles of the front wheels are computed using Eqs. (23) and (24) as a function of s. Then the steering error  $e_s$  is computed using Eq. (29).

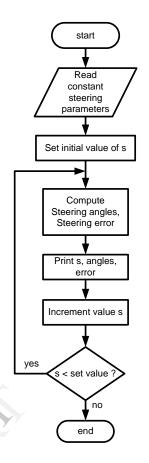


Fig.6 Flow chart of finding the steering error

<i>u</i> (in)	S (in)	$l_t$ (in)	$ heta_l$	$\theta_r$	$e_s$ (in)
			(rad)	(rad)	
2	2.683	16.514	0.580	0.746	-13.776
2	2.783	16.510	0.580	0.752	-12.756
2	2.883	16.507	0.580	0.758	-11.710
2	2.983	16.504	0.580	0.765	-10.636
2	3.083	16.502	0.580	0.771	-9.532
2	3.183	16.501	0.580	0.778	-8.393
2	3.283	16.500	0.580	0.786	-7.217
2	3.383	16.500	0.580	0.794	-6.000
2	3.483	16.501	0.580	0.802	-4.736
2	3.583	16.502	0.580	0.811	-3.420
2	3.683	16.503	0.580	0.820	-2.043
2	3.783	16.506	0.580	0.830	-0.596
2	3.883	16.508	0.580	0.840	0.933
2	3.983	16.512	0.580	0.852	2.560
2	4.083	16.516	0.580	0.864	4.307
2	4.183	16.521	0.580	0.879	6.209
2	4.283	16.526	0.580	0.895	8.315
2	4.383	16.532	0.580	0.913	10.710
2	4.483	16.539	0.580	0.936	13.565

Table 1 Steering angle error for different values of s

The result of determining the optimal steering error  $e_s$  is shown marked in Table 1. The minimum error is achieved with s = 3.78 in or s = 3.88 in, which are considered an optimal value of s. It should be noted that the length of the steering arm is 3.5 in.

### 5. Conclusion and Future Work

The rack and pinion steering system, designed for an SELU Mini Baja car, was constrained to use offthe-shelf commercial assemblies of a rack-and-pinion assembly and a steering knuckle assembly designed for go-carts. Due to this constraint the design cannot be Ackerman compliant, therefore the design must be optimized using the rest of the system geometry parameters.

The optimum value of the distance between the front axle and the steering rack axis has been systematically obtained so that the optimum value minimizes the Ackermann steering error on the axial line of the rear wheels.

This work serves as a guideline for the steering design of other Mini Baja cars or custom-built cars when production constraints do not make it possible to meet Ackerman compliance and thus optimization of other design parameters is necessary. The contribution is the methodology used to obtain the optimal design parameters using numerical methods.

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