

Detecting and Localising Damage In Bridge Structures From Vibration Measurements

Noel G. Nalitolola

Department of Mechanical and Industrial Engineering
University of Dar es Salaam

Abstract

Vibration characteristics of a structure correlate to the distribution of mass, stiffness and damping in the structure. Therefore, a change in any one of these three parameters will result in a change of the vibration characteristics of the structure. It is a fact that a structural damage results in a local reduction of stiffness of the structure and therefore a reduction of the natural frequencies. Methods to detect and possibly localise damage in structures using vibration measurements have been a subject of research interest among the structural dynamics community. The use of a vibration based methodology in detecting and localising damages in big structures like bridges will provide an efficient monitoring and diagnostic tool and overcome accessibility difficulties associated with physical inspection. The paper discusses a methodology proposed for the detecting and localising damage in structures with specific application to structural damage in bridges using vibration data.

1. INTRODUCTION

Efficient means of detecting and locating damage in structures is a subject of interest to engineers and technicians responsible for monitoring, maintenance and assurance of integrity of structures. Typical examples include aircraft structures, bridges and structures of key industrial machinery. The occurrence of a structural damage in aircraft structures or a bridge, for example, may pose a serious threat to the safety of the users. Moreover, in industrial machinery the damage may progress to an abrupt failure leading to costly unplanned breakdowns. Usually the damage start small and then progresses until failure occur. Therefore, to ensure the integrity of the structure, periodic inspection is usually conducted. The damage may be

caused by many factors, including stress as well as deterioration of the material. The damage may not be visible to the naked eye due to, either its size being small during the early stages or its location being not easily accessible. In the case of aircraft structures and industrial machinery physical inspection using non destructive test methods (NDT), eg. Ultrasonic flaw detection may be applied. Conventional NDT methods, however, are time consuming as they involve piecewise inspection over the entire structure. Likewise, a bridge will require piecewise inspection over its entire length while some areas may not be easily accessible. The vibration characteristics of a structure correlate to its mass, stiffness and damping distribution. A change in any one of these three parameters will result in a change of the vibration characteristics. It is known that a structural damage results in a local reduction of the stiffness and therefore a reduction of the natural frequencies. Therefore, monitoring the changes in vibration characteristics may tell something regarding the occurrence and location of the damage. A vibration based method is more attractive than the conventional NDT methods because vibration can be measured quickly and at only a few locations that are accessible.

Methods to detect and localise damage in structures based on vibration measurements have been a subject of research interest among the structural dynamics community from the early 1990's. The vibration based methods are based on measured natural frequencies and mode shapes. Richardson *et al* (1992) discusses a method for remote detection and localising defects in structures using vibration modal data. Fox (1992) compares the use of natural frequency and mode shape data in locating faults in structures. These methods attempt to localise damage using measurement information of natural frequencies and mode shapes before and after the occurrence of the damage. Narayana and Jebaraj (1999), and Abdo and Hori (2002) showed that mode shape measurement contains

displacement or strain information which is more intrinsically related to damage than natural frequency change. More recently, Koh and Ray (2003) investigated methods that attempt to reconstruct perturbations in the stiffness matrix from vibration responses of natural frequencies and mode shapes. A major challenge in the vibration based methods is that the modal data suffers the problem of incompleteness in the number of measured modes and measurement coordinates. Some coordinates are not accessible for measurement while rotational degrees-of-freedom are difficult to measure. As a result, erroneous localisation of the damage may occur. This paper investigates a technique for the localization of structural damage suited to bridge structures. It uses natural frequencies of the structure before and after the occurrence of the damage and it is based on model updating and strain energy.

2. THEORETICAL BACKGROUND

This section gives theoretical background of the proposed methodology to detect and localise damage in structures with a special focus on bridge structures. A bridge, Fig.1, is modelled as a simple beam with appropriate boundary conditions, for example simply supported or with clamped supports. Fig. 2 is an example of the bridge shown in Fig 1, modelled using 16 finite elements (E1 to E16) of 1m length each. S1 to S5 are simple supports. Once the model of the bridge is structured, its dynamic mathematical model, natural frequencies and modes of vibration are easily worked out. The low frequency modes of vibration are then used to work out strain energies of the elements of the finite element (FE) model.

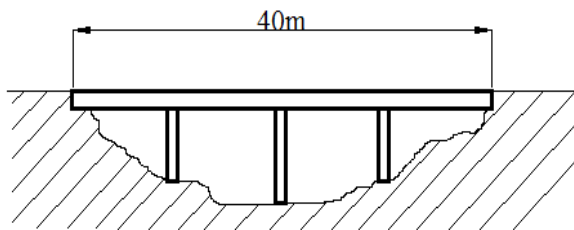


Fig.1: A 40m bridge with 4 sections

Let U be the strain energy of an element of the FE model. The strain energy is related to radius of curvature R as follows:

$$U = \frac{1}{2} \int_{element} EI \left(\frac{d^2 y}{dx^2} \right)^2 dx = \frac{1}{2} \int_{element} EI \left(\frac{1}{R} \right)^2 dx \quad (1)$$

Let's define reference strain energy as the strain energy per unit flexural rigidity. Thus, if we divide (1) by the flexural rigidity EI we obtain an expression for the reference strain energy U_{Ref} as:

$$U_{Ref} = \frac{1}{2} \int_{element} \left(\frac{1}{R} \right)^2 dx \quad (2)$$

The effect of damage in any particular element is to increase local flexibility and therefore lower the local stiffness and increase the curvature. Therefore, in an element with a damage, U_{Ref} after the damage is expected to be greater than U_{Ref} before damage. *i.e.*

$$U_{Ref}(\text{before damage}) - U_{Ref}(\text{after damage}) < 0 \quad (3)$$

Our task will therefore be to monitor natural frequencies of the bridge and detect any changes. Detecting changes in natural frequencies imply detecting the occurrence of a structural damage. Once damage has been detected, the next step is to localise it. Localisation can be with respect to an element of the finite element model or with respect to a section of the bridge. Localisation involves working out strain energies of each element or section of the bridge, before and after the damage has been detected

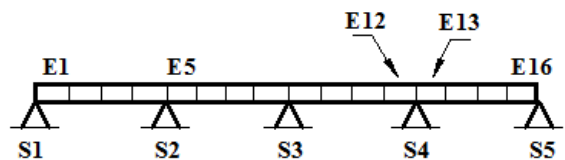


Fig. 2: Bridge modelled using 16 finite elements

Because the radius of curvature R is not easily measurable, the reference strain energy U_{Ref} cannot be

established directly using equation (2). The reference strain energy of the elements or sections of the beam is computed using the stiffness matrix of the finite element model of the respective element or section K_e , and the mode shape vectors of the respective element or section of the bridge, V_e . Thus,

$$U_{Ref} = \frac{1}{2} V_e^T K_e V_e \quad (4)$$

where

K_e portion of FE model covering the element or section of interest based on $EI = 1$.

V_e Portion of the mode shape vector covering element or section of interest

The stiffness matrix K_e , for a beam type structure is a well developed textbook material. For $EI = 1$ and based on a simple Euler-Bernoulli beam theory, the element stiffness matrix may be written as:

$$K_e = \frac{1}{L^3} \begin{bmatrix} 12 & 6L & -12 & 6L \\ 6L & 4L^2 & -6L & 2L^2 \\ -12 & -6L & 12 & -6L \\ 6L & 2L^2 & -6L & 2L^2 \end{bmatrix}$$

Since EI is taken as 1, the accuracy of mass and stiffness parameters of the FE model is not very important. What is important is that the mode shape vector used in the formulation of U_{Ref} before and after damage has been detected should be as correct as possible. Fortunately, mode shape vectors can be established from the dynamic FE model of the bridge. The model may be updated so that it reproduces the measured modes. Likewise, after the damage has been detected, the FE model parameters may be revised through updating algorithms (for example, Nalitolela, 2012) so that the model can reproduce measured natural frequencies and mode shapes of the damaged bridge. Literature on structural dynamics is rich with various model updating algorithms and therefore their discussion is not a subject of this paper.

Applying equation (3) for each finite element or section of the bridge, the damage is localized to the element or section(s) with large negative changes in strain energy.

3.0 SIMULATED EXAMPLES

Two examples to demonstrate the technique are presented. The first example is a study on the 16 elements FE model for the bridge with four sections as shown in Fig. 1, to localize the damage to within a section of the bridge and also attempting to localize the damage to within an element. The damage was simulated by the reduction of stiffness of one of the beam elements. The second example is the same bridge model but the damage simulated by introducing vertical flexibility on one of the supports.

3.1 Example 1

The 40m bridge of Fig. 1 was modelled with 16 beam elements of length 1m each, shown in Fig. 2, resulting in 29 degrees-of-freedom. The bridge was assumed to be simply supported at its ends and at intermediate supports. The flexural rigidity was taken as $EI = 50 \text{ kNm}^2$. The first three natural frequencies were computed from the FE model as follows:

$$f_1 = 2.195 \text{ Hz}, \quad f_2 = 2.562 \text{ Hz}, \quad f_3 = 3.43 \text{ Hz}$$

The corresponding mode shapes were also computed. Damage was simulated by reducing the stiffness of the fifth beam element, E5 in Fig. 2, by 10%. The new FE model generated the following natural frequencies of the damaged bridge:

$$f_{1d} = 2.193 \text{ Hz}, \quad f_{2d} = 2.553 \text{ Hz}, \quad f_3 = 3.41 \text{ Hz}$$

The corresponding mode shape vectors were also evaluated. Data of the undamaged and 'damaged' bridge were used to compute percentage changes in reference strain energy. Fig. 3 shows the results when the bridge was treated as to have four sections of interest. The sections are defined as follows:

- Section 1: Between support S1 and S2
- Section 2: Between support S2 and S3
- Section 3: Between supports S3 and S4
- Section 4: Between supports S4 and S5

The bar graph shows the percentage changes in reference strain energy of the sections summed over the first three modes. The graph tells us that the damage is located in section 2, between supports S2 and S3. The section containing the simulated damage has been correctly localised by having a large negative change in strain energy.

Fig. 4 shows the result when the bridge was treated to have as many sections as the number of finite elements (i.e. 16 sections).

The damaged element, element E5, has been localised as the element with a large negative change in strain energy.

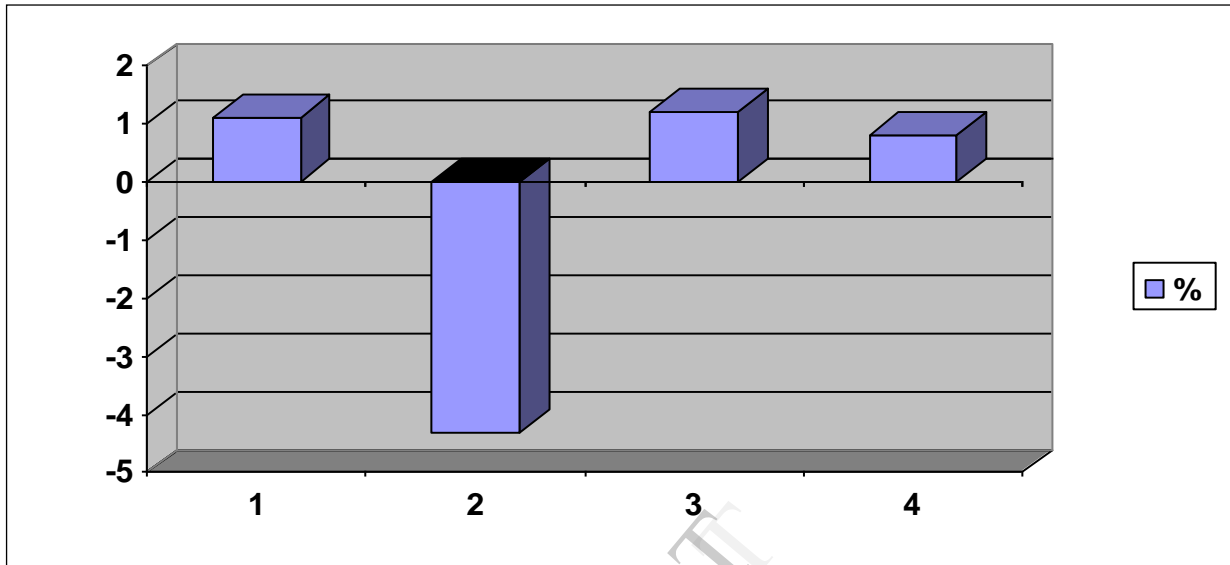


Fig. 3: Percentage change in reference strain energy of the beam sections summed for 3 modes

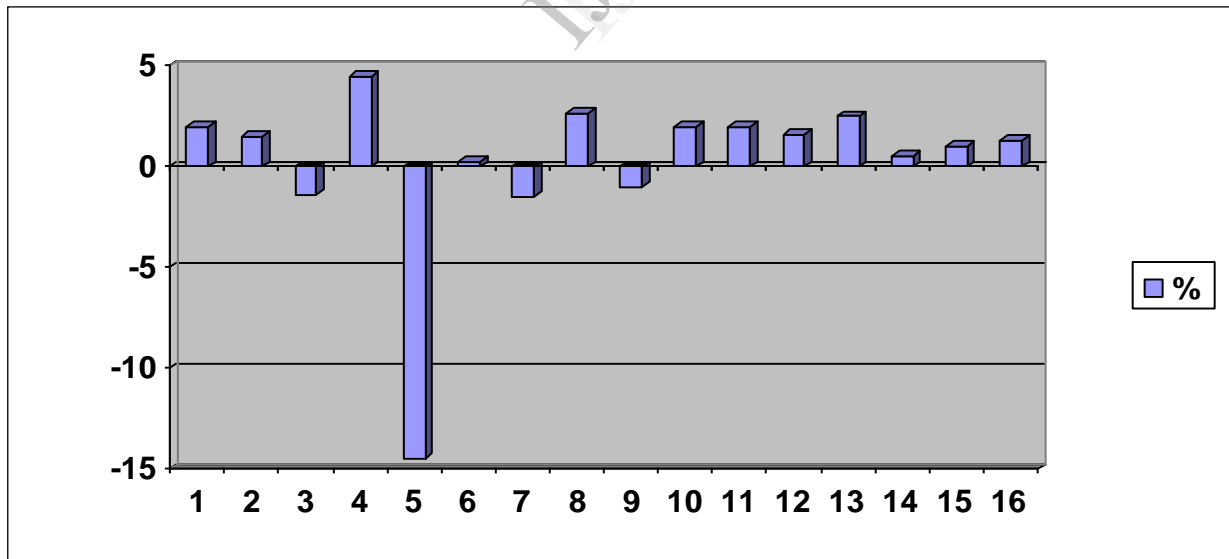


Fig. 4: Percentage change in reference strain energy of the beam elements summed for 3 modes

3.2 Example 2

In this example, damage was simulated by giving vertical flexibility to support S4. Giving the flexibility implied an additional degree-of-freedom. The flexibility was simulated by replacing support S4 by a linear stiffness of magnitude $k = 1 \text{ kN/mm}$, giving a flexibility of 1 mm/kN . Natural frequencies in this second damage case were as follows:

$$f_{1d} = 2.195 \text{ Hz}, \quad f_{2d} = 2.550 \text{ Hz} \quad f_3 = 3.341 \text{ Hz}$$

Corresponding mode shapes were computed and used in the evaluation of reference strain energies. Fig. 5 shows the result when the bridge was treated as to have four sections. The results tell us that damage is localized in sections 2 and 3. This is expected because damage was inflicted at the interface of the two sections. Fig. 6 shows the result when each of the 16 finite elements were treated as an individual section of interest. The result also tells us that damage is localized on beam element E12 and E13 because the damage was created at the interface of the two elements

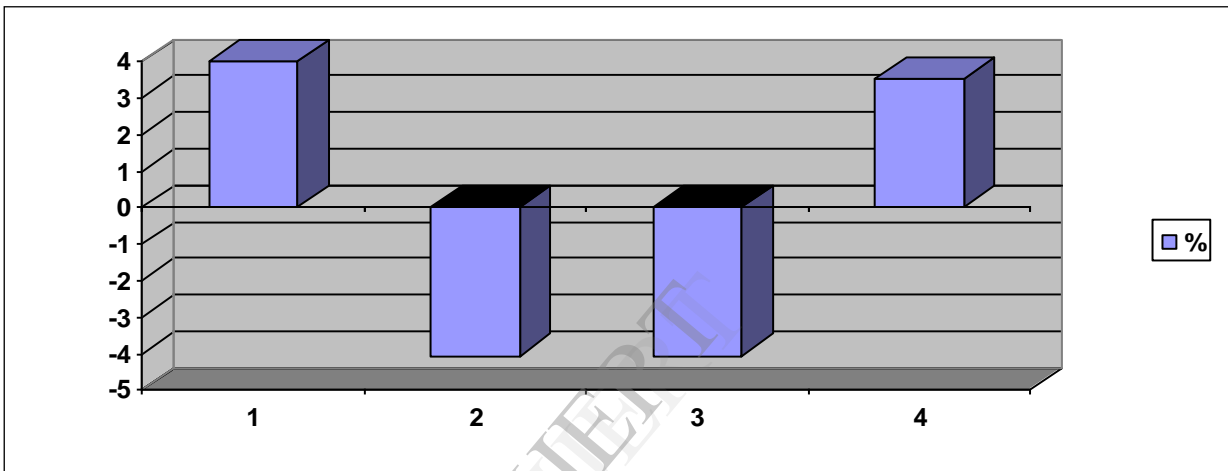


Fig. 5: Percentage change in reference strain energy of the beam sections summed for 3 modes

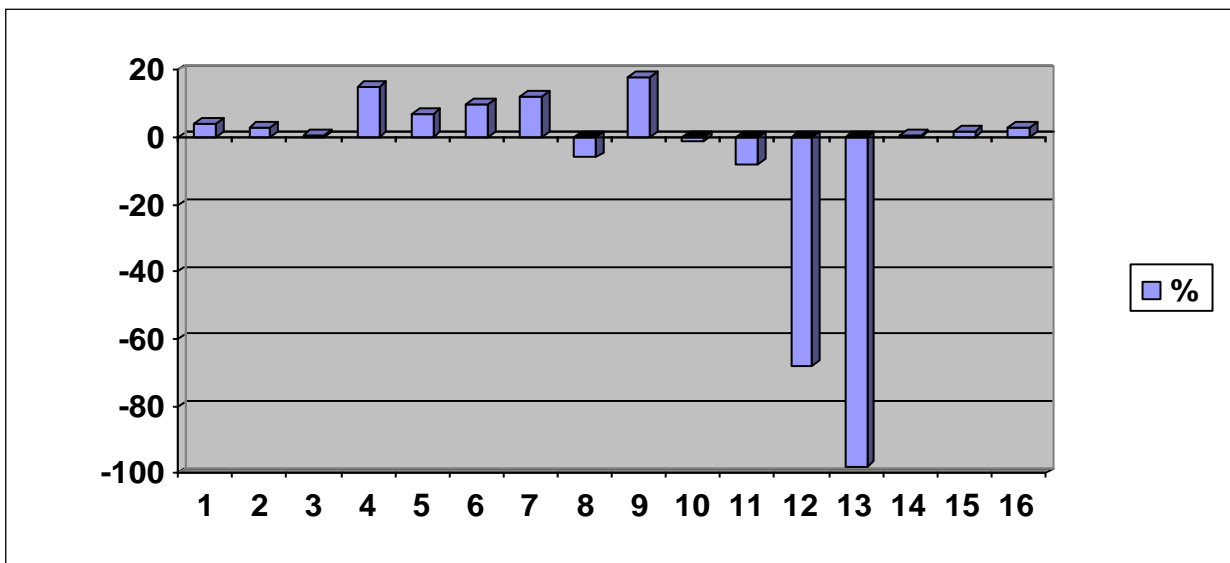


Fig. 6: Percentage change in reference strain energy of the beam sections summed for 3 modes

4.0 DISCUSSION

A methodology for the detection and localisation of damage in structures, with a focus on structural damage in bridge structures has been presented. The methodology is based on strain energy differences of the sections of the bridge before and after the damage has been detected. Detection of the damage is based on detecting changes in natural frequencies over a period of time. The methodology has been demonstrated on simulated examples of a bridge in 2 different damage cases. In both cases, the damage area was satisfactorily localized. In practice the natural frequencies will be measured on the bridge, for example by measurement of free response induced by traffic, and then identify natural frequencies from free response time domain data. Algorithms for the identification of natural frequencies from free response data are well developed. Measured natural frequencies can be used to update the theoretical finite element model and then use the model to calculate mode shape vectors. Alternatively, mode shape vectors may be established through measurements at several locations of the bridge, but this approach may prove time consuming. Once the damaged section(s) of the bridge are identified, a much closer physical inspection can then be conducted over the identified area. The methodology will greatly improve the effectiveness and efficiency of monitoring large bridges. However, more research is needed to evaluate the effectiveness of the methodology under real life environment.

5.0 CONCLUSION

A methodology to detect and localise damage in bridges using vibration measurements has been proposed and demonstrated using simulated examples with satisfactory results. The methodology is a potentially effective tool in the monitoring and maintenance of bridges. However, its effectiveness in a real life situation has to be explored further.

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