Determination of Transient Vibration Response of a Portal Frame Simulating Single Span Short Length Bridge

Ms. S. J. Modak
Assistant Professor in Civil Engg.,
Ramdeobaba College of Engineering & Management, Nagpur – 440 009

H. V. Hajare
Professor of Civil Engg.,
Priyadarshini College of Engg, Nagpur

S. S. Kulkarni
Ex-Professor of Structural Engg
VNIT, Nagpur

***Abstract*-The objective of the investigation is to mathematically simulate dynamics and vibrations of a portal frame subjected to a concentrated load moving on its horizontal member with a certain constant velocity. This portal frame is a basic structure of a low length single span bridge. The dynamic analysis of the portal frame is done by matrix method of structural analysis. The vertical members of the portal frame are subjected to time varying axial compressive load. The focus of the paper is to ascertain the transient vibration response of the vertical members treating the members represented by STOF vibratory system subjected to time varying axial compressive load.**

***Keywords:* *Bridge, Columns, Portal Frame, Transient Vibrations*.**

I. INTRODUCTION TO CONSTRUCTION OF A SHORT LENGTH BRIDGE

Figure 1 is a schematic presentation of a short length bridge. The length is so short that the basic structure of a bridge is a simple one span portal frame 0_1 AB 0_2. The width of the bridge is also fairly small so that it could be considered as a particular case of a girder bridge [4]•. The material of the frame is Mild Steel (M.S.). The philosophy of the analysis is explained through a representative small scale structure with dimensions length of AB = 1005 mm, width = 50 mm and thickness is = 0.01m. The vertical members 0_1A and 0_2B are geometry wise identical. The material of 0_1A & that of 0_2B is also M.S. A vehicle with total weight W is moving on AB with a constant velocity.

The objective of the investigation is to estimate vibration response of 0_1A & 0_2B.

II. ANALYSIS OF A PORTAL FRAME BY MATRIX METHOD

The specifications of the portal frame under consideration are as follows:

Material for all the members viz. 0_1A, AB, 0_2B is M.S. with modulus of Elasticity E = 2.0 x 10^8 kN/m2; Lengths of 0_1A = AB and 0_2B are equal each

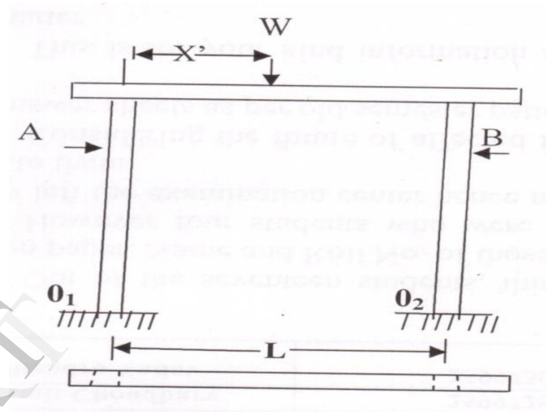


Figure 1. Schematics of a Portal Frame for a Short Bridge.

in turn equals to 1m, cross section rectangular 50 mm x 10mm; For member 0_1A, I = $(bd^3)/12.0$ where b = 0.01m and d = 0.05m. Dimension b is in the plane of the paper where as dimension d is at right angles to the plane of the paper. Geometry wise members 0_1A & 0_2B are identical. For member AB the thickness = 0.01m where as width = 0.05mm.

As far as joints 0_1, A, B, 0_2 are concerned the horizontal force components, vertical force components and rotation in vertical plane abut axis, passing through 01 and perpendicular to the plane of the paper are respectively $U0_1$, $V0_1$ and θ_{01}(positive C.W.). Similarly, the same quantities at remaining joints A, B and 02 are concerned are as follows.

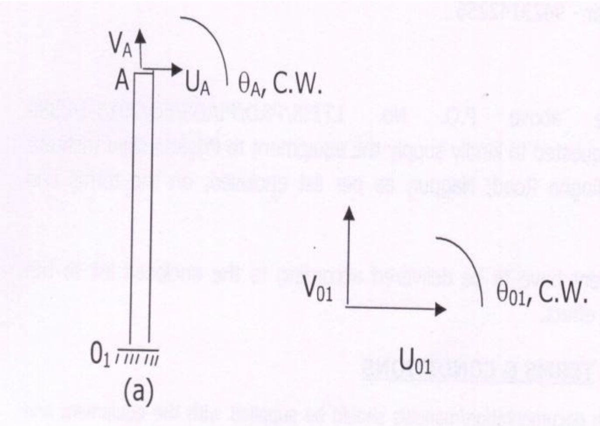
For A : U_A, V_A, θ_A
For B : U_B, V_B θ_B
And
For 0_2 : U_{02}, V_{02}, θ_2

For a concentrated load W = 1 kN acting at distance x = x' = 0.25m from A, these quantities (i.e. U_{01}, V_{01}, θ_{01}, U_A, V_A, θ_A, U_B, V_B, θ_B, U_{02}, V_{02}, θ_{02}) are estimated adopting Matrix Method of Structural Analysis [3].

Similar approach for the same load and the same location is applied for the members AB & 0_2B also. The findings are as shown in Table 1. Similar approach is adopted for the analysis of entire structure when the same load W = 1 kN changed its position x to 0.5m, 0.8m. Te complete findings are shown in Table-1. The graphic variation of V_{01} and V_{02} are decided off-course adopting the concept of interpolation [5]. These graphic plots are shown in Figure 3.

A. Determination Of Equivalent Axial Load For 0_1A And 0_2B

This is detailed in this article for the case when x =



0.25m & W = 1kN.

Figure 2. Results of Force Analysis of Various Members of Portal Frame

Effect of V_{01} & V_A is to create an axial compresive load equal to 0.879 x 10^3 kN

Effect of U_{01} & U_A is to create a c.c.w. couple = 0.195 x 10^3 kN-m

This stands to reason because of 1 kN load on AB at x = 0.25m a c.w. moment is created on AB of ≅ 0.25 kNm by 0_1A to which 0_1A should experience an anti clockwise moment.

This moment 0.195 x 10^3 kN-m c w on 0_1A will change it's magnitude as the load 1 kN changes it's position on AB. Thus, it will induce bending vibrations in 0_1A. Similarly, the net rotation of cross section at A with respect to cross section at 0_1 due to θ_A = 0.1263 radians c.c.w. and θ_0 = 0.0622 radians.

c.w. is 0.0641 radians c.c.w. which is also time variant has effect only to the extent of inducing bending vibrations in 0_1A. F

On the same lines Fig. 3 shows the Free Body Diagram (FBD) of member 0_2B of the same portal frame.

U_B = -101.99 N
V_B = 0.0458 kN = 45.8 N
θ_B = -0.0299 radius
$U_{(02)}$ = 101.99 N
$V_{(02)}$ = -0.0459 kN = 45.8 N
$\theta_{(02)}$ = 0.0153 radians

Effect of V_B = 45.8 N & V_{02} 4.8 kN is to create tension in the member 0_2B when x = 0.25m and W = 1kN. Similarly, the effect of $U_{(02),}$ +101.99 N and U_B = -101.99 N is to create c.c.w. moment on 0_2B. The magnitude of this moment is to change with positionof W. Thus, it is going to create bendign vibrations in the 0_2B. Similarly, net rotation of Section B w.r.to 0_2 is (-0.0299) + (0.0153) = -0.0146 c.w. which also changes with time and creates time varying bending vibrations in 0_2B.

Table 1. Findings of analysis of a portal frame

Sr. No	Loading Condition X,W	$\theta_{(01)}$	$U_{(01)}$	$V_{(01)}$	θ_A	U_A	V_A	θ_B	U_B	V_B	θ_{02}	U_{02}	V_{02}
1	X=0.25M W=1kN	0.0622 radians	0.195 x10^3	-0.879 x10^3	0.1263 rad	-0.195 x10^3	0.879 x10^3	-0.0299 rad	-101.99N	0.0458 kN	0.0153 rad	101.99N	-.0459kN
2	X=.05M W=1kN	-.0611	0.469 x10^3	-0.182 x10^3	-.122 rad	-0.489 x10^3	0.182 x10^3	0.121 rad	-0.489 x10^3	0.180 x10^3	0.060 rad	0.489 x10^3	-0.180 x10^3
3	X=0.8M W=1kN	0.042 x10^{-6}	0.0103kN	-0.044 kN	0.30 x10^{-1}	-.103 kN	0.44 kN	0.124	0.878kN	0.185	0.0621	-0.185kN	0.878

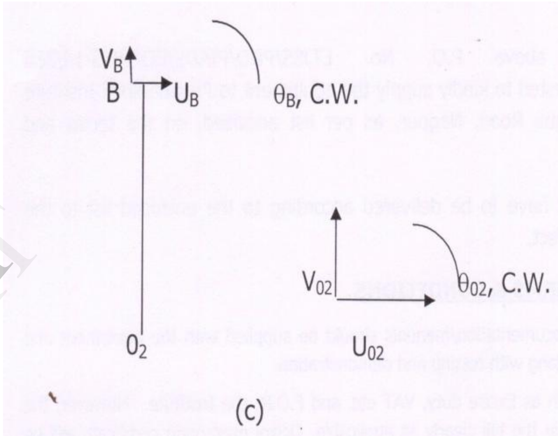


Figure 3. Free Body Diagram (FBD) of member 0_2B

As stated earlier, in this paper as emphasis on longitudinal vibrations of 0_1A and 0_2B is only considered, bending vibrations is not detailed further.

B. Influence Line For 0_1A And 0_2B

Figure 4(a) & 4(b) respectively shows the variation of V_{01} = V_A and V_{02} = V_B as the load i.e. the weight of the moving vehicle changes it's position on AB. This can be consideed as the influence lines [6] of member AB of the protal frame. This is off-course by the analysis of the concept influence line of a simply supported beam [6]. These figures 4 (a) and 4 (b) shows nonlinear variation of longitudinal load on the members 0_1A & 0_2B respectivelly of the portal frame 0_1AB0_2 under consideration. In other words as per matrix method of Structural Analysis, one gets nonlinear variation of compresive load on members 0_1A & 0_2B of the portal frame as some concentrated constant load changes it's position on portion AB of the portal frame. As stated earlier in this paper the single span short length bridge is treated annalogous to a single span portal frame, the loading on columns of the bridge would also be nonlinear. This nonlinearity is in terms of changed position of concentrated load. It is obvious that this is also interms of time because the load is nothing but the dead weight of the vehicle which is varying with constant velocity but the position of the vehicle is changing with respect to time . Hence, as per this approach of Structural Analysis [3]. it is concluded that the column reactions change nonlinearly [2] with time as against the linear variation as obtained in the previous analysis [1].

C. Approach To Estimation Of Vibration Response

Figure 4(a) shows influence line of 0_1A. This variation is having exponetial decay form.

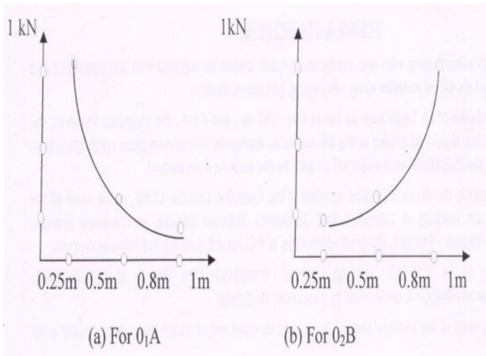


Figure 4.a &4 b Influences of 0_1A & 0_2B

Thus it can be stated as under, if $F_{(01A)}$ stands for external longitudinal load acting on the column 0_1A.

$F_{(01A)} = K_1 t^{-n1}$ ……………………… (I)

Whereas if $F_{(02B)}$ stands for external column load acting on the column 0_2B, then it is as described in Figure 3(b). Figure 3(b) shows the variation of $F_{(02B)}$ in exponential rising form. Hence, it can be stated as under

$F_{(02B)} = K_2 t^{n2}$ …………………………….. (II)

The parameters K_1, n_1, k_2, and n_2, can be decided by plotting these variations on log-log graph [5]

If one considers the entire coloumn represented by Single Degree of Freedom (SDoF) system, in which M, K, C represent the mass, elastic stiffness and damping co-eeficient of the structure of the column, then the governing equation of forced vibration phenomena of the column is presented as under.

$M\ddot{x} + c\dot{x} + kx = K_1 t^{-n1}$……………. (III)

For the column 0_1A where, as that for the column 0_2B would be

$$M\ddot{x} + c\dot{x} + kx = K_2 t^{-n2}$$

The initial conditions are off-course as under,

at $t = 0$; $x = 0$

and at $t = 0$; $\dot{x} = 0$

(a). Vibration Response Of 0_1a

Equation (III) presents the governing equation of vibrations motion of 0_1A. In Eq. (III) M, K, C are respectively Mass, Longitudinal Stifness and Damping co-efficient of material of 0_1A. These are estimated as under:

M = W/g; W = weight of 0_1A.

$\therefore$ W = ρ x L x b x d

$= 8 \times 10^3 \times 1 \times 0.01 \times 0.05$

$\therefore$ W = 40.0 Kgf

$\therefore$ M = 40.0/9.81

$\therefore$ M = 4.07 Kgf-m^{-1} – sec^2

K = Longitudinal Force per unit longitudinal displacment

$\therefore$ K = E x 1/L x bxd

Upon substitution of pentaniment numerical values; E = 2.0 x 10^8 KN/m^2, L = 1.0m; b = 0.01m; d = 0.05m

$\therefore$ K = 10^8 Kgf/m

Let ξ Damping ratio = C/C_c

For the member 0_1A, it will be appropriate to assume ξ = 0.01 [7] as the damping is going to be only due to material internal molecular friction or HYSTERISIS DAMPING. Hence, in view of K = 10.0^8 Kgf/m, M = 4.07 Kgf.m^{-1} sec^2, ξ = 0.01

C = 403.84 Kgf/m/sec

In Equation (III) the right hand side is $K_1 t^{-n1}$. The parameters K_1 & n_1 are decided by plotting the values of external longitudinal force to which 0_1A is subjected with respect to time as depicted in Figure 3(a). This variation if plotted on log-log paper, then it will farily approximately conform to the straight line. The slope of this approximate straight is the value $-n_1$ where as from the intersept of this line with ordinate, K_1 can be decided [5]. Accordingly, the external longitudinal load on 0_1A will come out to be

$F = K_1 t^{-n1} = (2.05 \times 10^{-3})\, t^{-2.5526}$ ………………. (V)

Accordingly, now Equation (III) gets transformed to

$4.07\,\ddot{x} + 403.484\dot{x} + 10.0^8 x = K_1 t^{-n1} = (2.05 \times 10^{-3})\, t^{-2.5526}$ ……………………. (III A)

upon substitution of numerical values of M, K, C, K_1 & n_1. This is an ordinary linear diferential equation with constant co-efficients. Hence, adopting the approach of Laplase Transformation [7] and in view of initial conditions t = 0, x = 0 & $\dot{x}$ = 0; the solution to this differential equation is obtained which is the vibration response of 0_1A.

The details of these calculations are elaborated in Annaxure-I.

(b) Vibration Response of 0_2B"

Adopting the procedure by which vibration response of 0_1A is decided, the vibration response of 0_2B is obtained. The governing equationof motion of 0_2B is

$4.07\,\ddot{x} + 403.484\dot{x} + 10.0^8 x = K_2 t^{-n2} = (3.096 \times 10^{-3})\, t^{5.19}$ ………………….. (IV B)

Equation (IV B) is also an ordinary linear differential equation with constant co-efficients. The initial conditions are at t=0, x=0 and $\dot{x}$=0. Again the approach of LAPLASE TRANSFORMATION is adopted to arrive at the solution of the differential equation (IV B). This is detailed in Annexure-I(B).

III. POSSIBLE EXTENSION

A more realistic solution could be by treating the entire column as a multidegree of freedom system represented by ¾ identical masses, stiffness of the springs and damping co-efficient.

On the same lines an entire column can be considered as a distributed mass, distributed elasticity and distributed damping system. Further all other possible configurations of vehicular trafic should be considered as detailed in the earlier papers of the authors [1 & 2].

IV. CONCLUSION

The paper reports on the possible approach to decide transient vibration response of two columns of a short length bridge which can be simulated by a single span portal frame. As this one is of the very few approaches towards this objective, it is based on large number of oversimplifying assumptions.

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Annexure – I
Vibration Response of 0_1A and 0_2B

Governing equation of vibratory motion of 0_1A is

$M\ddot{x} + c\dot{x} + Kx = K_1 t^{-n1}$ …………………… (III)

Substituting for

$M = 4.07$ Kgf-m^{-1}-sec^2

$K = 10.0^8$ Kgf/m

$C = 403.84$ Kgf/m/sec

$K_1 = 2.05 \times 10^{-3}$

$n_1 = 2.5526$

Rewriting the Equation (III A) again for the sake of ready ference

$4.07\ddot{x} + 403.484\dot{x} + 10.0^8 x = K_1 t^{-n1} = (2.05 \times 10^{-3})\, t^{-2.5526}$ …………………… (III.A)

$\therefore \ddot{x} + 99.136\dot{x} + (0.245) \times 10.0^8 x = (0.5036 \times 10^{-3}) t^{-2.5526}$

Performing Laplase Transformation of modified equation (III.A) given above, observing the initial condition tah at t = 0; x = 0 and $\dot{x} = 0$, one gets

$$S^2 x(S) + 99.136\, x(S) + (0.245)x\, 10^8 x(S) = \frac{(0.5036\, x\, 10^{-3})}{S^{1-2.5526}}$$

$$\therefore x(S)[S^2 + 99.136\, S + (0.245)x\, 10^8]x(S) = \frac{(0.5036\, x\, 10^{-3})}{S^{1-2.5526}}$$

$$\therefore x(S) = \frac{(0.5036\, x\, 10^{-3})S^{1.5526}}{[S^2 + 99.136S + (0.245)x\, 10^8]}$$

Now roots of $[S^2 + 99.136S + (0.245)x\, 10^8] = 0$ are $\propto_1$ and $\propto_2$ such that

$[S^2 + 99.136S + (0.245)x\, 10^8] = 0 = (S+\propto_1)(S+\propto_2)$

Solving the above quadratic equation one can get

$$\therefore x(S) = \frac{(0.5036\, x\, 10^{-3})S^{1.5526}}{(S+\propto_1)(S+\propto_2)} = \frac{C_1}{S+\propto_1} + \frac{C_2}{S+\propto_2}$$

To decide C_1:

$$x(S) = \frac{(0.5036\, x\, 10^{-3})S^{1.5526}}{(S+\propto_2)} = \frac{C_1(S+\propto_1)}{(S+\propto_1)} + \frac{C_2(S+\propto_1)}{(S+\propto_2)}$$

Now put $S = -\propto_1$

$$\therefore x(S) = \frac{(0.5036\, x\, 10^{-3})S^{1.5526}}{(-\propto_1 + \propto_2)} = C_1$$

$$\therefore C_1 = \frac{(0.5036\, x\, 10^{-3})(-49.140x10^6)S^{1.5526}}{(-10^3)}$$

$$= \frac{(0.5036\, x\, 10^{-3})(-49.140x10^6)^{1.5526}}{(10)^6}$$

$$\therefore = (0.5036)(49.140)^{1.5526} x\, 10^{(0.3156)}$$

On the same lines C_2 can be decided

$$C_2 = 0.0512(49.136)^{1.5526} x\, 10^{(0.3156)}$$

Then

$$x(S) = \frac{C_1}{S+\propto_1} + \frac{C_2}{S+\propto_2}$$

Now performing inverse Laplace Transformation,

$$x(t) = C_1\, e^{-\propto_1 t} + C_2\, e^{-\propto_2 t}$$

$$\therefore x(t) = C_1\, e^{(49.140\, x\, 10^6)t} + C_2\, e^{(-49.139\, x\, 10^6)t} \quad \text{……… (VI)}$$

Equation - (VI) is the vibration response of 01A. Similarly, the vibration response of 02B can be decided.

$$C_1 = 0.05125(49.140)^{1.5526} x\ (10)^{0.3156}$$

$$C_2 = 0.05125(49.140)^{1.5526} x\ (10)^{0.3156}$$

$$\propto_1 = -49.140\ x\ 10^{+6};\ \propto_2 = 49.139\ x\ 10^{+6}$$

Table 1. Findings of analysis of a portal frame O1A BO2, For W=1kN & X = 0.25, 0.50, 0.80 M

Sr. No	Loading Condition X,W	$\theta_{(01)}$	$U_{(01)}$	$V_{(01)}$	θ_A	U_A	V_A	θ_B	U_B	V_B	θ_{02}	U_{02}	V_{02}
1	X=0.25M W=1kN	0.0622 radians	0.195 x10^3	-0.879 x10^3	0.1263 rad	-0.195 x10^3	0.879 x10^3	-0.0299 rad	-101.99N	0.0458 kN	0.0153 rad	101.99N	-.0459kN
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3	X=0.8M W=1kN	0.042 x10^{-6}	0.0103kN	-0.044 kN	0.30 x10^{-1}	-.103 kN	0.44 kN	0.124	0.878kN	0.185	0.0621	-0.185kN	0.878