

# Development of Cognate Mechanisms to Generate Symmetric Coupler Curves to Eliminate Kinematic Singularities of Eight Bar 3-RRR Planar Parallel Manipulator

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**Abstract** - An eight bar 3-RRR planar parallel manipulator is an important role in machining process. Therefore selected eight bar 3-RRR planar parallel manipulator and its kinematic singularities are analyzed. To eliminate kinematic singularities of selected eight bar 3-RRR planar parallel manipulator, the cognate linkages are generated by using Roberts's chebychev technique. The cognate linkages, which will generate symmetric coupler curve of coupler point, which is on selected eight bar 3-RRR planar parallel manipulator. By using loop closer technique the mathematical equations are derived for the proposed approach. The purpose of generating cognate mechanisms to generate symmetric coupler curve of coupler point, to eliminate kinematic singularities of eight bar 3-RRR planar parallel manipulator for set of positions of all unconstrained limbs intersect at common point and also for set of positions of all unconstrained limbs are parallel to each other in the selected eight bar 3-RRR planar parallel manipulator.

**Keywords** – Cognate Mechanisms; Symmetric Coupler Curves; Kinematic Singularities; Kinematic Analysis; 3-Dof; Eight Bar Planar Parallel Mechanism

## I. INTRODUCTION

An eight bar 3-RRR planar parallel manipulator with binary and ternary limbs are selected. The selected architecture has three degrees of freedom. The design of cognate mechanisms for generating symmetric coupler curves of eight bar 3-RRR planar parallel manipulator are more complex due to their kinematic structure and its singularities. This paper presents identifying the kinematic singularities in workspace of eight bar 3-RRR planar parallel manipulator by analyzing its kinematic analysis, because the singularities are more attractive in several researches. The mathematical equations are derived by using loop closer technique. To eliminate the kinematic singularities of selected eight bar 3-RRR planar parallel manipulator, the cognate mechanism are developed by using Roberts's chebychev technique. The purpose of generating cognate mechanisms to generate symmetric coupler curves of coupler point which is selected on eight bar 3-RRR planar parallel manipulator.

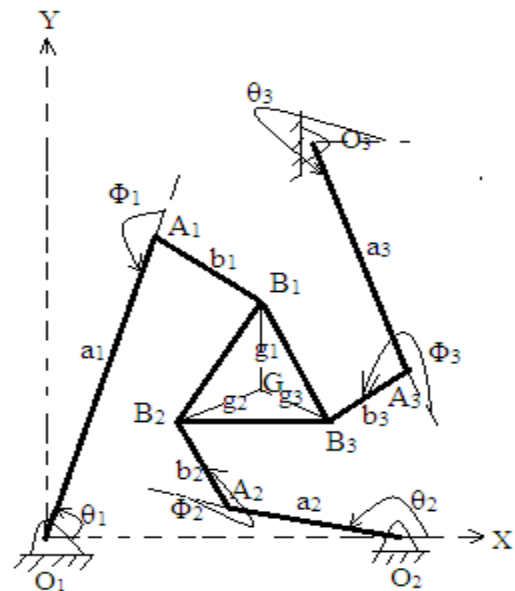


Fig (1) Eight bar 3-RRR planar parallel manipulator

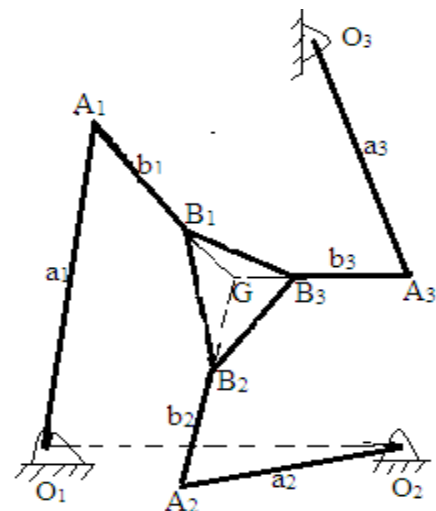


Fig (2) Kinematic Singularity of Eight bar 3-RRR planar parallel manipulator (All unconstrained limbs intersect at common point G)

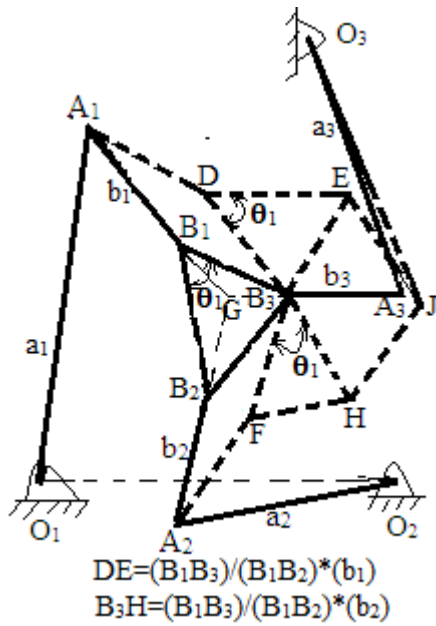


Fig (3) Development of Cognate mechanisms by using Roberts-Chebychev technique to generate Symmetric Coupler curves to eliminate Kinematic Singularity of Eight bar 3-RRR planar parallel manipulator (All unconstrained limbs intersect at common point G)

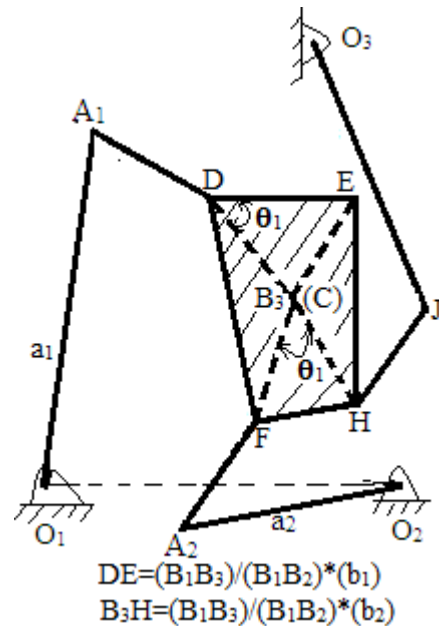


Fig (5) Symmetric Cognate mechanism to generate Symmetric Coupler curve of Coupler point 'B<sub>3</sub>(C)' to eliminate Kinematic Singularity of Eight bar 3-RRR planar parallel manipulator (Solution for all unconstrained limbs intersect at common point G)

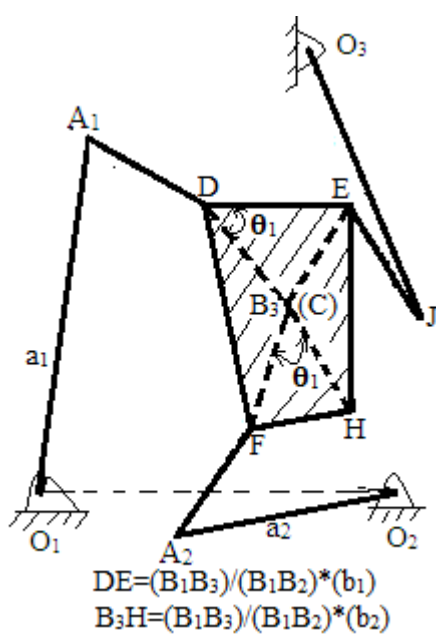


Fig (4) Cognate mechanism to generate Symmetric Coupler curve of Coupler point 'B<sub>3</sub>(C)' to eliminate Kinematic Singularity of Eight bar 3-RRR planar parallel manipulator (Solution for all unconstrained limbs intersect at common point G)

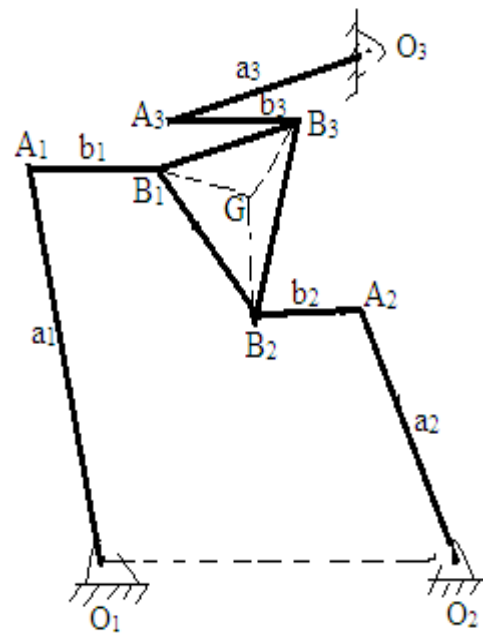


Fig (6) Kinematic Singularity of Eight bar 3-RRR planar parallel manipulator (All unconstrained limbs parallel to each other)

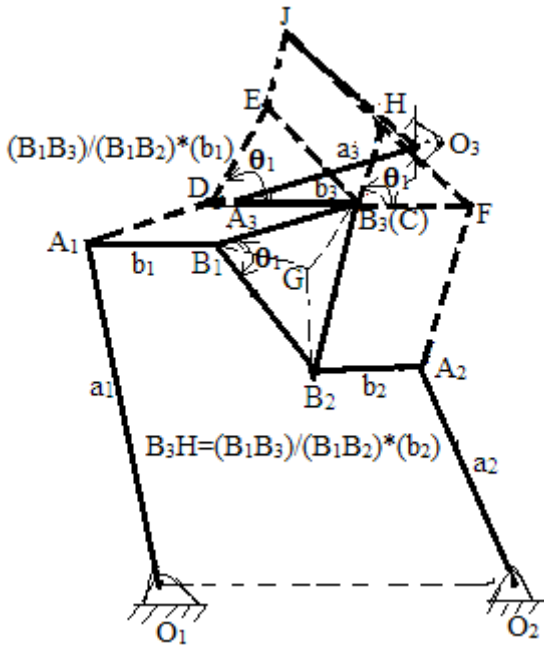


Fig (7) Development of Cognate mechanisms by using Roberts-Chebychev technique to generate Symmetric Coupler curves to eliminate Kinematic Singularity of Eight bar 3-RRR planar parallel manipulator (All unconstrained limbs parallel to each other)

Eight bar 3-RRR planar parallel manipulator (Solution for all unconstrained limbs parallel to each other)

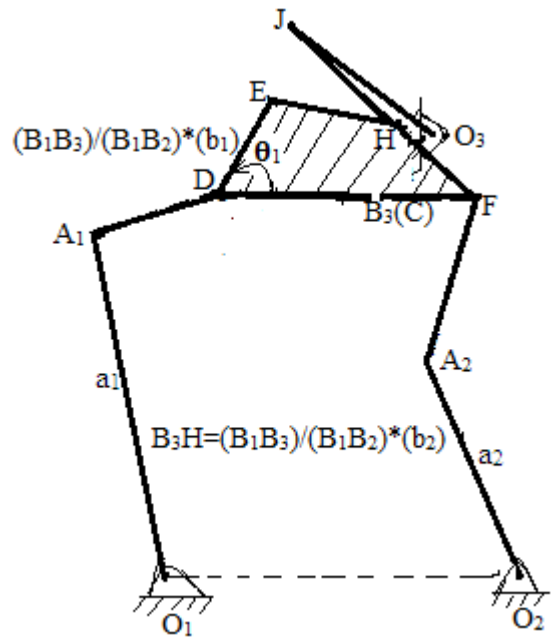


Fig (9) Symmetric Cognate mechanism to generate Symmetric Coupler curve of Coupler point 'B3(C)' to eliminate Kinematic Singularity of Eight bar 3-RRR planar parallel manipulator (Solution for all unconstrained limbs parallel to each other)

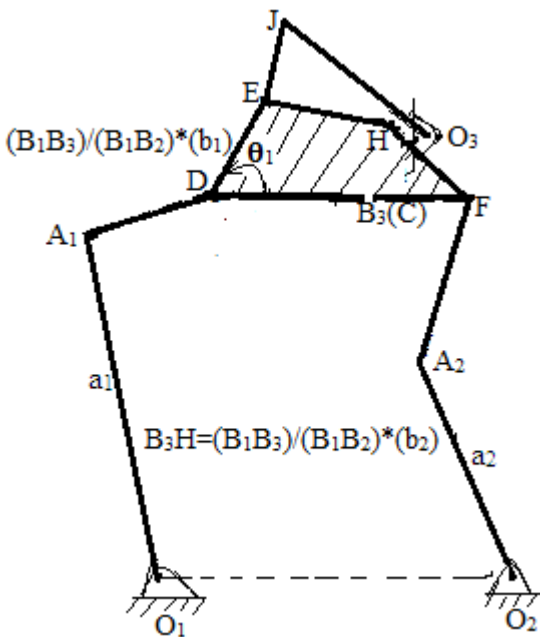


Fig (8) Cognate mechanism to generate Symmetric Coupler curve of Coupler point 'B3(C)' to eliminate Kinematic Singularity of

## II. KINEMATIC ARCHITECTURE OF EIGHT BAR 3-RRR PLANAR PARALLEL MANIPULATOR

The kinematic architecture of eight bar 3-RRR planar parallel manipulator is as shown in "Fig (1)". It consists of a movable triangular platform or circular lamina of its circumference passing through  $B_1B_2B_3$  which is connected with unconstrained three binary limbs  $B_iA_i|_{i=1,2,3}$  and, a fixed base  $O_1O_2O_3$  which is connected with three constrained limbs  $O_iA_i|_{i=1,2,3}$ . The constrained limbs are actuated by rotary actuators which are actuated by rotary variable differential transducers. The angular positions of input limbs  $O_iA_i|_{i=1,2,3}$  are measured with respect to

positive X- axis which represents the angular positions of three successive constrained limbs, which are denoted by  $\theta_i|_{i=1,2,3}$  for initial orientation of three constrained limbs and for different orientations it is denoted as  $\theta_{ij}|_{i=1,2,3 \text{ and } j=1,2,3,4 \text{ etc'}}$ , similarly the initial inclinations of three unconstrained limbs  $A_iB_i|_{i=1,2,3}$  with respect to  $O_iA_i|_{i=1,2,3}$  are denoted as  $\phi_i|_{i=1,2,3}$  and for different orientations of three unconstrained limbs are denoted as  $\phi_{ij}|_{i=1,2,3 \text{ and } j=1,2,3,4 \text{ etc'}}$

### III. JACOBIAN AND SINGULARITY ANALYSIS OF EIGHT BAR 3-RRR PLANAR PARALLEL MANIPULATOR

The degree of freedom of the selected moving triangular or circular lamina (platform) of eight bar 3-RRR planar parallel manipulator is three. The translational and rotational coordinates of the moving triangular or circular lamina centroid is denoted as  $x_G$ ,  $y_G$  and  $\psi_G$ . The input rotational vector of three constrained limbs are  $[\theta_1 \ \theta_2 \ \theta_3]^T$  and the output translational and rotational vector of moving triangular or circular platform centroid as  $[x_G \ y_G \ \psi_G]^T$ . The loop closer equation for each limb can be expressed as

$$[\vec{OG} + \vec{GB}_i]_{i=1,2,3} = [\vec{OA}_i + \vec{A_iB_i}]_{i=1,2,3} \quad (1)$$

The velocity vector loop equation can be obtained by making derivative of “(1)” with respect to time, then the velocity vector loop equations as

$$\begin{bmatrix} b_{x1} & b_{y1} & g_{x1}b_{y1} - g_{y1}b_{x1} \\ b_{x2} & b_{y2} & g_{x2}b_{y2} - g_{y2}b_{x2} \\ b_{x3} & b_{y3} & g_{x3}b_{y3} - g_{y3}b_{x3} \end{bmatrix} [\dot{x}_G \ \dot{y}_G \ \dot{\psi}_G]^T = \begin{bmatrix} a_{x1}b_{y1} - a_{y1}b_{x1} & 0 & 0 \\ 0 & a_{x2}b_{y2} - a_{y2}b_{x2} & 0 \\ 0 & 0 & a_{x3}b_{y3} - a_{y3}b_{x3} \end{bmatrix} [\dot{\theta}_1 \ \dot{\theta}_2 \ \dot{\theta}_3]^T \quad (4)$$

The forward kinematic singularities occur in “(4),” if

$$\begin{vmatrix} b_{x1} & b_{y1} & g_{x1}b_{y1} - g_{y1}b_{x1} \\ b_{x2} & b_{y2} & g_{x2}b_{y2} - g_{y2}b_{x2} \\ b_{x3} & b_{y3} & g_{x3}b_{y3} - g_{y3}b_{x3} \end{vmatrix} = 0 \quad (5)$$

By the inspection of “(5),” the forward kinematic singularities are possible, if  $[g_{xi}b_{yi} - g_{yi}b_{xi}]_{i=1,2,3} = 0$ . That means the limbs  $A_iB_i|_{i=1,2,3}$  are in the line with  $GB_i|_{i=1,2,3}$  as shown in “Fig (2)”. By the inspection of “Fig (2),” all unconstrained limbs intersect at common point G. At this position the moving platform  $GB_i|_{i=1,2,3}$  require

$$\begin{vmatrix} a_{x1}b_{y1} - a_{y1}b_{x1} & 0 & 0 \\ 0 & a_{x2}b_{y2} - a_{y2}b_{x2} & 0 \\ 0 & 0 & a_{x3}b_{y3} - a_{y3}b_{x3} \end{vmatrix} = 0 \quad (6).$$

By the inspection of “(6),” the inverse kinematic singularities are possible in one of the diagonal elements are zero, that means  $[a_{xi}b_{yi} - a_{yi}b_{xi}]_{i=1 \text{ or } 2 \text{ or } 3} = 0$ . This type of problem will be raised in “Fig (1),” if one of the limbs  $A_iB_i|_{i=1,2,3}$  is fully stretched out or folded completely back. In this case the manipulator losses 1 or 2 or 3 degrees of freedom, this depends on whether  $A_1B_1$  or

$$V_G + \psi_G(k * g_i)|_{i=1,2,3} = [\dot{\theta}_i(k * a_i) + (\dot{\theta}_i + \dot{\phi}_i)(k * b_i)]_{i=1,2,3} \quad (2)$$

Where  $V_G$  is the velocity of the centroid of the moving triangular or circular platform: ‘k’ is the unit vector pointing in positive Z- direction and  $\dot{\phi}_i|_{i=1,2,3}$  is the passive variables. In “(2)” the passive variables can be eliminated by making dot product of above equation by  $b_i|_{i=1,2,3}$  then “(2)” can be written as

$$[b_i V_G + \dot{\psi}_i k \cdot (g_i * b_i)]_{i=1,2,3} = \dot{\theta}_i \cdot k(a_i * b_i)|_{i=1,2,3} \quad (3)$$

That means the jacobian matrix representation as

infinitesimal rotation about point of contact G. In this case the degree of freedom of entire structure is converted into four that means the moving platform gains one degree of freedom and it cannot withstand any external moment about point G. Therefore the rotary actuators are locked. Similarly in “Fig (6),” the three vectors  $A_iB_i|_{i=1,2,3}$  are parallel to each other then another type of forward kinematic singularities will occur. At this position moving platform require infinitesimal translational motion along the translational direction. Therefore again the three actuators are locked. Same way the inverse kinematic singularities will occur in “(4),” if

$A_2B_2$  or  $A_3B_3$  or combination of these limbs are fully stretched out or folded completely back. There is no output motion of moving triangular or circular platform for an infinitesimal rotation of input limbs. Therefore all three rotary actuators are again locked. To eliminate the above problems cognate mechanisms are selected.

#### IV. DESIGN AND DEVELOPMENT OF COGNATE MECHANISMS TO GENERATE SYMMETRIC COUPLER CURVES TO ELIMINATE KINEMATIC SINGULARITIES OF EIGHT BAR 3 – RRR PLANAR PARALLEL MANIPULATOR

The kinematic architecture of eight bar 3-RRR planar parallel manipulator is as shown in “Fig (1)”. Suppose all unconstrained limbs intersect at common point G as shown in “Fig (2),” then the moving triangular or circular platform  $GB_i|_{i=1,2,3}$  require infinitesimal rotation about point of contact G. Therefore the rotary actuators are locked. Similarly, if all unconstrained limbs  $A_iB_i|_{i=1,2,3}$  are parallel to each other as shown in “Fig (6),” then again the three rotary actuators are locked. To eliminate the above problems cognate eight bar 3-RRR planar parallel manipulators are to be designed. The kinematic singularities of selected eight bar 3-RRR planar parallel manipulator are occur if all unconstrained limbs intersect at common point G as shown in “Fig (2),” then the cognate linkages are to be developed by using Roberts’s chebychev technique. Consider the coupler point  $B_3(C)$  which is on the vertex of the moving triangular platform or circumference of the circular platform. The development of cognate linkages by using Roberts’s chebychev technique is as shown in “Fig (3),” which will generate symmetric coupler curves of selected coupler point  $B_3(C)$ . The designed and developed cognate mechanism as shown in “Fig (4)” and symmetric cognate mechanism as shown in “Fig (5),” which will generate symmetric coupler curves of coupler point ‘ $B_3(C)$ ’ to eliminate kinematic singularity of selected eight bar 3-RRR planar parallel manipulator . That means the cognate mechanisms as shown in “Fig (4)” and “Fig

(5),” are the solutions for all unconstrained limbs intersect at common point G. Similarly, if all unconstrained limbs  $A_iB_i|_{i=1,2,3}$  are parallel to each other as shown in “Fig (6),” then again the cognate linkages are to be developed by using Roberts’s chebychev technique. Consider the coupler point  $B_3(C)$  which is on the vertex of the moving triangular platform or circumference of the circular platform of “Fig (6)”. The development of cognate linkages by using Roberts’s chebychev technique is as shown in “Fig (7),” which will generate symmetric coupler curves of selected coupler point  $B_3(C)$ . The designed and developed cognate mechanism as shown in “Fig (8)” and symmetric cognate mechanism as shown in “Fig (9),” which will generate symmetric coupler curves of coupler point ‘ $B_3(C)$ ’ to eliminate kinematic singularity of selected eight bar 3-RRR planar parallel manipulator . That means the cognate mechanisms as shown in “Fig (8)” and “Fig (9),” are the solutions for all unconstrained limbs  $A_iB_i|_{i=1,2,3}$  are parallel to each other. In “Fig (4)” and in “Fig (8)” moving quaternary laminas or circular laminas passing through DEF points are to be selected and choose coupler point on “Fig (4)” and in “Fig (8)” as per design procedure of “Fig (3)” and in “Fig (7)”. Similarly in “Fig (5)” and in “Fig (9)” moving quaternary laminas or circular laminas passing through DFH points are to be selected and choose coupler point on “Fig (5)” and in “Fig (9)” as per design procedure of “Fig (3)” and in “Fig (7)”.

#### CONCLUSION

This paper presents identifying the kinematic singularities in the workspace of selected eight bar 3 – RRR planar parallel manipulator. To eliminate kinematic singularities of selected eight bar 3-RRR planar parallel manipulator, the cognate linkages are generated by using Roberts’s chebychev technique. The cognate linkages, which will generate symmetric coupler curve of coupler point, which is on selected eight bar 3-RRR planar parallel manipulator. By using loop closer technique the mathematical equations are derived for the proposed approach. The purpose of generating cognate mechanisms to generate symmetric coupler curve of coupler point, to eliminate kinematic singularities of eight bar 3-RRR planar parallel manipulator

for set of positions of all unconstrained limbs intersect at common point or for set of positions all unconstrained limbs in the selected 3-RRR planar parallel manipulator are parallel to each other or the position of one set of constrained and unconstrained limbs of eight bar 3-RRR planar parallel manipulator are on the same line of action. Then the all rotary actuators are locked. To eliminate the above problems the cognate mechanisms are generated to eliminate the singularities of eight bar 3 – RRR planar parallel manipulator. The future work can be extended by generating different cognate linkages, to generate symmetric coupler curves for eight bar 3- PPP spatial parallel manipulators, to eliminate its singularities.

#### REFERENCES

- [1] Pundru Srinivasa Rao, “Forward and Inverse Kinematic Singularity analysis of Eight bar 3 – RRR Planar Parallel Manipulator,” IJERT, Vol.6, Issue 05, pp. 295-297, May 2017.
- [2] Pundru Srinivasa Rao, “Forward and Inverse Kinematic Singularity analysis of Eight bar 3 – PPP Spatial Parallel Manipulator,” IJERT, Vol.6, Issue 06, pp. 31-34, June 2017.
- [3] C. M. Gosselin, I.A. Bonev, D. Zlatanov, “Singularity Analysis of 3-DOF Planar Parallel Mechanisms via Screw Theory,” ASME J. Mech. Des. 125(3), pp. 573-581, 2003.
- [4] C.M. Gosselin, S. Lemieux and J. P.Merlet, “A New Architecture of Planar 3-DOF Parallel Manipulator,” Proc. IEEE Int. Conf. on Robotics and Automation, Minneapolis , MN, pp. 3738-3743, April 1996.
- [5] J.M. McCarthy, C.L. Collins, “The Singularity Loci of Two Triangular Paralle Manipulators,” Proc. IEEE Int. Conf. ICAR, Monterey, CA, pp. 473-478, July 1997.
- [6] R.P. Podhorodeski, M.A. Nahon, J.A. Carretero, “Workspace Analysis and Optimization of a Novel 3-DOF Parallel Manipulator,” IEEE J. Robotics and Automation, Vol. 15, No. 4, pp. 178-188, 2000.

- [7] J. Wang, Q. Zhang, W. Chou, "Inverse Kinematics and Dynamics of the 3-RRS Parallel Platform," Proc. IEEE Int. Conf. Robotics and Research, Seoul, Korea, pp. 2506- 2511, 2001.
- [8] C.M. Gosselin and J. Angeles, "Singularity Analysis of Closed – Loop Kinematics Chains," IEEE Transactions, Robotics and Automation, 6(3), pp. 281-290, 1990.
- [9] R. Boudreau, M. Arsenault, "The Synthesis of 3-DOF Planar Parallel Mechanisms with Revolute Joints for an Optimal Singularity Free Workspace," J. Rob. Syst. 21(5), pp. 259-274, 2004.
- [10] R. Boudreau, M. Arsenault, "Synthesis of Planar Parallel Mechanisms While Considering Workspace, Dexterity, Stiffness, and Singularity Avoidance," ASME J. Mech. Des., 128(1), pp. 69-78, 2006.
- [11] C. Gosselin, M.T. Masouleh, "Determination of Singularity Free Zones in the Workspace of Planar 3-PRR Parallel Mechanisms," Vol. 129, pp. 649-652, June 2007.