

Direction Of Arrival (DOA) Estimation Using Smooth Music Algorithm

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ABSTRACT

An array antenna system with innovative signal processing can enhance the resolution of a signal direction of arrival (DOA) estimation. Super resolution algorithms take advantage of smart antenna structures to better process the incoming signals. Thus, the smart antenna system becomes capable to locate and track signals by the both users and interferers and dynamically adapts the antenna pattern to enhance the reception in Signal-Of-Interest direction and minimizing interference in Signal-Of-Not-Interest direction. They also have the ability to identify multiple targets. This paper explores the Eigen-analysis category of super resolution algorithm. A class of Multiple Signal Classification (MUSIC) algorithms known as a Smooth-MUSIC algorithm is presented in this paper. The Smooth-MUSIC method is based on the eigenvectors of the sensor array correlation matrix. It obtains the signal estimation by examining the peaks in the spectrum. Statistical analysis of the performance of the processing algorithm and processing resource requirements are discussed in this paper. Extensive computer simulations are used to show the performance of the algorithms. The resolution of the DOA techniques improves as number of array elements increases.

Keywords --MUSIC, Adaptive Beamforming, DOA, Smooth-MUSIC, Smart antenna.

1. Introduction

The high demand on the usage of the wireless communication system calls for higher system capacities. The system capacity can be improved either enlarging its frequency bandwidth or allocating new portion of frequency spectrum to wireless services. But since the electromagnetic spectrum is a limited resource, it is not easy to get new spectrum allocation without the international coordination on the global level[1,7]. One of the approaches is to use existing spectrum more efficiently, which is a challenging task. Efficient source and channel coding as well as reduction in transmission power or transmission bandwidth or both are possible solutions to the challenging issue. With the advances in digital techniques, the frequency efficiency can be improved by multiple access technique (MAT), which gives mobile users access to scarce resource (base station) and hence improves the system's capacity. Family of existing Frequency Division Multiple Access (FDMA),

Time Division Multiple Access (TDMA) and Code Division Multiple Access (CDMA) can be enlarged by adding a new parameter "space" or "angle", which results in MAT known as "Space Division Multiple Access" (SDMA)[9,3,5]. At the receiver's side, the transmitted signal is received with its multipath components plus interferers signal, as well as with present noise. Thus, detection of the desired signal is a challenging task.

The Smart Antenna System (SAS) employs the antenna elements and the digital signal processing which enables it to form a beam to a desired direction taking into account the multipath signal components. This can be achieved by the adaptive beamforming technique. In this way, Signal-to-Interference-and-Noise Ratio (SINR) improves by producing nulls towards the interferers Signal-Of-No-Interest (SONI). The performance of SAS greatly depends on the performance on DOA estimation.

In this paper, we are investigating the performance of MUSIC, ROOT-MUSIC and Smooth-MUSIC algorithms simulated by using MATLAB as a tool. The performance of these algorithms is analysed by considering parameters like number of array elements, user space distribution, which results in optimum array design in SAS.

2. Adaptive beam forming

As the name indicates, an adaptive beam former is able to automatically adapt its response to different situations. Some criterion has to be set up to allow the adaption to proceed such as minimizing the total noise output. Because of the variation of noise with frequency, in wide band systems it may be desirable to carry out the process in the frequency domain. Beam forming can be computationally intensive. Sonar phased array has a data rate low enough that it can be processed in real-time in software, which is flexible enough to transmit and/or receive in several directions at once. In contrast, radar phased array has a data rate so high that it usually requires dedicated hardware processing, which is hard-wired to transmit and/or receive in only one direction at a time. However, newer field programmable gate arrays are fast enough to handle radar data in real-time, and can be quickly re-programmed like software, blurring the hardware/software distinction. The above mentioned

concept is related to the process of beam forming and it doesn't exploit the possibility to steer the nulls of the antenna beam in the direction of the RFIs (adaptive beam forming). These evolutions of standard beamformers are conceived to separate (analogically, digitally or both) a desired signal from one or more interfering signals (Spatial Filtering) by means of automatic and continuous characterization of the components of the weighting vector. This can be performed using a wide variety of different algorithms designed for many specific applications.

3. DOA ESTIMATION

Adaptive signal processing sensor arrays, known also as smart antennas, have been widely adopted in third-generation (3G) mobile systems because of their ability to locate mobile users with the use of DOA estimation techniques. Adaptive antenna arrays also improve the performance of cellular systems by providing robustness against fading channels and reduced collateral interference. The goal of direction-of-arrival (DOA) estimation is to use the data received on the downlink at the base-station sensor array to estimate the directions of the signals from the desired mobile users as well as the directions of interference signals. The results of DOA estimation are then used by to adjust the weights of the adaptive beamformer so that the radiated power is maximized towards the desired users, and radiation nulls are placed in the directions of interference signals. Hence, a successful design of an adaptive array depends highly on the choice of the DOA estimation algorithm which should be highly accurate and robust.

There are three methods to find Direction Of Arrival (DOA)

1. Spectral based method,
2. Parametric method,
3. Sub space-based method.

3.1 Spectral Based Method:

The DOA estimation can be done by computing spatial spectrum and then determining local maximas.

- (i) Conventional
- (ii) Capon's & Bartlett

Conventional:

The conventional beam former is a natural extension of classical Fourier-based spectral analysis to sensor array data. For an array of arbitrary geometry, this algorithm maximizes the power of the beam forming output for a given input signal.

Capon's & Bartlett:

In this method a rectangular window of uniform weighting is applied to the time series data to be analyzed.

3.2 Parametric Method:

It requires simultaneous search of all parameters. It requires prior information about data to be generated. Eliminates need for window functions and assumptions. In this section, we review the maximum likelihood methodology for the case of additive Gaussian noise of zero mean and variance matrix.

The two methods as follows:

- (i) Deterministic Maximum Likelihood
- (ii) Stochastic Maximum Likelihood

Deterministic maximum likelihood:

According to the Deterministic ML (DML) the signals are considered as unknown, deterministic quantities that need to be estimated in conjunction with the direction of arrival. This is a natural model for digital communication applications where the signals are far from being normal random variables, and where estimation of the signal is of equal interest.

Stochastic maximum likelihood:

In general, the SML estimate (SML) cannot be found analytically. Hence, numerical procedures must be employed to carry out the required optimization. Several optimization methods have been proposed in the literature, including the Alternating Projection method, several Newton type techniques and the Expected Maximization (EM) method. The SML likelihood function is regular and the SML estimator is consistent and asymptotically efficient, i.e., the covariance of the estimates asymptotically attains the stochastic Cramer Rao Bound.

3.3 Sub Spaced-Based Method:

In this method the DOA estimation is based on the Eigen values and depends upon the steering vectors.

- (i) MUSIC Family algorithms
- (ii) ESPRIT Algorithm

In this paper we focussed on MUSIC Family algorithms. The MUSIC family consists of

- MUSIC Algorithm
- ROOT-MUSIC Algorithm
- SMOOTH-MUSIC Algorithm

The mathematical models of MUSIC family as follows:

3.3.1 MUSIC:

Consider the signal model of M signals incident on the array, corrupted by noise, i.e.

$$x = \sum_{m=1}^M \alpha_m S(\theta_m) + n$$

Using the above data

$$x = S\alpha + n$$

$$S = [S(\theta_1) \quad S(\theta_2) \quad \dots \quad S(\theta_m)]$$

$$\alpha = [\alpha_1 \quad \alpha_2 \dots \alpha_m]^T$$

n=noise

S is a matrix of M steering vectors. Assuming that the different signals to be uncorrelated the correlation matrix of x can be written as

$$\begin{aligned} R &= E[xx^H] \\ &= E[S\alpha\alpha^H S^H] + E[nn^H] \\ &= SAS^H + \sigma^2 I \\ &= R_s + \sigma^2 I \end{aligned}$$

Where

$$R_s = SAS^H$$

$$A = \begin{bmatrix} E[\det \alpha_1]^2 & 0 & \dots & 0 \\ 0 & E[\det \alpha_2]^2 & \dots & 0 \\ 0 & 0 & 0 & E[\det \alpha_m]^2 \end{bmatrix}$$

The signal covariance matrix Rs is clearly a N×N matrix with rank M. It therefore has N-M Eigen vectors corresponding to the Eigen value. Let qm be such an Eigen vector.

Therefore,

$$\begin{aligned} R_s q_m &= SAS^H q_m = 0 \\ \Rightarrow q_m^H SAS^H q_m &= 0 \\ \Rightarrow q_m^H q_m &= 0 \end{aligned}$$

Where this final equation is valid since the matrix A is clearly positive definite. The above equation implies that all N-M Eigen vectors (qm) of Rs corresponding to the zero Eigen value are orthogonal to all M signal steering vectors.

This is basic of MUSIC. Call Qn the N×(N-M) matrix of these Eigen vectors. MUSIC plots the pseudo spectrum.

$$\begin{aligned} P_{MUSIC}(\phi) &= \frac{1}{\sum_{m=1}^{N-M} \det(S^H(\phi)q_m)^2} \\ &= \frac{1}{S^H(\phi)Q_n Q_n^H S(\phi)} \\ P_{music} &= \frac{1}{\|Q_n^H S(\phi)\|^2} \end{aligned}$$

Eigen vectors making up Qn are orthogonal to the signal steering vectors. The denominator becomes zero when Φ is a signal direction.

Therefore estimated signal directions are the M largest peaks in pseudo spectrum.

Eigen vectors in Qn can be estimated from the Eigen vector of R

For any Eigen vector $q_m \in Q$,

$$\begin{aligned} R_s q_m &= \lambda q_m \\ R q_m &= R_s q_m + \sigma^2 I q_m \\ &= (\lambda_m + \sigma^2) q_m \end{aligned}$$

i.e., any Eigen vector of Rs is also an Eigen vector of R with corresponding Eigen value $\lambda + \sigma^2$.

Let $R_s = Q\Lambda Q^H$

$$R = Q \begin{bmatrix} \lambda_1 + \sigma^2 & 0 & \dots & 0 & 0 & \dots & 0 \\ 0 & \lambda_2 + \sigma^2 & \dots & 0 & 0 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \dots & \lambda_M + \sigma^2 & 0 & \dots & 0 \\ 0 & 0 & \dots & 0 & \sigma^2 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \dots & 0 & 0 & \dots & \sigma^2 \end{bmatrix}$$

$$R = Q(\Lambda + \sigma^2 I)Q^H$$

Based on Eigen decomposition we can partition the Eigen vector matrix Qs with M columns, corresponding to M signal Eigen values, and a matrix Qn with (N-M) columns, corresponding the noise Eigen values (σ^2).

Note that Qn, N×(N-M) matrix of Eigen vectors corresponding to the noise Eigen value (σ^2) is exactly the same as the matrix of Eigen vectors of Rs corresponding to the zero Eigen value. Qs defines the signal subspace, while Qn defines the noise subspace.

3.3.2 ROOT-MUSIC:

In music the accuracy is limited by the discretization at which the MUSIC function $P_{MUSIC}(\phi)$ is evaluated. More importantly, it requires either human interaction to decide on the largest M peaks or a comprehensive search algorithm to determine these peaks. This is an extremely computationally intensive process. Therefore, MUSIC by itself is not very practical. We require a methodology that results directly in numeric values for the estimated directions. This is where Root-MUSIC comes in. Note that MUSIC is a technique that estimates the spectrum of the incoming data stream, i.e., it is a spectral estimation technique. The end product is a function $P_{MUSIC}(\phi)$ as a function of the DOA, Φ. Root-MUSIC, on the other hand, is an example of model-based parameter estimation (MBPE) technique. We use a model of the received signal as a function of the DOA here; the model is the steering vector. The DOA, Φ, is a parameter in this model. Based on this model and the received data, we will estimate this parameter. A crucial aspect of MBPE is that the estimation technique is valid only as much as the model itself is valid. For example, our steering vector model is not valid when we take mutual coupling into account or for a circular array. For now we define

$$Z = e^{jkdcos\phi}$$

Then assuming the receiving antenna is linear array of equi spaced, isotropic elements,

$$S(\phi) = [1, Z, Z^2, \dots, Z^{N-1}]^T$$

$$S = \sum_{n=0}^{N-1} q_{mn}^* Z^n = q_m(Z)$$

i.e., inner product of Eigen vector q_m and steering vector $S(\Phi)$ is equivalent to a polynomial in Z . Since we are looking for the directions (Φ) where $q_m \perp S(\Phi)$, $m = (M + 1), \dots, N$, we are looking for the roots of a polynomial.

To find the polynomial whose roots we wish to evaluate, we use

$$P_{MUSIC}^{-1}(\phi) = S^H(\phi) Q_n Q_n^H S(\phi) \\ = S^H(\phi) C S(\phi)$$

$$C = Q_n Q_n^H \\ P_{MUSIC}^{-1}(\phi) = \sum_{m=0}^{N-1} \sum_{n=0}^{N-1} Z^n C_{mn} Z^{-m} \\ = \sum_{m=0}^{N-1} \sum_{n=0}^{N-1} Z^{(n-m)} C_{mn}$$

The final double summation can be simplified by rewriting it as a single sum by setting

$l = n - m$. The range on l is set by the limits on n and m , i.e. $-(N - 1) \leq l \leq (N - 1)$ and

$$P_{MUSIC}^{-1} = \sum_{l=-(N-1)}^{(N-1)} C_l Z^l \\ C_l = \sum_{n-m=l} C_{mn}$$

i.e., C_l is the sum of the elements of C on the n^{th} diagonal. Eqn. (6) defines a polynomial of degree $(2N - 2)$ with $(2N - 2)$ zeros. However, we can show that not all zeros are independent. If z is a

zero of the above polynomial, and of $P_{MUSIC}^{-1}(\phi) \frac{1}{z^*}$, $\frac{1}{z^*}$ is also a zero of the polynomial. The zeros of $P_{MUSIC}^{-1}(\phi)$ therefore come in pairs. Since z and $\frac{1}{z^*}$ have the same phase and reciprocal magnitude, one zero is within the unit circle and the other outside. Note that we are using this root to estimate the signal angle. From the definition of z , only the phase carries the desired information, i.e., both z and $\frac{1}{z^*}$ carry the same desirable information. Also, without noise, the roots would fall on the unit circle.

MUSIC ALGORITHM:

1 Consider the input signal as

$$x = \sum_{m=1}^M \alpha_m S(\phi_m) + n$$

2 Estimate the correlation matrix R using input signal x . Find its Eigen decomposition $R = Q \Lambda Q^H$

3 Partition Q to obtain Q_n , corresponding to the $(N - M)$ smallest Eigen values of Q , which spans the noise subspace.

4 Plot, as a function of Φ , the MUSIC function $P_{MUSIC}(\Phi)$ in Eqn.

$$P_{MUSIC}(\phi) = \frac{1}{\sum_{m=1}^{N-M} \det(S^H(\phi) q_m)^2} \\ = \frac{1}{S^H(\phi) Q_n Q_n^H S(\phi)}$$

5 The M signal directions are the M largest peaks of $P_{MUSIC}(\Phi)$

3.2.3 ROOT-MUSIC ALGORITHM:

1 Estimate the correlation matrix R . Find its Eigen decomposition

$$R = Q \Lambda Q^H$$

2 Partition Q to obtain Q_n , corresponds to the $(N - M)$ smallest Eigen values of Q , which spans the noise subspace.

3 Find $C = Q_n * Q_n^H$

4 Obtain C_l by summing the l^{th} diagonal of C .

5 Find the zeros of the resulting polynomial in terms of $(N-1)$ pairs.

6 Of the $(N - 1)$ roots within the unit circle, choose the M closest to the unit circle

$(Z_m, m = 1, \dots, M)$.

7 Obtain the directions of arrival using

$$\phi_m = \cos^{-1} \left[\frac{3 \ln Z_m}{kd} \right], m=1,2,\dots,M$$

4 PROPOSED METHOD (SMOOTH-MUSIC):

There are several variants of the MUSIC algorithm, including Cyclic-MUSIC and Smooth-MUSIC. Smooth-MUSIC is interesting because it overcomes the MUSIC assumption that all incoming signals are uncorrelated (we had set the matrix A to be diagonal). In a communication situation, assuming flat fading, there may be multipath components from many directions. These components would be correlated with each other. Correlated components reduce the rank of the signal correlation matrix R_s , resulting in more than $(N - M)$ noise Eigen values. In smooth-MUSIC, the N elements are subdivided into L overlapping sub arrays, each with P elements. For example, sub array 0 would include elements 0 through $P - 1$, sub array 1 elements 1 through P , etc. Therefore, $L = N - P + 1$. Using the data from each sub array, L correlation matrices are estimated, each of dimension

$P \times P$. The MUSIC algorithm then continues using a smoothed correlation matrix correlation matrix.

$$R_L = \frac{1}{L} \sum_{l=0}^{L-1} R_l$$

This formulation can detect the DOA of up to $L - 1$ correlated signals. This is because the signal correlation matrix component of R_L becomes full rank again.

SMOOTH-MUSIC ALGORITHM:

- 1 Divide the array into K overlapping sub arrays.
- 2 Estimate the correlation matrix R using input signal for each sub array
- 3 Find its Eigen decomposition $R = Q\Lambda Q^H$ for each sub array
- 4 Estimate the number of received signals.
- 5 Partition Q to obtain Q_n , corresponding to the (N – M) smallest Eigen values of Q, which spans the noise subspace.

6 Plot, as a function of Φ , the MUSIC function $P_{MUSIC}(\phi)$ in Eqn. for each sub array

$$P_{MUSIC}(\phi) = \frac{1}{\sum_{m=1}^{N-M} \det(S^H(\phi)q_m)^2}$$

$$= \frac{1}{S^H(\phi)Q_n Q_n^H S(\phi)}$$

7. The M signal directions are the M largest peaks of $P_{MUSIC}(\phi)$

5. RESULTS AND DISCUSSION:

Let us consider

- Number of incident signals = 2
- Number of elements = 20
- Given Angles [-10, 40]

The figure 1 presents Spectrogram of estimation of two signals located at (-10, 40), respectively, receiving at the uniform linear array. The Direction of Arrival is estimated by MUSIC method using 100 snapshots. The figure shows two peaks coinciding with the real DOA with a high precision, which proves the validity of the system.

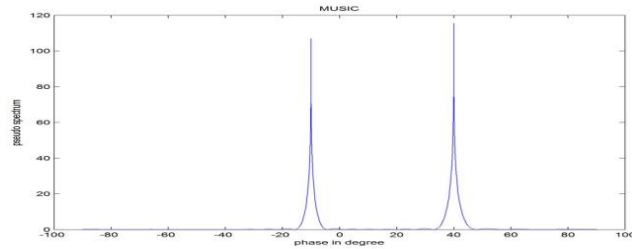


Figure1: Music spectrum

The figure 2 represents the roots of the polynomial equation which are lying in the Z-Plane. Among them, two roots are lying on the unit circle which represents the incident signals at the uniform linear array. By using these two roots we can able to find the direction of arrival estimation. The simulation results of the root-MUSIC algorithm clearly demonstrate the ability to resolve multiple targets with separation angles smaller than the main lobe

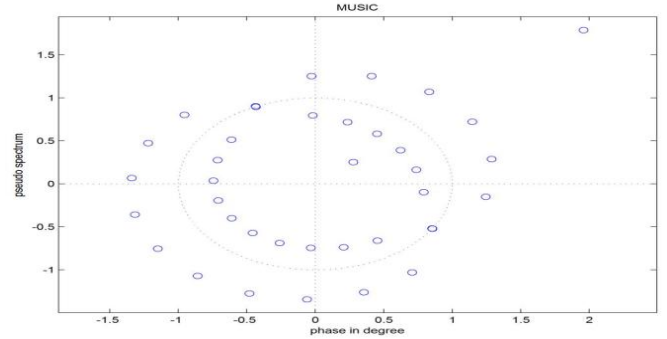


Figure2: Roots in Z-plane(ROOT-MUSIC)

Consider

- Number of incident signals = 2
- Number of elements = 20
- Length of the sub-array = 7
- Number of sub-arrays = 7

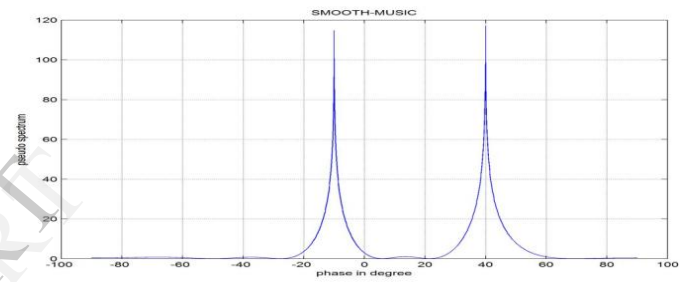


Figure3: Smooth-MUSIC spectrum

The figure 3 presents Spectrogram of estimation of two signals located at (-10, 40), respectively, receiving at the uniform linear array. The Direction of Arrival is estimated by SMOOTH-MUSIC method using 100 snapshots. The figure shows two peaks coinciding with the real DOA with a high precision, which proves the validity of the proposed system.

5.1 Comparision:

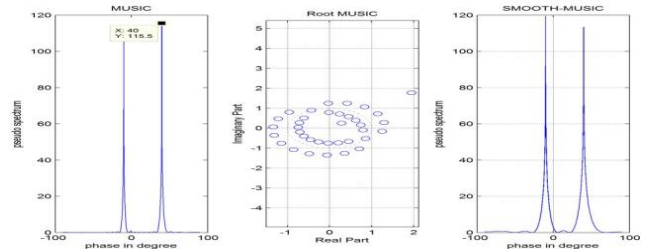


Figure 4: Spectrum for MUSIC, ROOT-MUSIC AND SMOOTH-MUSIC

The figure 4 shows that the simulation results for the MUSIC, ROOT-MUSIC, SMOOTH-MUSIC.

5.2 Performance Analysis:

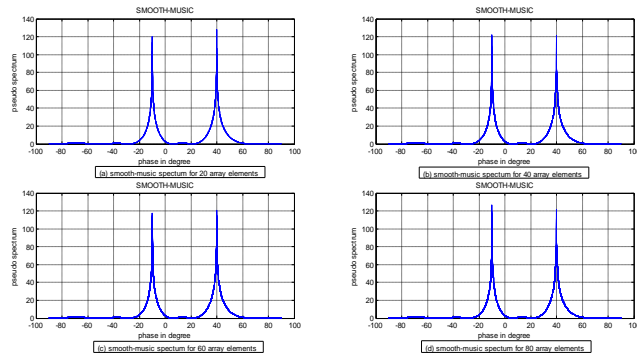


Figure 5: Music for varying number of array elements

The figure 5 indicates that as array size increases from 20 to 80 elements, peaks in the spectrum become sharper and hence resolution capability of MUSIC increases

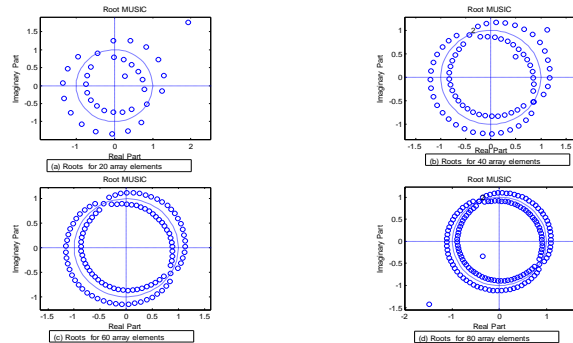


Figure 6: Root-MUSIC for varying number of array elements

The figure 6 indicates that as array size increases from 20 to 80 elements, the roots in the Z-plane are becoming closer to the Unit Circle and hence resolution capability of Root-MUSIC increases.

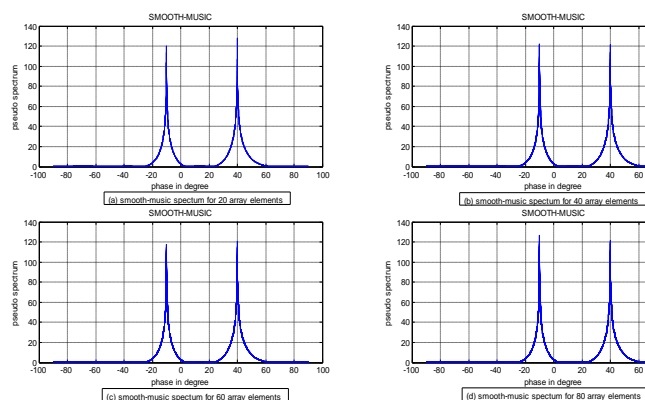


Figure 7: Smooth-MUSIC for varying number of elements

The figure 7 indicates that as array size increases from 20 to 80 elements, peaks in the spectrum become sharper and hence resolution capability of Smooth-MUSIC increases.

6. CONCLUSION:

When we seen the results, as the number of elements in the array increases, then the peaks in the spectrum become sharper and the resolution capability of the MUSIC, ROOT-MUSIC and SMOOTH MUSIC increases. And, the peaks in the spectrum is more sharper for Smooth-MUSIC when compared to the MUSIC. So, the resolution capability for the Smooth-MUSIC is high when compared to the MUSIC and Root-MUSIC in DOA estimation.

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