Discrete-Time Chebyshev Neural Observer for Twin Rotor MIMO System

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***Abstract*-- In this paper, we investigate the problem of designing a Neural Network (NN) observer for the Euler discretized model of Twin Rotor Multi-Input-Multi-Output (MIMO) system which belongs to a class of nonlinear system. The observer is based on a Chebyshev Neural Network (CNN), trained by using Extended Kalman Filter (EKF) learning algorithm. The state estimation error and output error are guaranteed to be semiglobally uniformly ultimately bounded (SGUUB) and neural network weights to be bounded. Simulation results are also included to illustrate the applicability of the proposed observer.**

Keywords-- Chebyshev Neural Network, Discrete–Time Nonlinear System, Extended Kalman Filtering, Neural Observer, Twin Rotor MIMO System.

I. INTRODUCTION

During the past four decades, nonlinear state estimation has been a very important topic for nonlinear control. The concept of an observer for a dynamic process was introduced by D. Luenberger [1]. The generic Luenberger Observer, however, appeared several years after the Kalman Filter [2], which infact an important case of a Luenberger Observer – an observer optimized for the noise present in the observation and in the input to the process. Furthermore, state estimation has been studied by many authors, who have obtained interesting results in different directions [3]-[6]. Most of the approaches need the previous knowledge of the plant dynamics. Recently, neural observers [7]-[8] has emerged for unknown plant dynamics. Now a days neural networks are very important methodology for solving some very difficult problems in engineering, as exemplified by their applications in control nonlinear and complex systems.

In this paper, we develop a Luenberger - like observer for the Euler discretized model of Twin Rotor Multi-Input-Multi-Output (MIMO) system (TRMS) [9] which belongs to a class of nonlinear system. The observer is based on a Chebyshev Neural Network (CNN) [10]-[12], which estimates the state vector of the unknown plant dynamics. In CNN, for functional expansion of the input pattern, we have chosen the Chebyshev polynomials and the network is also named as Chebyshev-Functional Link Artificial Neural Network (CFLANN) [12]. The learning algorithm for the CNN is based on an extended Kalman filter (EKF) [7]-[8], [13], [18]. With the EKF based algorithm, the learning convergence is improved as compared to other previously used algorithms [13]. The state estimation error and output error are guaranteed to be semiglobally uniformly ultimately bounded (SGUUB) and neural network weights to be bounded [7].

This paper presents the following main contributions: In Section II the TRMS system is introduced and its discrete model is obtained. In Section III, CNN structure is given. The EKF training algorithm is given in Section IV. The proposed CNN observer is introduced in Section V. The observer performance is demonstrated in Section VI by providing simulation results. Finally concluding remarks are made in the last section.

II. 2-DOF TRMS MODEL

The mechanical setup of the twin rotor MIMO system is shown in Fig.1.

A. *Continuous-Time Model*

The complete dynamics of the TRMS can be approximately represented in the state space form as follows [9]

$$\left.\begin{aligned}
&\frac{d}{dt}\psi = \dot{\psi} \\
&\frac{d}{dt}\dot{\psi} = \frac{a_1}{I_1}\tau_1^2 + \frac{b_1}{I_1}\tau_1 - \frac{M_g}{I_1}\sin\psi + \frac{0.0326}{2I_1}\sin(2\psi)\dot{\phi}^2 \\
&\qquad - \frac{B_{1\psi}}{I_1}\dot{\psi} - \frac{K_{gy}}{I_1}a_1\cos(\psi)\dot{\phi}\tau_1^2 - \frac{K_{gy}}{I_1}b_1\cos(\psi)\dot{\phi}\tau_1 \\
&\frac{d}{dt}\phi = \dot{\phi} \\
&\frac{d}{dt}\dot{\phi} = \frac{a_2}{I_2}\tau_2^2 + \frac{b_2}{I_2}\tau_2 - \frac{B_{1\phi}}{I_2}\dot{\phi} - \frac{1.75}{I_2}k_c a_1\tau_1^2 - \frac{1.75}{I_2}k_c b_1\tau_1 \\
&\frac{d}{dt}\tau_1 = -\frac{T_{10}}{T_{11}}\tau_1 + \frac{k_1}{T_{11}}u_1 \\
&\frac{d}{dt}\tau_2 = -\frac{T_{20}}{T_{21}}\tau_2 + \frac{k_2}{T_{21}}u_2
\end{aligned}\right\} \quad (1)$$

The output is given by

$$y = [\psi \quad \phi]^T \quad (2)$$

where

ψ : Pitch (Elevation) Angle,
ϕ : Yaw (Azimuth) Angle,

τ_1 : Momentum of Main Rotor,
τ_2 : Momentum of Tail Rotor,
u_1 and u_2 : Inputs,
and, the system parameters of the TRMS are given in Table I.

TABLE I. TRMS PARAMETERS

Parameters	Values
I_1 – Moment of Inertia of Vertical Rotor	6.8x10^{-2} kg-m^2
I_2 – Moment of Inertia of Horizontal Rotor	2x10^{-2} kg-m^2
a_1 – Static Characteristic Parameter	0.0135
b_1 – Static Characteristic Parameter	0.0924
a_2 – Static Characteristic Parameter	0.02
b_2 – Static Characteristic Parameter	0.09
M_g – Gravity Momentum	0.32 N-m
$B_{1\psi}$ – Friction Momentum Function Parameter	6x10^{-3} N-m-s/rad
$B_{1\phi}$ – Friction Momentum Function Parameter	1x10^{-1} N-m-s/rad
K_{gy} – Gyroscopic Momentum Parameter	0.05 s/rad
k_1 – Motor 1 Gain	1.1
k_2 – Motor 2 Gain	0.8
T_{11} – Motor 1 Denominator Parameter	1.1
T_{10} – Motor 1 Denominator Parameter	1
T_{21} – Motor 2 Denominator Parameter	1
T_{20} – Motor 2 Denominator Parameter	1
T_p – Cross Section Momentum Parameter	2
T_0 – Cross Section Momentum Parameter	3.5
k_c – Cross Reaction Momentum Gain	-0.2

Figure 1. The Twin Rotor MIMO System (TRMS).

Consider the state space form

$$\left.\begin{aligned} \dot{x} &= Ax + f(x) + Bu \\ y &= Cx \end{aligned}\right\} \qquad (3)$$

where $x \in \Re^n$, $u \in \Re^m$, $y \in \Re^P$, $A \in \Re^{nxn}$, $B \in \Re^{nxm}$, $C \in \Re^{pxn}$ and $p \geq m$. The function $f(x)$ can be constructed as uncertainties or nonlinearities in plant.

For the state space representation of TRMS by (3)

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & -\frac{B_{1\psi}}{I_1} & 0 & 0 & \frac{b_1}{I_1} & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -\frac{B_{1\phi}}{I_2} & -1.75\frac{k_c b_1}{I_2} & \frac{b_2}{I_2} \\ 0 & 0 & 0 & 0 & -\frac{T_{10}}{T_{11}} & 0 \\ 0 & 0 & 0 & 0 & 0 & -\frac{T_{20}}{T_{21}} \end{bmatrix}_{(6x6)};$$

$$B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ \frac{k_1}{T_{11}} & 0 \\ 0 & \frac{k_2}{T_{21}} \end{bmatrix}_{(6x2)};$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix}_{(2x6)};$$

and

$$f(x) = \begin{bmatrix} 0 \\ \{\frac{a_1}{I_1}\tau_1^2 - \frac{M_g}{I_1}\sin\psi + \frac{0.0326}{2I_1}\sin(2\psi)\dot{\phi}^2 \\ -\frac{K_{gy}}{I_1}a_1\cos(\psi)\dot{\phi}\tau_1^2 - \frac{K_{gy}}{I_1}b_1\cos(\psi)\dot{\phi}\tau_1\} \\ 0 \\ \frac{a_2}{I_2}\tau_2^2 - \frac{1.75}{I_2}k_c a_1 \tau_1^2 \\ 0 \\ 0 \end{bmatrix}_{(6x1)}$$

where

$$x = \begin{bmatrix} \psi & \dot{\psi} & \phi & \dot{\phi} & \tau_1 & \tau_2 \end{bmatrix}^T_{(6x1)}$$

For existence purpose, one requires that $f(x)$ be continuous in x.

B. Discrete-Time Model

The discrete-time model of TRMS, obtained using Euler forward discretization method, is given by

$$\left.\begin{aligned} x(k+1) &= Fx(k) + g(x(k)) + Gu(k) \\ y(k) &= Cx(k) \end{aligned}\right\} \quad (4)$$

where
k : Sampling Step,
$x(k) \triangleq x[(kT_s)]$,
$x(k+1) \triangleq x[(k+1)T_s]$,
$u(k) \triangleq u[(kT_s)]$,
$F = I_6 + AT_s$,
$G = BT_s$,
$g(x(k)) = f(x)T_s$,
T_s : Sampling Time,
and, I_6 is a 6x6 identity matrix.

III. CNN STRUCTURE

Consider a discrete-time MIMO nonlinear system

$$\left.\begin{aligned} x(k+1) &= Fx(k) + g(x(k)) + Gu(k) \\ y(k) &= Cx(k) \end{aligned}\right\} \quad (5)$$

where $x(k) \in \Re^n$, $u(k) \in \Re^m$, $y(k) \in \Re^P$, $F \in \Re^{nxn}$, $G \in \Re^{nxm}$, $C \in \Re^{pxn}$ and $p \geq m$. The function $g(x(k)) \in \Re^n$ is a nonlinear function, which is unknown with entries $g_i(.)$ (i=1, …, n).

Here, a single layer CNN is used to approximate the unknown nonlinear function. The output of the CNN is given by [14]

$$\hat{g}_i(\hat{x}(k)) = w_i^T \varphi(\hat{x}(k)) \qquad (i = 1, \ldots\ldots, n) \quad (6)$$

where $\hat{g}_i(\hat{x}(k))$ is the output of i^{th} neuron, n is the output dimension, w_i is the respective online adapted weight vector, given by

$$w_i = \begin{bmatrix} w_{i1} & w_{i2} & \ldots\ldots\ldots\ldots & w_{iL_i} \end{bmatrix}$$

where L_i is the respective number of higher order connections, and $\varphi(\hat{x}(k))$ is the basis function which is formed using Chebyshev polynomials.

The Chebyshev polynomials can be generated by the following recursive formula

$$T_{r+1}(x) = 2xT_r(x) - T_{r-1}(x), \qquad T_0(x) = 1$$

where $T_r(x)$ is a Chebyshev polynomial, r is the order of polynomials chosen and x is a scalar quantity.

The following theorem states the function approximation capability of CNN

Theorem 1: Assume a feed forward MLP neural network with only one hidden layer and linear activation functions of the output layer. If all the activation functions of the hidden layer satisfy the Riemann integrable condition, then the feed forward neural network can always be represented as a Chebyshev neural network. [11]

The CNN structure is shown in Fig. 2. The basis function is given as

$$\varphi(\hat{x}(k)) = \begin{bmatrix} 1 & T_r(\hat{x}_1(k)) & T_r(\hat{x}_2(k)) & \ldots\ldots\ldots\ldots & T_r(\hat{x}_n(k)) \end{bmatrix}$$

In this paper, the order of r is taken as 2.

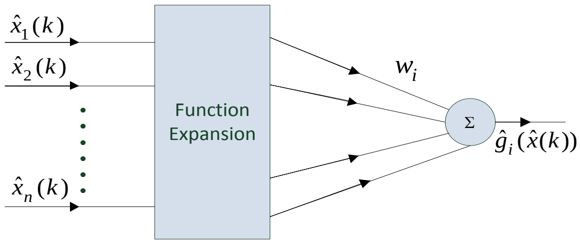

Figure 2. CNN Structure.

A smooth nonlinear function $g(x(k))$ can be approximated by CNN [14]

$$g_i(x(k)) = w_i^{*T} \varphi(x(k)) + \varepsilon_i \quad (7)$$

where $g_i(x(k))$ is the nonlinear function in i^{th} plant state, ε_i is the bounded approximation error [14].

Assume that there exists an ideal weight vector w_i^* such that $\|\varepsilon_i\|$ can be minimized on a compact set $\Omega_i \subset \Re^{L_i}$. The weight estimation error is defined as

$$\tilde{w}_i(k) = w_i^* - w_i(k) \quad (8)$$

where w_i^* is the ideal weight vector and w_i its estimate. Since w_i^* is constant, one has

$$\tilde{w}_i(k+1) - \tilde{w}_i(k) = w_i(k) - w_i(k+1)$$

IV. EKF TRAINING ALGORITHM

In this paper we use the following modified EKF based training algorithm, for designing the CNN observer [7]-[8], [13], [18]

$$w_i(k+1) = w_i(k) + K_i(k)e(k) \qquad i = 1, \ldots\ldots\ldots, n \quad (9)$$

$$K_i(k) = P_i(k)H_i(k)M_i(k) \quad (10)$$

$$P_i(k+1) = P_i(k) - K_i(k)H_i^T(k)P_i(k) + Q_i(k) \quad (11)$$

with

$$M_i(k) = [R_i(k) + H_i^T(k)P_i(k)H_i(k)]^{-1} \quad (12)$$

$$e(k) = y(k) - \hat{y}(k) \quad (13)$$

where $e(k) \in \Re^p$ is the output error and $P_i(k) \in \Re^{L_i x L_i}$ is the weight estimation error covariance matrix at step k, $w_i(k) \in \Re^{L_i}$ is the weight state vector, $y(k) \in \Re^p$ is the plant output, $\hat{y}(k) \in \Re^p$ is the neural observer output, n is the number of plant states, $K_i(k) \in \Re^{L_i x p}$ is the Kalman gain matrix, $Q_i(k) \in \Re^{L_i x L_i}$ is the NN weight estimation noise covariance matrix, $R_i(k) \in \Re^{pxp}$ is the error noise covariance matrix and $H_i(k) \in \Re^{L_i x p}$ is a matrix, in which each entry H_{ij} is the derivative of the i^{th} neural output with respect to ij^{th} neural network weight given as follows

$$H_{ij}(k) = \left[\frac{\partial \hat{y}(k)}{\partial w_{ij}(k)} \right]^T \quad (14)$$

where j=1, ………, L_i and i=1, ………, n. The ij^{th} NN weight is selected according to the output $\hat{y}(k)$. If $\hat{y}(k) = \hat{x}_n(k)$, then H_{ij} is given as

$$H_{ij}(k) = \left[\frac{\partial \hat{y}(k)}{\partial w_{nj}(k)} \right]^T$$

where w_{nj} is the nj^{th} NN weight.

$P_i(k)$, $Q_i(k)$ and $R_i(k)$ matrices are initialized as diagonal matrices with entries $P_i(0)$, $Q_i(0)$ and $R_i(0)$ respectively. It is to be noted that $P_i(k)$, $Q_i(k)$ and $R_i(k)$ for EKF are bounded [15].

V. NEURAL OBSERVER DESIGN USING CNN

Consider an observable discrete-time system given by (5). Equation (5) can be be rewritten as

$$\left.\begin{aligned} x(k) &= [x_1(k) \ldots\ldots\ldots x_i(k) \ldots\ldots\ldots x_n(k)]^T \\ x_i(k+1) &= F_i x(k) + g_i(x(k)) + G_i u(k) \\ y(k) &= Cx(k) \end{aligned}\right\} \quad (15)$$

For the system (15) the proposed CNN observer, shown in Fig. 3, is given as

$$\left.\begin{aligned} \hat{x}(k) &= [\hat{x}_1(k) \ldots\ldots\ldots \hat{x}_i(k) \ldots\ldots\ldots \hat{x}_n(k)]^T \\ \hat{x}_i(k+1) &= F_i \hat{x}(k) + w_i^T \varphi(\hat{x}(k)) + G_i u(k) + D_i e(k) \\ \hat{y}(k) &= C\hat{x}(k) \qquad i = 1, \ldots\ldots\ldots, n \end{aligned}\right\} \quad (16)$$

where $D_i \in \Re^{1xp}$, $F_i \in \Re^{1xn}$ and $G_i \in \Re^{1xm}$.

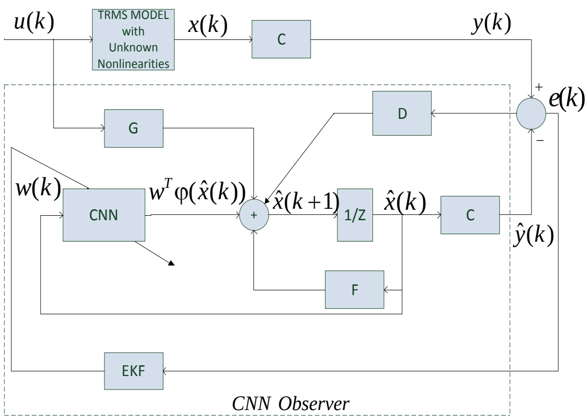

Figure 3. CNN Observer Scheme.

The observer gain matrix $D \in \Re^{nxp}$ is chosen such that *F-DC* is convergent. (Note that a matrix P is called convergent if all the eigen values of P lie inside the open unit circle in complex plane.). The weight vectors are updated online with a modified EKF algorithm (9)-(14). The output error is defined by

$$e(k) = y(k) - \hat{y}(k) \quad (17)$$

and the state estimation error as

$$\tilde{x}(k) = x(k) - \hat{x}(k) \quad (18)$$

The dynamics of (18) can be given as

$$\tilde{x}_i(k+1) = \tilde{w}_i(k)\varphi(x(k)) + \varepsilon_i^* - J_i \tilde{x}(k) \quad (19)$$

with

$$J_i = -(F_i - D_i C)$$

$$\tilde{w}_i = w_i^* - w_i(k)$$

$$\varepsilon_i^* = w_i(k)[\varphi x(k) - \varphi \hat{x}(k)] + \varepsilon_i$$

where ε_i^* is a bounded error term [16].

The dynamics of (8) is given by

$$\tilde{w}_i(k+1) = \tilde{w}_i(k) - K_i(k)e(k) \quad (20)$$

The main result is establish as the following theorem

Theorem 2: For system (15), the nonlinear observer (16) trained with the EKF-based algorithm (9)-(14), ensures that the output error (17) and the estimation error (18) are semi-globally uniformly ultimately bounded (SGUUB). [7]

Proof: Refer to [7].

VI. SIMULATION RESULTS

A detailed simulation study of the proposed observer is carried out with the inputs as $u_1(k) = u_2(k) = 0.2\sin(0.5kT_s) + 0.3\sin(0.6kT_s)$. The sampling time T_s is .001s. All initial values of states are set to zero. All the NN weights are initialized as zero. The covariance matrices are initialized as diagonals and the non-zero elements are $P_i(0)$=100, $Q_i(0)$=.000001 and $R_i(0)$=100 respectively (i=1,……,n).

The simulation results are shown in Fig. 4 and Fig. 5. Fig. 4 shows the actual states and observed states respectively. Fig. 5 shows the state estimation errors.

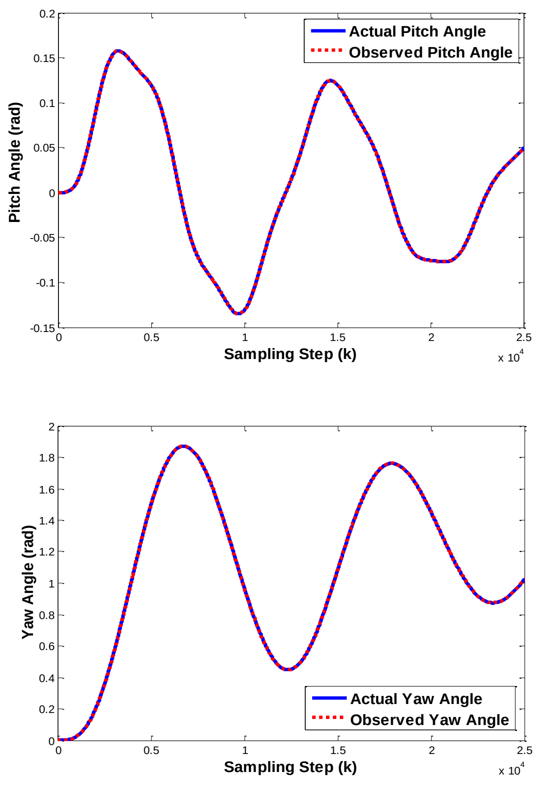

Figure 4. Actual and Observed States of TRMS with CNN Observer.

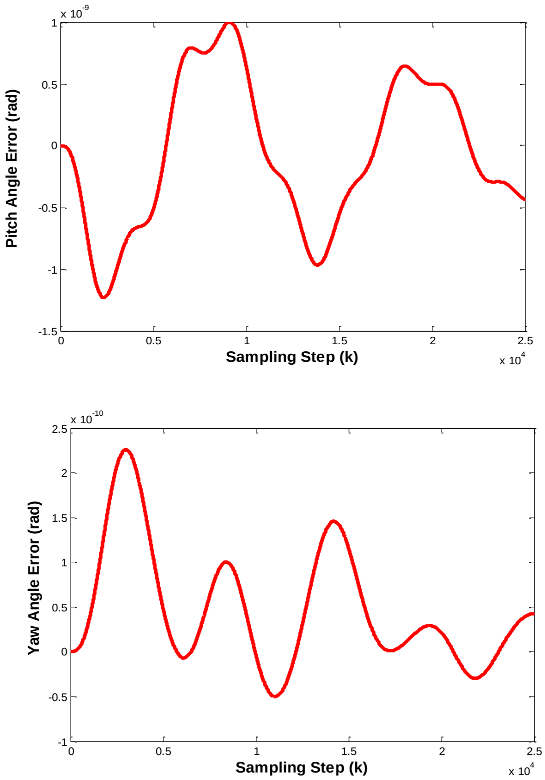

Figure 5. State Estimation Errors in TRMS with CNN Observer.

In the neural observer the unknown nonlinearities are estimated using CNN. It can be seen from Fig. 4 that the response of the observer is good using neural network. Fig. 5 shows that the observer error of TRMS with CNN observer is very small and bounded.

VII. CONCLUSIONS

A neural observer for Euler discretized model of 2-DOF Twin Rotor MIMO System (TRMS) is presented. A CNN is used to design a Luenberger-like observer for a class of MIMO discrete-time nonlinear system. The CNN Observer proposed is trained with an EKF based algorithm. With the EKF based algorithm, the learning convergence is improved as compared to other previously used algorithms. Simulation results show the effectiveness of the proposed CNN Observer.

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