Dispersion Model of Air Pollutant from Continuous Single Point Source

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Abstract

The dispersion models are used to estimate or to predict the downwind concentration of pollutants or toxins emitted from sources such as industrial plants, urban waste, vehicular traffic or accidental chemical releases, for emergency planning of accidental chemical releases. A study was made on the wind for the dispersion of pollutants from a single point source. The concentration of a pollutant in the two regions is governed by the processor of molecular diffusion and convection equation. In the atmosphere, dispersion depends upon the types and numbers of sources, various meteorological factors and topography of the terrain.

The main purpose of this work is to have a model that allows write evolution of pollutant concentration from continuous single point sources having variable wind profile and diffusivities with removal parameter and know the impact on the urban area. We considered that the concentration of pollutant in this cases is governed by a steady state three dimensional convective diffusion equation in which wind profile and vertical diffusivity is a function of z whereas the crosswind diffusivity is considered to be a function of x and z only.

Keywords: *Dispersion, air pollutant, variable wind profile, diffusivities.*

1. Introduction

Dispersion models are now commonly used to evaluate the impacts of air pollution sources, design sampling networks, calculate concentrations in ambient air, etc... However, their use requires some standardization so that the results can be analyzed and used in the best possible.

The air pollution has become a major concern for people living in large urban centres because of the often negative effects generated by industrial development, urbanization and population growth. The political and administrative authorities are already experiencing and will experience a higher level of demand for access to clean, breathable air for all. The city of Tangier for example as urban and industrial metropolis is not immune to threats of pollution that will grow with the commissioning of the port of Tangier Med. These threats are increasing with the current locations of the municipal landfill and industrial area which lie east of the city of Tangier when the east wind is dominant.

The modelling of atmospheric pollution has been studied by Alam and Seinfeld (1981), Reda and Carmichael (1982). The development of a second generation mathematical model for urban air pollution has been presented by Gregory et al. (1982). The effect of a foggy environment on reversible absorption of a pollutant from an area source has recently been studied by Shukla et al. (1982). Several studies have been conducted to understand the process of pollutant dispersion by including some of above mentioned factors (Smith (1957)), (Pasquill (1962)), (Hoffert (1972)) et (Ermak (1977)). A review of atmospheric deposition and plant assimilation of gases and particles has been presented by Smith (1981) and Hosker (1982) wherein a mathematical model for aerosol and deposition on forests employs a modified form of convective of diffusion equation with reaction terms.

Dispersion models are programs that use mathematical algorithms to simulate the dispersion of pollutants in the environment and in some cases how they can react chemically in the atmosphere, require as input various geometrical parameters to calculate the flow field and dispersion characteristics in the urban environment. The dispersion model can be used to assess and predict the concentration of atmospheric pollutants from point sources such as landfill, industrial facilities or vehicular traffic.

In this paper, we study the dispersion of air pollutant from point source. We focus on dry deposition on the ground when the wind velocity and diffusion coefficient of elevation. The exact solution of the diffusion equation with a reaction term is obtained by dividing the inversion layer into two parts. The effect of a green belt on the reduction of concentration of pollutant due to a removal mechanism is then discussed.

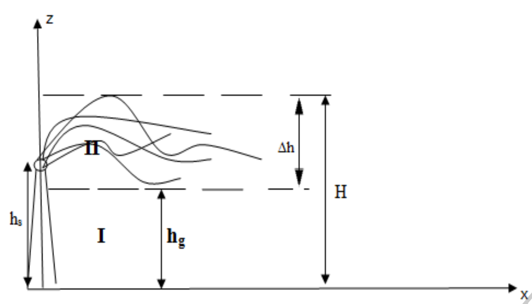

Figure 1: A dispersion model at an effective stack height H

Most of us have observed that the visible plumes from power plants, factories and smokestacks tend to rise and then become horizontal. Plume rise buoyantly because of:

i) They are hotter than the surrounding air.

ii) They exit the stack with vertical velocity.

They stop rising because as they mix with the surrounding air, they lose the velocity and cool by mixing. Finally, they level off when the temperature is the same with atmosphere.

To calculate the plume rise (Δh), we applied this equation,

$$\Delta h = \frac{v_s D}{u}\left(1.5 + 2.68PD\frac{T_s - T_a}{T_s}\right)$$

where

Δh is the plume rise, v_s is the stack exit velocity, D is the stack diameter , u is the wind speed, P is the pressure, T_s is the stack gas temperature and T_a is the atmospheric temperature.

2. MATHEMATICAL FORMULATION

The differential equation governing the concentration of pollutant in the atmosphere can be written as

$$u_i \frac{\partial C_i}{\partial x} = K_{zi}\frac{\partial^2 C_i}{\partial z^2} + K_{yi}\frac{\partial^2 C_i}{\partial y^2} - \alpha C_i \qquad (1)$$

where the diffusion in the downwind direction has been neglected as compared to the advection. This hypothesis is widely used in connection with diffusion from a continuous source.

The dispersion of reactive pollutants in the air from a point source located at a height h_s from the ground in the presence of inversion layer (0 < z <h) of height H. We consider the inversion layer is assumed to be divided into two layers:

- Region I : (0 < z < h_g)
- Region II : (h_g < z < h)

where h_g is the height of the lower layer.

When the direction is taken to be the prevalent wind direction, the equilibrium equations for the dissemination of state governing the pollutant concentration in the two regions can be written as follows:

- Region I: 0 < z < h_g

$$u_1 \frac{\partial C_1}{\partial x} = K_{z1}\frac{\partial^2 C_1}{\partial z^2} + K_{y1}\frac{\partial^2 C_1}{\partial y^2} - \alpha C_1 \qquad (2)$$

with the following boundary conditions:

$$C_1 = 0 \quad for \quad x = 0 \ ; \qquad (3)$$

$$C_1 = 0 \quad for \quad y \to \pm\infty \ ; \qquad (4)$$

$$K_{z1}\frac{\partial C_1}{\partial z} = v_d C_1 \quad for \quad z = 0 \ ; \qquad (5)$$

$$K_{z1}\frac{\partial C_1}{\partial z} = K_{z2}\frac{\partial C_2}{\partial z} \ ; \ C_1 = C_2 \quad for \quad z = h_g \ ; \ (6)$$

- Region II : h_g < z < H

$$u_2 \frac{\partial C_2}{\partial x} = K_{z2}\frac{\partial^2 C_2}{\partial z^2} + K_{y2}\frac{\partial^2 C_2}{\partial y^2} - \alpha C_2 \qquad (7)$$

with

$$C_2 = \frac{Q}{u_2}\delta(y)\delta(z-h_s) \quad for \quad x=0 \quad ; \qquad (8)$$

$$C_2 = 0 \quad for \quad y \to \pm\infty \quad ; \qquad (9)$$

$$\frac{\partial C_2}{\partial z} = 0 \quad for \quad z = H \quad ; \qquad (10)$$

$$K_{z2}\frac{\partial C_2}{\partial z} = K_{z1}\frac{\partial C_1}{\partial z} \;;\; C_2 = C_1 \quad for \quad z = h_g ; \qquad (11)$$

where C_1 et C_2 are the concentrations of the pollution of the air in the two regions, u_i is the average speed in each region, K_{yi}, K_{zi} are the diffusion coefficient, v_d is the deposition velocity, α is the coefficient of chemical reaction and δ is the Dirac function.

3. Solution

To solve differential equation (2) and (7), we use dimensionless quantities

$$\bar{x} = \frac{K_{z\max}x}{uH^2} \;;\; \bar{z} = \frac{z}{H} \;;\; \bar{u}_i = \frac{u_i}{u_{\max}} \;;$$

$$\bar{K}_{zi} = \frac{K_{zi}}{K_{z\max}} ;\; \bar{v}_d = \frac{v_d h}{K_{z\max}} \;;\; \bar{h}_g = \frac{h_g}{H} ;$$

$$\bar{\alpha} = \frac{kH^2\alpha}{K_{z\max}} \;;\; \bar{C}_i = \frac{u_{\max}H^2}{Q}C_i \;;$$

These quantities in the above equations are introduced.

The partial differential equation (2) becomes:

$$\frac{\partial C_1}{\partial x} = \beta_1\frac{\partial^2 C_1}{\partial z^2} + \gamma_1\frac{\partial^2 C_1}{\partial y^2} - \alpha_1 C_1 \;; \qquad (12)$$

with the boundary conditions:

$$C_1 = 0 \quad for \quad x = 0 \;;$$

$$C_1 = 0 \quad for \quad y \to \pm\infty \quad ;$$

$$K_{z1}\frac{\partial C_1}{\partial z} = v_d C_1 \quad for \quad z = 0 \;;$$

$$K_{z1}\frac{\partial C_1}{\partial z} = K_{z2}\frac{\partial C_2}{\partial z} \;;\quad C_1 = C_2 \quad for \quad z = h_g ;$$

The partial differential equation (7) becomes

$$\frac{\partial C_2}{\partial x} = \beta_2\frac{\partial^2 C_2}{\partial z^2} + \gamma_2\frac{\partial^2 C_2}{\partial y^2} - \alpha_2 C_2 \;; \qquad (13)$$

with the boundary conditions:

$$C_2 = \frac{Q}{u_2}\delta(y)\delta(z-h_s) \quad for \quad x=0 \quad ;$$

$$C_2 = 0 \quad for \quad y \to \pm\infty \quad ;$$

$$\frac{\partial C_2}{\partial z} = 0 \quad for \quad z = H \;;$$

$$K_{z2}\frac{\partial C_2}{\partial z} = K_{z1}\frac{\partial C_1}{\partial z} \;;\; C_2 = C_1 \quad for \; z = h_g \;,$$

where

$$\gamma_i = \frac{K_{zi}}{u_i} ;\; \alpha_i = \frac{\alpha}{u_i} \;;$$

$$N_1 = \frac{v_d}{Kz_1} \;;\; \beta_i = \frac{K_{yi}}{K_{z\max}u_i} ;$$

Assuming that $\beta_1 = \beta_2 = \beta$ and the variable separation method is used:

$$C_1 = A_1(x,y) * B_1(x,z) \;;$$

then by substituting in the equation (12) and (13), we obtained:

$$B_1\frac{\partial A_1}{\partial x} + A_1\frac{\partial B_1}{\partial x} = \beta_1 B_1\frac{\partial^2 A_1}{\partial y^2} + \gamma_1 A_1\frac{\partial^2 B_1}{\partial z^2} - \alpha_1 A_1 B_1 \;;$$

where

$$\begin{cases} \dfrac{\partial A_1}{\partial x} = \beta_1\dfrac{\partial^2 A_1}{\partial y^2} & (14) \\ \dfrac{\partial B_1}{\partial x} = \gamma_1\dfrac{\partial^2 B_1}{\partial z^2} - \alpha_1 B_1 & (15) \end{cases}$$

using Poisson formula, the equation (14) gives us:

$$A_1 = \frac{\exp(\frac{-y^2}{4\beta x})}{\sqrt{4\beta\pi x}};$$

We used the 2nd time a separation method for variable B (x, z)

$$B_1 = X_1(x) * Z_1(z);$$

using equation (15), we have:

$$\begin{cases} \dfrac{\partial X_1}{\partial x} + \lambda^2 X = 0 & (16) \\ \dfrac{\partial^2 Z_1}{\partial z^2} + \dfrac{\lambda^2 - \alpha_1}{\gamma_1} Z_1 = 0 & (17) \end{cases}$$

where λ is the solution of the following equation:

$$\frac{K_{z1}\left(N_1 * \cos(a_{1n}h_g) - a_{1n} * \sin(a_{1n}h_g)\right)}{\left((N_1 / a_{1n}) * \sin(a_{1n}h_g) + \cos(a_{1n}h_g)\right)} = \frac{K_{z2}\left(a_{2n} * \tan(a_{2n}) * \cos(a_{2n}h_g) - a_{2n} * \sin(a_{2n}h_g)\right)}{\left(\tan(a_{2n}h_g) * \sin(a_{1n}h_g) + \cos(a_{1n}h_g)\right)} \quad (18)$$

The solution general can be obtained with the assumption $\dfrac{K_{y1}}{u_1} = \dfrac{K_{y2}}{u_2}$, as

$$\boxed{C_i = \frac{e^{(\frac{-y^2}{4\beta_i x})}}{\sqrt{4\pi\beta_i x}} \sum_{n=1}^{\infty} e^{-\lambda_n^2 x} R_n \frac{\frac{v_d}{a_{in}K_{zi}}\sin(a_{in}z) + \cos(a_{in}z)}{\frac{v_d}{a_{in}K_{zi}}\sin(a_{in}h_g) + \cos(a_{in}h_g)}} \quad (19)$$

where

$$R_n = \frac{\tan(a_{in})\sin(a_{in}h_s) + \cos(a_{in}h_s)}{\tan(a_{in})\sin(a_{in}h_g) + \cos(a_{in}h_g)} \quad ,$$

$$a_{in} = \sqrt{(\lambda_n^2 - \alpha_i) / \gamma_i} \quad .$$

The Raphsen Newton's method is used to solve equation (18). We find the graphe

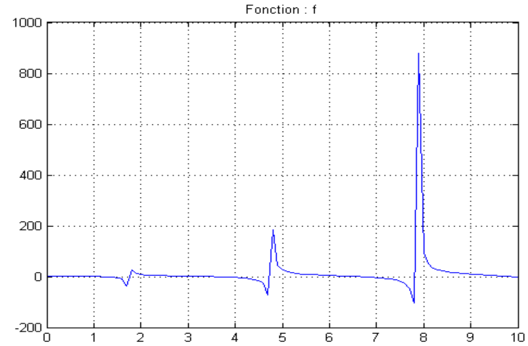

Figure 2: Raphsen Newton's profile solution

4. RESULTS AND DISCUSSION

A set of graphs have been plotted corresponding to single point sources by using equations (12) and (13). All physical parameters considered here are dimensionless.

We considered that a pollutant source is located at the origin at a height $z = h_s$.

To study the effects of various parameters are chosen by keeping in view that the wind velocity and diffusion coefficient near the grounds are smaller:

$u_1 = 0.55$, $u_2 = 1.0$, $K_{z1} = 0.55$, $K_{z2} = 1.0$, $\alpha = 0.55$, $h_s = 0.1$, $h_g = 0.05$ and $\beta_1 = \beta_2 = 10.0$

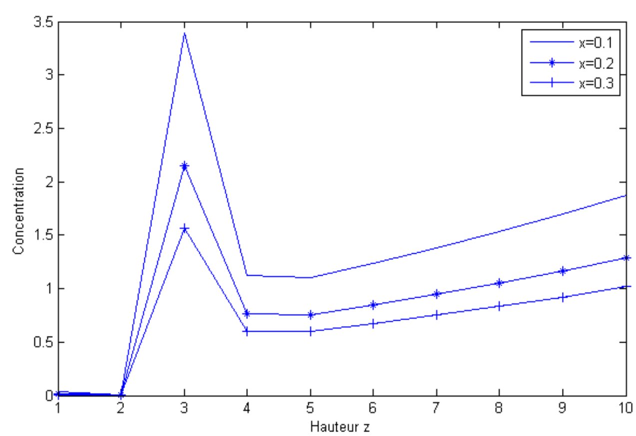

Fig 3 : Concentration of the pollutant is plotted against vertical height ($0 \leq z \leq 10$) for different downwind distance with fixed value of crosswind distance y = 0

The graph were plotted corresponding to x = 0.1, 0.2, 0.3. It can be seen that the concentration profile increases up to z = 3 at which it attain its peak and then steadily decreases. For z > 4 the profile becomes a bit uniform except which is due to the closeness to the source. It may also be concluded that as the downwind distance increases, the peak of the concentration curve decreases that implies greater danger from the pollutant near to the surface at the point where the emission falls to the ground

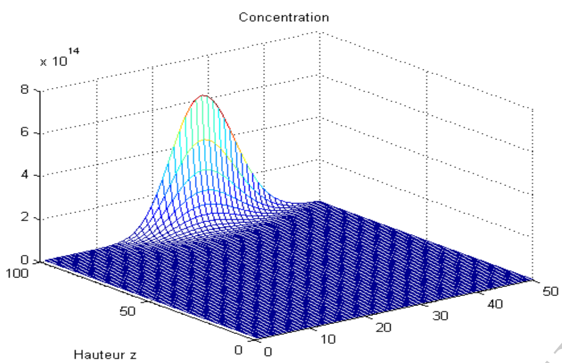

Figure4: 2D Concentration of the pollutant is plotted against crosswind distance for the different value of downwind distance with fixed value of z.

In figure 4 It can be seen that the figure the concentration profile are symmetric with peak concentration along the centreline of the plume. For single point source, the centreline plume is at y = 0 corresponding to which curve attain its peak whereas. Also it may be concluded that the concentration level is low & approaches to uniform distribution throughout the region. Therefore to reduce the effect of concentration of pollutant one has to establish a greenbelt (or artificial removal) were the concentration level is maximum.

5. Conclusion

A mathematical model has been developed on the effect of dispersion of pollutants from single point sources having variable wind profile and variable diffusivities. We obtained the analytical solution by considering similar boundary conditions. Thus the concentration level is high and varies significantly from the corresponding variation of the vertical distance. This implies a greater risk of pollution in the vicinity of where the issue falls to the ground. It is also seen that in this account the level of pollutant concentration is important to certain height and then decreases when you cannot see the wind profile and diffusivity is constant Shukla et al. (Shukla et al. (1991) where the pollutant concentration decreases significantly from the surface to the cloud base. Therefore the impact of the pollutant is highest in a region close to the surface, which can directly or indirectly affect humans and the environment. The results of dispersion modeling can provide an estimate of the location of the affected areas and ambient concentrations. Thus, and can be used to determine the appropriate protective measures to the accident.

6. References

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