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Dissipation Of Energy In Viscous Liquid Through Porous Region

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Abstract

In the paper, the flow of a viscous liquid is considered through a cylinder containing porous region. The mechanical energy dissipated in the fluid is calculated. The boundary of the tube performs harmonic oscillations. The effect of the permeability coefficient on the flow and the dissipation of energy are examined and discussed completely.

Key words and phrases: Newtonian Fluid, dissipation of energy, Porous medium, Permeability.

1. Introduction

In the paper, the study of flow through porous medium has many interesting applications in the diverse fields of science, engineering and technology. The particular applications which are well-known include the percolation of water through soil, extraction and filtration of oils from wells, the drainage of water, irrigation and sanitary engineering and also in the inter-disciplinary fields such as medical and bio-medical engineering etc. The lung alveolar is an example that finds application in the animal body. The classical Darcy's law musakat (3) states that the pressure gradient pushes the fluid against the body forces exerted by the medium which can be expressed as.

$$\bar{v} = -\left(\frac{K}{\mu}\right) \nabla \, p \qquad \text{(with usual notion)}$$

The classical Darcy's law gives good results in the situations when the flow is uni-directional or at low speed. In general, the specific discharge in the medium need not be always low. As the specific discharge increases, the convective forces get developed and the internal stress generates in the fluid due to its viscous nature and produces distortions in the velocity field. In the case of highly porous medium such as fiberglass, papus of dandelion the flow occurs even in the absence of the pressure gradient.

Modifications of the classical Darcy's law were considered by the Beverse and Joseph [1], saffman [9] and other. A generalized Darcy's law proposed by Brinkman [2] in given by

$$O = -\nabla p - \left(\frac{\mu}{K}\right)\bar{v} + \mu \nabla^2 \bar{v}$$

Where μ and K are co-efficient of viscosity of the fluid and permeability of the porous medium respectively.

The generalized equation of momentum for the flow through the porous medium is

$$\rho\left[\frac{\partial \bar{v}}{\partial t} + (\bar{v} \cdot \nabla)\bar{v}\right] = -\nabla p + \mu \nabla^2 \bar{v} - \left(\frac{\mu}{K}\right)\bar{v}$$

The classical Darcy's law helps in studying flows through porous medium. In the case of highly porous medium such as papus of dandelion etc., The Darcy's law fails to explain the flow near the surface in the absence of pressure gradient. The non-Darcian approach is employed to study the problem of flow through highly porous medium by several investigators. Narsimha charyulu and pattabhi Rama Charyulu [4, 5] Narsimha Charyulu [6] and singh [7] etc, studied the flow employing Brinkman law [2] for the flow through highly porous medium.

The problem of flow of the Newtonian fluid in the presence of transverse magnetic field, find application in nuclear engineering and other fields. The rotatory flow of the fluid has special applications in various engineering field such as mechanical, petroleum and chemical in addition to the geophysical fluid dynamics, which helps in explaining the phenomena like oceanic circulation [8]. Several investigations are made in the study of flow of viscous fluid in the presence of transverse magnetic field under the assumption that the induced magnetic effect is negligible on the flow of the fluid e.g. see Greenspan [10] and Herbut [10] etc. But some investigations are made by considering the effect of the induced magnetic field on the flow; e.g. see somdalgekar [12] and pop [13] etc.

The flow through porous medium in the presence of transverse magnetic field is studied in the past by several investigators. The non-Darcian flow in the presence of transverse magnetic field is investigated by Nassimha charyulu [14]. Venkat Raman and Narsimha Charyulu [15, 16, 17, and 18] have studied the flow employing Brinkman's law [2] for the flow through a rotating porous duct and highly porous medium.
In the paper, we considered the flow of a viscous liquid through a cylinder containing porous region. The mechanical energy dissipated in the fluid is calculated. The boundary of the tube performs harmonic oscillations. The effect of the permeability coefficient on the flow and the dissipation of energy are examined and discussed completely.

2. Formulation and solution of the problem

Consider the flow of an incompressible, viscous liquid through porous region contained by an infinite circular tube of radius a. Let (r,θ,x) be the co-ordinate system such that the x co-ordinate is along the axis of the tube. Let the velocity of the fluid is given by $\vec{V}(u,0,0)$ which satisfies the equation of continuity

$$\nabla \cdot \vec{V} = 0. \tag{2.1}$$

The physical quantities are independent of x and also independent of θ because of symmetry of the flow. The Navier-Stokes equation for the flow problem will be

$$\frac{\partial u}{\partial t} = \nu\left(\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r}\right) - \frac{\nu}{k}u \tag{2.2}$$

where, ν the kinetic viscosity, t is the time and k is the permeability of the medium.

The boundary conditions are given by

$$u(a,t) = 0, \qquad t < 0 \tag{2.3}$$

$$u(a,t) = U\cos pt, \qquad t > 0 \tag{2.4}$$

where, U is the amplitude of the velocity fluctuation and p the frequency of the motion of the tube.

Flow model

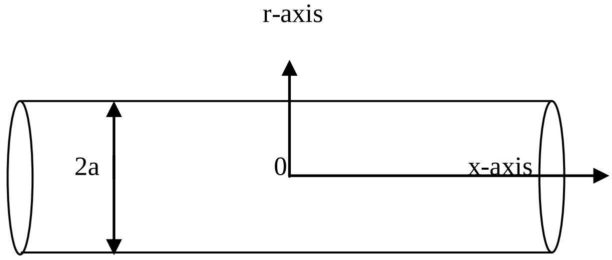

Figure 1. Flow of Newtonian fluid through porous region

By applying Laplace transform on equations (2.2), (2.3) and (2.4), we get the transformed equations as

$$\frac{d^2\bar{u}}{dr^2} + \frac{1}{r}\frac{d\bar{u}}{dr} - \alpha^2\bar{u} = 0 \tag{2.5}$$

and

$$\bar{u}(a,s) = \frac{Us}{s^2 + p^2} \tag{2.6}$$

where, $\alpha^2 = \frac{s}{\nu} + \frac{1}{k}$.

The solution of equation (2.5) satisfying the condition at the origin and the boundary condition (2.6) is

$$\bar{u}(r,s) = \frac{Us}{s^2 + p^2}\,\frac{I_0(\alpha r)}{I_0(\alpha a)} \tag{2.7}$$

The inverse Laplace transform of $\vec{u}(r,s)$, gives

$$u(r,t) = \frac{U}{2\pi i}\int_{\beta - i\infty}^{\beta + i\infty} \frac{s}{s^2 + p^2}\,\frac{I_0(\alpha r)}{I_0(\alpha a)}\exp(st)\,ds \tag{2.8}$$

where, β is a constant such that all poles lie to its left. It can easily be verified that the integrand is a single-valued function of s. the inversion may be performed by summing the residues at the simple poles s = ± ip, and s + (ν /k) = - ν α_n^2 where ± α_n (n = 1, 2, 3 …) are the roots of $J_0(\alpha_n a) = 0$. It can be shown that there are no branch points. Evaluating the residues and using Cauchy's integral theorem gives,

$$u(r,t) = \frac{U}{2}\frac{I_0[\sqrt{(l)}r]}{I_0[\sqrt{(l)}a]}\exp(ipt) + \frac{U}{2}\frac{I_0[\sqrt{(-l)}r]}{I_0[\sqrt{(-l)}a]}\exp(-ipt)$$

$$-\frac{2\nu^2 U}{a}\sum_{n=1}^{\infty}\frac{\alpha_n{}^3}{\alpha_n{}^4 + p^2}\,\frac{J_0(\alpha_n r)}{J_1(\alpha_n a)}\exp(-\nu\alpha_n{}^2 t) \tag{2.9}$$

where, $l = i\left(\frac{p}{\nu} + \frac{1}{k}\right)$. (2.10)

2.1 Dissipation of Energy

The time rate of energy dissipated per unit length along the axis of the tube in viscous flow is obtained from

$$\frac{dE}{dt} = -2\pi\mu\int_0^a r\left(\frac{\partial u}{\partial r}\right)^2 dr \qquad (2.11)$$

where, μ denotes the coefficient viscosity and the negative sign is inserted because the integral on the right represents a loss of energy. The velocity gradient obtained from the equation (2.9) as

$$\frac{\partial u}{\partial r} = \frac{U}{2}\sqrt{(l)}\,\frac{I_1[\sqrt{(l)}r]}{I_0[\sqrt{(l)}a]}\exp(ipt) + \frac{U}{2}\sqrt{(-l)}\,\frac{I_1[\sqrt{(-l)}r]}{I_0[\sqrt{(-l)}a]}\exp(-ipt)$$

$$+\frac{2\nu^2 U}{a}\sum_{n=1}^{\infty}\frac{\alpha_n^{\,4}}{\alpha_n^{\,4}+p^2}\,\frac{J_1(\alpha_n r)}{J_1(\alpha_n a)}\exp(-\nu\alpha_n^{\,2}t)\cdot \qquad (2.12)$$

Using the equations (2.11), (2.12) and carrying out the integrations gives,

$$\frac{1}{2}\frac{dE}{dt} = \frac{U^2\mu\pi}{8}\,\frac{(i\gamma)\exp(i2pt)}{I_0^{\,2}[\sqrt{(i\gamma)}]}\left\{I_0^{\,2}(i\gamma) - 2\frac{I_0[\sqrt{(i\gamma)}]I_1[\sqrt{(i\gamma)}]}{\sqrt{(i\gamma)}} - I_1^{\,2}[\sqrt{(i\gamma)}]\right\}$$

$$+\frac{U^2\mu\pi}{8}\,\frac{(-i\gamma)\exp(-i2pt)}{I_0^{\,2}[\sqrt{(-i\gamma)}]}\left\{I_0^{\,2}(-i\gamma) - 2\frac{I_0[\sqrt{(-i\gamma)}]I_1[\sqrt{(-i\gamma)}]}{\sqrt{(-i\gamma)}} - I_1^{\,2}[\sqrt{(-i\gamma)}]\right\}$$

$$+2U^2\mu\pi\sum_{n=1}^{\infty}\frac{r_n^{\,2}}{(r_n^{\,2}+\gamma^2)^2}\exp\left(\frac{-pr_n^{\,2}t}{\gamma}\right) + \frac{U^2\mu\pi}{4}\,\frac{i}{I_0[\sqrt{(i\gamma)}]I_0[\sqrt{(-i\gamma)}]}$$

$$\times\left\{\sqrt{(i\gamma)}I_0[\sqrt{(i\gamma)}]I_1[\sqrt{(-i\gamma)}] - \sqrt{(-i\gamma)}I_0[\sqrt{(-i\gamma)}]I_1[\sqrt{(i\gamma)}]\right\}$$

$$-\frac{2U^2\mu\pi\sqrt{(-i\gamma)}\exp(-ipt)}{I_0[\sqrt{(-i\gamma)}]}\sum_{n=1}^{\infty}\exp\left(\frac{-pr_n^{\,2}t}{\gamma}\right)$$

$$\times\frac{r_n^{\,4}\left\{\sqrt{(-i\gamma)}J_1(r_n)I_2[\sqrt{(-i\gamma)}] + r_n I_1[\sqrt{(-i\gamma)}]J_2(r_n)\right\}}{(r_n^{\,2}-i\gamma)(r_n^{\,4}+\gamma^2)}$$

$$-\frac{2U^2\mu\pi\sqrt{(i\gamma)}\exp(ipt)}{I_0[\sqrt{(i\gamma)}]}\sum_{n=1}^{\infty}\exp\left(\frac{-pr_n^{\,2}t}{\gamma}\right)$$

$$\times\frac{r_n^{\,4}\left\{\sqrt{(i\gamma)}J_1(r_n)I_2[\sqrt{(i\gamma)}] + r_n I_1[\sqrt{(i\gamma)}]J_2(r_n)\right\}}{(r_n^{\,2}+i\gamma)(r_n^{\,4}+\gamma^2)} \qquad (2.13)$$

where, $r_n = \alpha_n a$ and

$$\gamma = \left(\frac{p}{\nu}+\frac{1}{k}\right)a^2. \qquad (2.14)$$

The energy dissipation per unit length of the tube at the end of the m[th] cycle is obtained by integrating the equation (2.13) over m cycles. The resulting expression is

$$E = \frac{2U^2\gamma\mu\pi}{p}\sum\frac{r_n^{\,6}}{(r_n^4+\gamma^2)^2}\left[\exp\left(\frac{-4r_n^2 m\pi}{\gamma}\right)-1\right]$$

$$+\frac{U^2\mu m\pi^2 i}{p}\left\{\frac{\sqrt{(i\gamma)}I_0[\sqrt{(i\gamma)}]I_1[\sqrt{(-i\gamma)}] - \sqrt{(-i\gamma)}I_0[\sqrt{(-i\gamma)}]I_1[\sqrt{(i\gamma)}]}{I_0[\sqrt{(i\gamma)}]I_0[\sqrt{(-i\gamma)}]}\right\} \qquad (2.15)$$

3. SPECIAL CASES

3.1 Case (i). For the highly porous medium (i.e. k is very large)

The energy dissipation is given by

$$E = \frac{-U^2 m\mu\pi^2}{p}\left[\sum_{n=1}^{\infty}\frac{r_n^8}{(r_n^4+\lambda^2)^2} + \frac{\left(1+\frac{\lambda^2}{192}\right)}{8\left(1+\frac{\lambda^2}{32}+\frac{\lambda^4}{4096}\right)}\right] \qquad (3.1)$$

3.2 Case (ii). For the flow through clear medium (i.e. k→∞)

The dissipation energy is given by

$$E = \frac{-U^2 m\mu\pi^2}{p}\left[\sum_{n=1}^{\infty}\frac{r_n^8}{(r_n^4+\frac{p^2a^4}{\nu^2})^2} + \frac{\left(1+\frac{p^2a^4}{192\nu^2}\right)}{8\left(1+\frac{p^2a^4}{32\nu^2}+\frac{p^4a^8}{4096\nu^4}\right)}\right] \qquad (3.2)$$

3.3 Case (iii). When the permeability of the medium is very small (i.e. k→0)

The energy dissipation is given by

$$E = \left[\sqrt{i\lambda} - \sqrt{-i\lambda}\right]\times\frac{\left[1-\frac{3}{64\lambda}\left\{1-8(\sqrt{i\lambda}+\sqrt{-i\lambda})\right\}\right]}{\left[1+\frac{1}{64\lambda}\left\{8(\sqrt{i\lambda}+\sqrt{-i\lambda})+1\right\}\right]} \qquad (3.3)$$

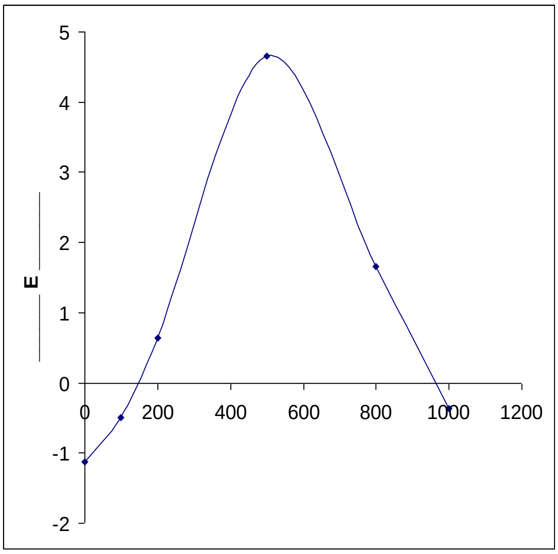

Figure.1 Variation of E for different values of γ. (r_n=1)

4. Results and Conclusion

In the present problem, an attempt is made to estimate the mechanical energy dissipated in the fluid through a circular tube whose boundary performs harmonic motion. Expression for the energy is obtained when the tube is filled with highly porous medium. The energy dissipated per unit length of the tube is obtained in terms of γ which involves the permeability coefficient. The graph depicting the variation of E for different values of the permeability coefficient is drawn in fig.1.

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