Double Closed Loop Coordinated Control of Quadrotor Based on PID / LADRC

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Abstract- For control system of a quadrotor aircraft with under-driven, strong coupling and nonlinear characteristics. This paper presents a double closed loop collaborative control method of PID / LADRC to control the position and attitude of the control system. Among them, the attitude loop is the inner loop of the control system, and the linear automatic disturbance rejection control method is adopted. The position loop is the outer loop of the control system, and the PID control method is adopted. Due to the under-driven nature of the control system of quadrotors aircraft itself, the expected values of pitch and roll angle are given by the outer loop controller. Finally, through the MATLAB simulation of the control system, and then to verify the effectiveness of the control system of quadrotor using the PID / LADRC double closed-loop coordinated control method.

Keywords: Quadrotors; Position Control; Attitude Control; Automatic Disturbance Rejection Control; PID Control

I. INTRODUCTION

In recent years, the continuous development and advancement in the field of control have promoted the development of the quadrotors aircraft. In terms with the traditional unmanned aerial vehicle, the quadrotors aircraft have the pros of small size, light weight and flexible maneuver performance. Therefore, a considerable number of domestic and foreign universities and researcher's attention. As the quadrotors aircraft can achieve translation and rotation during the movement, and then can do pitch, roll, yaw, hover and vertical movement, before and after the movement and lateral movement of the six basic movement states. Through the different combinations of the six basic movement states, it can make any trajectories and motions in any three-dimensional space and is widely used in civil and military areas. For example, on the military side, the quadrotors aircraft can perform control tasks such as detecting battlefields, targeting and tracking targets, and delivering weapons as carriers; On the civilian side, the quadrotors aircraft can be used in aerial photography, environmental surveys and search and rescue after a major disaster [1], and can carry on scientific experiments by carrying a variety of devices.

Due to the typical features of under-driven, strong coupling and non-linearity, the quadrotors aircraft bring tremendous difficulties to the control of quadrotor aircraft [2]. At the same time, the mass of quadrotors aircraft is lighter, so during the movement in the extremely vulnerable to external environment and other factors. Thus, we need to be strong anti-interference performance in the design of the controller.

The continuous development and advancement of control theory has given rise to an increasing number of control methods being applied to the analysis and design of the quadrotors control system. In the meantime, the control system can be controlled more optimally by using the above control methods. For example, PID control [3-4], adaptive control

[5], fuzzy control [6], sliding mode control [7-8]and ADRC [9]. Simple sliding mode control has the significant pros of strong robustness. When the control system is disturbed by the outside world, the steady state error will be generated, which will result in lower control accuracy. The adaptive control is generally used in conjunction with other methods, and through the integrated control technology to achieve the stability of its control system. In the case of model uncertainty and external disturbance, adaptive fuzzy control based on fuzzy CMAC is applied to the control system. At the same time, adaptive parameters are used to make the control system have good steady state performance.

In this paper, based on a dual closed-loop control method combining PID and LADRC to control the position and attitude of the quadrotors, the inner loop adopts the method of LADRC to control the attitude of the aircraft, while the outer loop adopts the method of PID control to control the position of the aircraft, the controller has strong anti-jamming performance and strong robustness.

II. THE QUADROTOR AIRCRAFT DYNAMICS MODEL AND ANALYSIS

As the quadrotor aircraft is a nonlinear, multivariable and strongly coupled under-driven system, in order to further control such under-actuated control systems, such as the quadcopter, firstly, the dynamics model of the control system must be established. In order to establish the dynamics model of quad-rotor control system, first and foremost, we must select the appropriate coordinate system, which is the ground coordinate system and the airframe coordinate system. When the quadrotor aircraft is flying, the constant change of the three Euler angles will directly result in the change of the flight state of the quadrotor aircraft in the airframe coordinate system, but the ground coordinate system will remain unchanged all the time. Four-rotor aircraft flying attitude in the air is usually reflected by three Euler angles. which is rolling angle ϕ , pitch

angle θ and yaw angle Ψ . Euler angle is used to accurately determine the role of rigid body at a fixed point where the specific location. The selected fixed point is the origin of the airframe coordinate system, and the three attitude angles - roll angle ϕ , pitch angle θ and yaw angle Ψ are used to determine the coordinate of the axis of motion in the spatial direction. It can also indicate the rigid body around its corresponding rotation angle of the shaft. In order to describe the attitude of quadrotors aircraft more clearly, three Euler angles can be used to express the concrete conversion relationship between attitude matrix and Euler rotation, and then the concrete conversion relation between ground coordinate system and body coordinate system can be derived. Fig. 1 shows the relationship between the ground coordinate system and the airframe coordinate system.



Figure 1 the relationship between the ground coordinate system and the airframe coordinate system

The ground coordinate system to the airframe coordinate system conversion matrix as (1) shown, that is

$$R = \begin{pmatrix} \cos\psi\cos\theta & \cos\psi\sin\phi\sin\theta-\sin\psi\sin\phi & \cos\psi\sin\theta\cos\phi+\sin\psi\sin\phi \\ \sin\psi\cos\theta & \sin\psi\sin\theta\sin\phi+\cos\psi\cos\theta & \sin\psi\sin\theta\cos\phi-\sin\phi\cos\psi \\ -\sin\theta & \cos\theta\sin\phi & \cos\theta\cos\phi \end{pmatrix}$$
(1)

In order to further simplify the dynamic model of the qu adrotors control system, according to Newton's second law, th e dynamic equations of the quadrotors can be expressed as:

$$F = m \frac{dv}{dt}$$
(2)

$$M = \frac{dH}{dt}$$
(3)

In equation (2) and (3), F is the resultant force of quadrotors aircraft flying in space, M is the mass of the quadcopter itself, V is the speed of the quadrotors in space motion, M is the external moment of rotation of the quadrotors, and H is the absolute moment of momentum of the quadrotors with respect to the ground coordinate system.

The four-rotor aircraft in flight during the specific force situation is as follows: that the four rotor generated by the lift F_i (i = 1, 2, 3, 4), The body's own gravity mg and air resistance, quadrotors aircraft lift in the body coordinate system as follows: $F = \sum_{i=1}^{4} F_i$, The force components along the provise and gravity in three directions are:

the x-axis, y-axis and z-axis in three directions are:

$$\begin{cases} F_x = (\cos\psi\sin\theta\cos\phi + \sin\psi\sin\phi)\sum_{i=1}^4 F_i \\ F_y = (\cos\phi\sin\theta\sin\psi - \sin\phi\cos\psi)\sum_{i=1}^4 F_i \\ F_z = (\cos\phi\cos\theta)\sum_{i=1}^4 F_i \end{cases}$$
(4)
And because the x-axis, y-axis, z-axis acceleration are:

$$\begin{cases} x = (F_x - K_1 x)/m \\ \ddot{y} = (F_y - K_2 \dot{y})/m \\ \ddot{z} = (F_z - mg - K_3 \dot{z})/m \end{cases}$$
(5)

In reference [11], the control inputs of the quadrotors are defined as follows:

$$\begin{cases} U_{1} = \sum_{i=1}^{4} F_{i} / m \\ U_{2} = (-F_{1} - F_{2} + F_{3} + F_{4}) / I_{1} \\ U_{3} = (-F_{1} + F_{2} + F_{3} - F_{4}) / I_{2} \\ U_{4} = C (F_{1} - F_{2} + F_{3} - F_{4}) / I_{3} \end{cases}$$
(6)

Where U_1 is the control input acting on the quadrotors in the z-axis direction; U_2 and U_3 denote the control inputs of the pitch angle θ and roll angle ϕ respectively; U_4 denotes the yaw moment; C denotes the force-to-torque conversion factor.

Therefore, we can draw:

 $\left(\ddot{x} = U_1 \left(\cos\psi\sin\theta\cos\phi + \sin\psi\sin\phi\right) - K_1 \dot{x}/m\right)$

$$\begin{cases} \ddot{y} = U_1 \left(\cos \phi \sin \theta \sin \psi - \sin \phi \cos \psi \right) - K_2 \dot{y} / m \quad (7) \end{cases}$$

$$\ddot{z} = U_1 (\cos\phi\cos\theta) - g - K_3 \dot{z}/m$$

The quadrotors body in rotation, the moment balance equation can be drawn:

$$\begin{cases} I_{1}\ddot{\theta} = l\left(-F_{1} - F_{2} + F_{3} + F_{4} - K_{4}\dot{\theta}\right) \\ I_{2}\ddot{\phi} = \left(-F_{1} + F_{2} + F_{3} - F_{4} - K_{5}\dot{\phi}\right) \\ I_{3}\ddot{\psi} = C\left(F_{1} - F_{2} + F_{3} - F_{4}\right) - K_{6}\dot{\psi} \end{cases}$$
(8)

Combined with equation(6) can be drawn:

$$\begin{split} \phi &= U_2 - lK_5 \phi / I_2 \\ \ddot{\theta} &= U_3 - lK_4 \dot{\theta} / I_1 \\ \ddot{\psi} &= U_4 - K_6 \dot{\psi} / I_3 \end{split}$$
(9)

In conclusion, the nonlinear dynamics model of the quadrotor obtained is:

$$\begin{vmatrix} \ddot{x} = U_1(\cos\psi\sin\theta\cos\phi + \sin\psi\sin\phi) - K_1\dot{x}/m \\ \ddot{y} = U_1(\sin\psi\sin\theta\cos\phi - \sin\phi\cos\psi) - K_2\dot{y}/m \\ \ddot{z} = U_1(\cos\phi\cos\theta) - g - K_3\dot{z}/m \\ \ddot{\phi} = U_2 - lK_5\dot{\phi}/I_2 \\ \ddot{\theta} = U_3 - lK_4\dot{\theta}/I_1 \\ \ddot{\psi} = U_4 - K_6\dot{\psi}/I_3 \end{vmatrix}$$
(10)

In the dynamics model of quadrotor aircraft, (x, y, z) represents the position of the quadrotor aircraft; (ϕ, θ, ψ) represents the attitude of the aircraft, that is, roll angle, pitch angle and yaw angle; U_i (i = 1, 2, 3, 4) indicates the amount of control; K_i is the drag coefficient; I_i represents the moment of inertia of each axis; m is the mass of the aircraft body itself; l is the distance between the center of mass of the aircraft body and the axis of rotation of the rotor; g is the gravitational acceleration of the earth's surface.

III. THE DESIGN OF CONTROL SYSTEM

A. The Structure of Control System

Because the four-rotor aircraft dynamics model has four control inputs and six control outputs, the control system has a strong coupling, highly nonlinear. The rotation speed of the four rotors plays a decisive role in the three position coordinates of the four-rotor and the three position coordinates of the centroid in the inertial coordinate system. The change of the three position coordinates of the quadruped's center of mass in the inertial coordinate system will give rise to the change of the three attitude angles, but the change of the three attitude angles will not cause the changes of the coordinates of the three positions. According to the strong coupling between the three position coordinates of the quadrocopter's center of mass in the inertial coordinate system and the three attitude angles of the quadrotors, most of the control methods adopted by the quad-rotors are the double closed-loop coordinated control method. Double closed-loop control system is composed of inner and outer loop, the outer ring is a position control subsystem, the use of PID control method, and the inner loop is the attitude control subsystem, the use of linear automatic disturbance rejection control. Fig. 2 shows the block diagram of a double closed-loop control system composed of PID control and linear automatic disturbance rejection control system composed of PID control and linear automatic disturbance rejection control.



Figure 2 The double closed loop control system structure

B. The Design of Outer Loop Pid Controller

The position control loop of the quadrotors can be divided into two independent parts: height control and horizontal position control. Height control and horizontal position control are shown in Equations. (11), (12) and (13), respectively

 $\ddot{z} = U_1 \left(\cos \phi \cos \theta \right) - g - K_3 \dot{z} / m \tag{11}$

$$\ddot{x} = U_1 \left(\cos \psi \sin \theta \cos \phi + \sin \psi \sin \phi \right) - K_1 \dot{x} / m \quad (12)$$

 $\ddot{y} = U_1 \left(\sin \psi \sin \theta \cos \phi - \cos \psi \sin \phi \right) - K_2 \dot{y} / m \quad (13)$

Supposing that the expected value of the position coordinate (x, y, z) is (x_d, y_d, z_d) , according to PID control algorithm can be obtained:

$$\ddot{z} = K_{pz} \left(z_d - z \right) + K_{ix} \int \left(z_d - z \right) dt + K_{dx} \left(\dot{z}_d - \dot{z} \right)$$
(14)

$$\ddot{x} = K_{px} \left(x_d - x \right) + K_{ix} \int \left(x_d - x \right) dt + K_{dx} \left(\dot{x}_d - \dot{x} \right) \quad (15)$$

$$\ddot{y} = K_{py} (y_d - y) + K_{iy} \int (y_d - y) dt + K_{dy} (\dot{y}_d - \dot{y})$$
(16)

C. The Design of Inner Loop Linear Automatic Disturbance Rejection Controller

Linear automatic disturbance controller (*LADRC*) is mainly composed of linear PD controller, linear expansion state observer (*LESO*) and error compensation control law (*LNESEF*). Second-order linear automatic disturbance rejection controller schematic diagram shown in Fig. 3.



Figure 3 The schematic of second-order linear automatic disturbance rejection controller

Based on the feedback error signal, the design of a LADRC is independent of the exact math model of the control system. The input signal is V_0 , which is given as a state reference. z_1 and z_2 are the observations of the linear dilatometer, z_3 is the dilatational state variable, that is, the

observation of the total disturbance, W is the disturbance and u is the amount of control acting on the object.

According to the mathematical model of the control system of quadrotors, a LADRC is designed. Because its attitude angle satisfies the following relation:

$$\begin{cases}
\phi = U_2 - lK_5 \phi / I_1 \\
\ddot{\theta} = U_3 - lK_4 \dot{\theta} / I_2 \\
\ddot{\psi} = U_4 - K_6 \dot{\psi} / I_3
\end{cases}$$
(17)

Therefore, you can order

$$\begin{cases} \ddot{\theta} = U_3 - lK_4 \dot{\theta} / I_1 + \xi_1 \\ \ddot{\phi} = U_2 - lK_5 \dot{\phi} / I_2 + \xi_2 \\ \ddot{\psi} = U_4 - K_6 \dot{\psi} / I_3 + \xi_3 \end{cases}$$
(18)

Among them, ξ_i (i = 1, 2, 3) is the uncertainty of the system internal disturbance, the actual flight process also need to consider the external disturbance. d_i (i = 1, 2, 3), let $f_i = \xi_i + d_i$, f_i be the total perturbation of the control system, and f_i can lead, then

$$\begin{cases} \ddot{\theta} = U_{3} - lK_{4}\dot{\theta}/I_{1} + f_{1} \\ \ddot{\phi} = U_{2} - lK_{5}\dot{\phi}/I_{2} + f_{2} \\ \ddot{\psi} = U_{4} - K_{6}\dot{\psi}/I_{3} + f_{3} \end{cases}$$
(19)

Take pitch channel as an example, design a LADRC,

Set $x_{1\theta} = \theta$, $x_{2\theta} = \dot{\theta}$, $x_{3\theta} = f_1 \cdot x_{3\theta}$ is the expansion state variable of pitch angle θ , so $\ddot{\theta} = U_3 - lK_4\dot{\theta}/I_1 + f_1$ can be rewritten as

$$\begin{cases} \dot{x}_{1\theta} = x_{2\theta} \\ \dot{x}_{2\theta} = x_{3\theta} - lK_4 / I_1 \cdot x_{2\theta} + U_3 \\ \dot{x}_{3\theta} = \dot{f}_1 \end{cases}$$
(20)

Linear expansion of the pitch channel state observer design:

Let $z_{1\theta} - x_{1\theta} = e_{\theta}$, for its pitch channel linear expansion state observer design, the process is as follows:

$$\begin{cases} \dot{z}_{1\theta} = z_{2\theta} - \beta_{1\theta} \cdot e_{\theta} \\ \dot{z}_{2\theta} = z_{3\theta} - \beta_{2\theta} \cdot e_{\theta} - a_{\theta} \cdot z_{2\theta} + b_{\theta}U_{3} \\ \dot{z}_{3\theta} = -\beta_{3\theta} \cdot e_{\theta} \end{cases}$$
(21)

Where $z_{i\theta}$ (i=1,2,3) is the observed value of $x_{i\theta}$ (i=1,2,3), $\beta_{i\theta}$ (i=1,2,3) is the observer gain, and the poles of the extended state observer are all configured to $-w_{0\theta}$, where a is the observer bandwidth, and $\beta_{1\theta} = 3w_{0\theta}$, $\beta_{2\theta} = 3w_{0\theta}^2$, $\beta_{3\theta} = w_{0\theta}^3$, $a_{\theta} = lK_4/I_1 = 0.00192$, $b_{\theta} = b_0 = 1$.

The design of linear state error feedback controller for pitch channel (*NLSEF*)

$$\begin{cases} U_3 = \frac{y_{\theta} - z_{3\theta}}{b_{\theta}} \\ y_{\theta} = K_{P\theta}(\theta_d - z_{1\theta}) - K_{d\theta} \cdot z_{2\theta} \end{cases}$$
(22)

Where θ_d represents the expected value of pitch angle, $K_{P\theta}$ and $K_{d\theta}$ both represent the gain of the pitch channel controller, and let $K_{P\theta} = w_{c\theta}^2$, $K_{d\theta} = 2w_{c\theta}$, where $w_{c\theta}$ represents the bandwidth of the controller and y_{θ} represents the amount of feedback control.

By the same token, the design of the LADRC for the roll path and the yaw path can be performed.

IV. SIMULATION RESULTS

The body parameters of a quadrotor aircraft are given by reference [10], and the body parameters of a quadrotor aircraft are set as: $K_1 = K_2 = K_3 = 0.010$, $K_4 = K_5 = K_6 = 0.012$, $I_1 = I_2 = 1.25$, $I_3 = 2.5$, m = 2kg, l = 0.2m, $g = 9.8m/s^2$.

Proportional coefficient K_p , integral coefficient K_i and differential coefficient K_d values of PID controller can be obtained by trial and error method, while the design of the main parameters of the LADRC, so that the LADRC bandwidth: $w_0 = 28$, the pitch, yaw and roll the observer gain of these three channel β_1 , β_2 , β_3 respectively, the value is $\beta_{1\phi} = \beta_{1\theta} = \beta_{1\psi} = 3w_0 = 84$, $\beta_{2\phi} = \beta_{2\theta} = \beta_{2\psi} = 3w_0^2 = 2352$, $\beta_{3\phi} = \beta_{3\theta} = \beta_{3\psi} = w_0^3 = 21952$. Let

controller bandwidth $w_c = 2.8$, the parameters of PD controller are configured as $k_{d\phi} = k_{d\theta} = k_{d\psi} = 2w_c = 5.6$, $k_{p\phi} = k_{p\theta} = k_{p\psi} = w_c^2 = 7.84$, $b_{0\phi} = b_{0\theta} = b_{0\psi} = b_0 = 1$.

According to the above derivation process, a block diagram of the PID / LADRC double closed-loop coordinated control system of the quad-rotor aircraft control system can be set up, as shown in Fig. 4.



Figure 4 PID / LADRC double closed-loop coordinated control simulation module diagram

By running the control system simulation block diagram in Fig. 4, we can get the curve of the position coordinates and attitude angle of the quad-rotor, as shown in Fig.5 and Fig.6.



16

Through the simulation of quadruped rotor dual-loop coordinated control system by MATLAB/ simulink.As you can see, the response curve of the position coordinate (x, y, z)can reach the expected value in about 8 seconds and remains unchanged at all times, at the same time, the response curve of its vaw angle Ψ can reach a constant value in about 3 seconds and remains unchanged, The response curve is almost no overshoot, while the pitch angle and roll angle can be restored to the desired value of 0 degrees after a period of time, which can make the quadrotors aircraft operating conditions remain stable. It is demonstrated that the coordinated control of quadrotors with PID and linear automatic disturbance rejection control can make the position response and attitude response have good dynamic performance with fast performance and stable performance. The experimental results show the effectiveness of the dual closed-loop control which combines the PID control of quad-rotor with the linear automatic disturbance rejection control.

time(sec)

Figure 6 The response curve of attitude angle

V. CONCLUSION

In this paper, a quad-rotor aircraft of four-input and sixoutput is proposed, which is based on PID and LADRC. The inner loop is designed with a linear automatic disturbance rejection controller control the attitude of the quadrotors aircraft, and the outer loop is designed PID controller to control the position of the quadrotors aircraft. For the linear automatic disturbance rejection controller, the linear state of expansion observer is a core part of the system, which can estimate and compensate the total disturbance generated by the quadcopter in real time, according to the simulation results in this paper, we can know that the designed PID / LADRC double closed-loop cooperative controller can make sure the control system of quadrotors aircraft can track and control the position and attitude better, and the control system has strong robustness.

VI. REFERENCES:

- Erginer B, Altug E. Design and Implementation of a Hybird Fuzzy Logic Controller for a Quad-rotor VTOL Vehicle[J]. International Journal of Control, Automation and Systems(S1598-6446), 2012, 10(1): 61-70.
- [2] YU Yali, SUN Feng, WANG Yuanxi. Controller design of quad-rotor aerial robot [J]. Physics Procedia, 2012, 33: 1254-1260.
- [3] Cavalcante Sa R, De Araujo A L C, Varela A T, et al. Construction and PID Control for Stability of an Unmanned Aerial Vehicle of the Type Quadrotor[C]// Proceedings of 2013 latin American Robotics Symposium and Competition. USA: IEEE, 2013: 95-99.
- [4] LI Junder, LI Yuntang. Four rotor aircraft dynamics modeling and PID control[J]. Journal of LIANNIAN Technology University, 2012, 31(1): 114-117.
- [5] NICOL C, MACNAB C J B, RAMIREZ-SERRANO A. Robust adaptive control of a quadrotor helicopter[J]. Mechatronics, 201121(6):927-938.
- [6] Cozab C, Nicol C, Macnab C J B, et al. Adaptive fuzzy control for a quad-rotor helicopter robust to wind buffeting[J]. Joural of Intelligent and Fuzzy Systems, 2011, 22(5): 267-283.
- [7] BOUABDALLAH S, SIEGWART R. Backstepping and sliding-mode techniques applied to an indoor micro quadrotor[C]//Proceedings of the 2015 IEEE International Conference on Robotics and Automation. Barcelona, Spain: IEEE, 2005: 2247-2252.
- [8] DIKMEN I C, ARISOY A, TEMELTAS H. Attitude control of a quadrotor[C]//Proceedings of the 4th International Conference on Recent Advances in Space Technologies. Istanbul, Turkey: IEEE, 2009: 722-727.
- [9] XIA Guoyan, LIU Zhenghua. Quad-rotor unmanned helicopter attitude control based on improved ADRC[C]//Proceedings of the 2014 IEEE Chinese Guidance, Navigation and Control Conference. Yantai, China: IEEE, 2014: 916-921.
- [10] Rong Xu and Umit Ozguner. Sliding Mode Control of a Quadrotor Helicopter[C]. Proceedings of the 45th IEEE Conference on Decision and Control Manchester Grand Hyatt Hotel San Diego, CA, USA, December 13-15, 2006.