Dynamic Analysis of Beam under the Moving Mass for Damage Assessment

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***Abstract*— In this paper, the dynamic behavior of the structures subjected to moving mass is analyzed by modeling the structure as a simply supported beam under the moving mass. Fourth order Runge-Kutta numerical method is used to solve the resulting set of differential equations and a crack is modeled as a torsional spring with local flexibility connecting two segments of beam. A mathematical modeling and simulation is carried out to find the responses of healthy and damaged beam by writing a code in Matlab-R2009b.The responses of healthy and damaged beam under the moving mass are compared with increase in velocity of the moving mass. The response of the beam with moving sensor method is compared to that with fixed sensor at center. A midpoint deflection is observed to be large compared to that for moving sensor and at critical velocity, midpoint deflection is increasing continuously even when the moving mass reach to the end of beam. A very small difference can be observed between the deflection response of the healthy and damaged beam even with crack size 25% of the beam depth. Vibration acceleration of the beam is also investigated to identify the crack presence and its location. Deflection signals contain a small discontinuity at the crack location but it is not visible while this discontinuity is significant in the acceleration signals. The acceleration signals are helpful to identify the crack presence and its location more significantly the signals of beam under the travelling mass with lower velocity.**

Keywords*— *Simply supported beam; moving mass; cracked; dynamic analysis; acceleration

I. INTRODUCTION

Modeling, computation and analysis of structural response of bridge type systems is very crucial to determine the service life of the structures. A vast research is carried out to analyze the midpoint deflection of the beam type structures under the moving load, moving mass and also moving vehicle. An extensive research is also carried out to develop the new methods to solve the resulting differential equation of motion. Akin and Mofld [1] presented numerical solution to find the response of beam under moving mass for different boundary conditions. In the field of structural health monitoring, detecting a crack, its location and severity is very important to avoid the failure of structures. Ariaei *et al.* [2] presented discrete element technique and finite element method to determine the dynamic response of the un-damped Euler–Bernoulli beams with open and breathing cracks under a point moving mass with different boundary conditions. Bilello and Bergman [3, 4] presented theoretical and experimental study to find the response of a damaged Euler–Bernoulli beam subjected to the moving mass. Damage is modeled as rotational spring whose compliance is evaluated using linear elastic fracture mechanics. A small scale model of a prototype bridge structure is prepared for conducting the experiments to validate the analytical solution. Bulut and Kelesoglu [5] presented various analytical–numerical methods to determine the dynamic behavior of beams carrying a moving mass with different boundary conditions. A continuous model for vibration analysis of a beam with an open edge crack is presented. A quasi-linear displacement filed is suggested for the beam and the strain and stress fields are calculated. The equation of motion of the beam is calculated using the Hamilton principle and calculated equation of motion is solved with a modified weighted residual method. The natural frequencies and mode shapes are obtained [6, 7, 16]. Esen [8] used moving finite element approximation to investigate the dynamic response of a beam due to an accelerating moving mass. The effect of mass ratio and acceleration on dynamic response of beam is analyzed. Ichikawa *et al.* [9] carried out the vibration analysis of the continuous multi span beam subjected to a moving mass to see the effect of velocity of the moving mass and mass ratio. Lee *et al.* [10] and Lin and Chang [11] analyzed the dynamic response of Euler-Bernoulli beam with a single-sided crack subjected to a moving load. The beam is modeled as two separate beams divided by the crack. Mahmoud and Abou Zaid [12] investigated the effect of crack of size 50% on the dynamic response of the beam under the moving mass by using an iterative modal analysis approach. He found out that crack presence resulted in higher deflections and also changes the beam response patterns. Mehri *et al*. [13] presented response of beam under the moving load with different boundary conditions. Using the green dynamic function, he analyzed the results for different boundary conditions and velocity of moving load.

Ouyang [14] presented the tutorial which covers the different topics of structural dynamics problems caused by moving mass. Pala and Reis [15] studied the effects of inertial, centripetal and Coriolis forces on the dynamic response of a simply supported beam with a single crack under moving mass load for the different velocity of the moving mass. To solve the governing differential equation of motion, a convolution integration method is used. Yavari *et al.* [17] analyzed discrete element analysis of dynamic response of Timoshenko beams under moving mass. Yang *et al.* [18] presented the effect of the crack, material property, and axial compression on the deflection response of the beam with different boundary conditions. Kozar and Stimac [19] studied the deflection response of beam carrying a moving load. Average acceleration method has been employed since direct use of finite differences had shown as being practically unusable.

An extensive study of dynamic midpoint deflection of the beam under the moving load, moving mass and moving vehicle is done for analyzing the effect of crack, velocity of the moving mass, mass ratio and crack location. Also beam with different boundary conditions are also considered for these problems. In this paper, in addition to the midpoint deflection of beam (fixed sensor at center) a moving sensor approach is also used to evaluate the complex behavior of the structures under the moving mass. The aim is to evaluate the deflection and acceleration response of the beam under the moving mass for the effect of crack and velocity. The acceleration graphs provide the qualitative picture for the identification of the crack presence and its location.

II. THEORY AND FORMULATING THE SOLUTION

A. Undamaged Beam

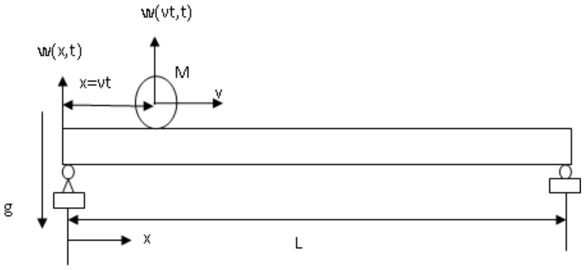


Fig. 1 Simply supported beam under the moving mass

A simply supported undamaged beam with moving mass is shown in Fig.1. It is assumed that the mass travels from left to right end in the direction x and the beam vibrates only in the *y* direction. Neglecting damping, rotary inertia, and shearing force effects, the governing equation of motion of the beam under the moving mass *M* can be written as [24]

$$EI\frac{\partial^4 y(x,t)}{\partial x^4} + \rho A\frac{\partial^2 y(x,t)}{\partial t^2} = \left[Mg - M\frac{\partial^2 y(vt,t)}{\partial t^2}\right]\delta(x - vt) \quad (1)$$

with initial conditions:

$$y(x,0) = 0 \text{ and } \frac{dy}{dt}(x,0) = 0$$

where I = Constant moment of inertia.
m =Constant mass per unit length of the beam = ρA
g = Uniform gravitational field and
M= constant moving mass
As the load is moving on the beam with a constant velocity v, then

$$\frac{dy}{dt} = \frac{\partial y}{\partial t} + v\frac{\partial y}{\partial x} \quad (2)$$

and

$$\frac{d^2 y}{dt^2} = \frac{\partial^2 y}{\partial t^2} + 2v\frac{\partial y}{\partial x}\frac{\partial y}{\partial t} + v^2\frac{\partial^2 y}{\partial x^2} \quad (3)$$

Therefore Eq. (1) can be written as

$$EI\frac{\partial^4 y(x,t)}{\partial x^4} + \rho A\frac{\partial^2 y(x,t)}{\partial t^2} + M\left[\frac{\partial^2 y}{\partial t^2} + 2v\frac{\partial y}{\partial x}\frac{\partial y}{\partial t} + v^2\frac{\partial^2 y}{\partial x^2}\right] = Mg.\delta(x - vt) \quad (4)$$

Using the mode superposition principle, Eq. (4) can be written as

$$\sum_{n=1}^{\infty}\omega_n^2\,\rho A Y_n(x) q_n(t) + \sum_{n=1}^{\infty}\rho A\frac{d^2 q_n(t)}{dt^2}Y_n(x) + M\left[\sum_{n=1}^{\infty}\frac{d^2 q_n(t)}{dt^2}Y_n(x) + \sum_{n=1}^{\infty}2v\frac{dY_n(x)}{dx}\frac{dq_n(t)}{dt}Y_n(x) + \sum_{n=1}^{\infty}v^2\frac{d^2 Y_n(x)}{dx^2}q_n(t)\right]$$

$$\delta(x - vt) = Mg\delta(x - vt) \quad (5)$$

Where, $Y_n(x)$ is the mode shape of the beam. Multiplying with $Y_m(x)$ and integrating it over the beam length (0 to L) and using the orthogonality condition, Eq. (5) becomes

$$\omega_n^2 q_n(t)\delta_{mm} + \frac{d^2 q_n(t)}{dt^2}\delta_{mm} + M\sum_{n=1}^{\infty}Y_n(vt)\frac{d^2 q_n(t)}{dt^2}Y_m(vt) + 2Mv\sum_{n=1}^{\infty}\frac{dY_n(vt)}{dx}\frac{dq_n(t)}{dt}Y_m(vt) + Mv^2\sum_{n=1}^{\infty}Y_m(vt)q_n(t) = MgY_m(vt) \quad (6)$$

Eq. (6) is solved numerically using fourth order Runge-Kutta method considering the series expansion limited to first three modes only.

B. Damaged Beam

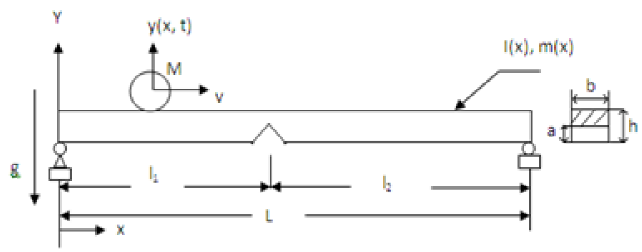


Fig. 2 Simply supported cracked beam under the moving mass

Considering a simply supported beam of length L with open crack located at $x=l_1$ in Fig. 2. A mass M is moving on the beam with constant velocity v. The beam is divided into two segments by the crack.

According to Euler-Bernoulli beam theory, the equation of motion for each part for free vibration can be written as

$$EI\frac{\partial^4 y_i(x,t)}{\partial x^4} + \rho A\frac{\partial^2 y_i(x,t)}{\partial t^2} = 0$$

$$x_{i-1} < x < x_i \quad i = 1,2 \tag{7}$$

Where, $y_1$and y_2 are vertical displacements for first and second part of beam. c_f is local crack flexibility, and it is a function of the crack depth (α).

$$c_f = \frac{(1-\vartheta^2)}{EI}\, 6\pi h\phi(\alpha) \tag{8}$$

Where, (α=a/h) is the crack-depth ratio and for a single sided open crack [7]

$$\phi(\alpha) = 0.655563\alpha^2[0.9566 - 1.5944\alpha + 7.008\alpha^2$$
$$-15.21\alpha^3 + 30.9534\alpha^4 - 50.38657\alpha^5$$
$$+71.8488\alpha^6 - 62.1624\alpha^7 + 29.89486\alpha^{10}] \tag{9}$$

Now applying the boundary conditions for the simply supported beam

$$1)\ w|_{x=0} = 0, \qquad 2)\ w''|_{x=0} = 0,$$
$$3)\ w|_{x=L} = 0, \qquad 4)\ w''|_{x=L} = 0 \tag{10}$$

And the continuous conditions to enforce the continuities of the displacement, bending moment, and shear force across the crack are given by

$$1)\ w_1(l_1,t) - w_2(l_1,t) = 0$$
$$2)\ w_1''(l_1,t) - w_2''(l_1,t) = 0$$
$$3)\ w_1'''(l_1,t) - w_2'''(l_1,t) = 0$$
$$4)\ w_2'(l_1,t) - w_1'(l_1,t) = w_2''(l_1,t)\left[\frac{(EIc_f)}{l_1}\right]l_1 \tag{11}$$

Using the method of separation of variables as

$$y(x,t) = Y(x)\eta(t)$$

in Eq. (7), we get

$$Y_i^{IV}(x) - \beta^4 Y_i(x) = 0, \quad l_{i-1} < x < l_i \qquad i = 1,2 \tag{12}$$

where
$$\beta^4 = \frac{m\omega_n^2}{EI} \tag{13}$$

The solution of Eq. (12) for each part of beam can be written as

$$Y_1(x) = A_1\sin(\beta x)\, B_1\cos(\beta x) +$$
$$C_1\sinh(\beta x) + D_1\cosh(\beta x) \qquad 0 < x < l_1 \tag{14}$$
$$Y_2(x) = A_2\, sin\beta\,(x-l_1) + B_2\cos\beta(x-l_1) +$$
$$C_2\sinh\beta(x-l_1) + D_2\cosh\beta(x-l_1) \qquad l_1 < x < L \tag{15}$$

Using the boundary conditions in Eq. (14)-(15), leads to

$$B_1 = D_1 = 0 \tag{16}$$

And

$$A_2\sin\beta(L-l_1) + B_2 cos\,\beta(L-l_1) +$$
$$C_2 sinh\,\beta(L-l_1) + D_2 cosh\,\beta(L-l_1) = 0 \tag{17}$$

$$-A_2\sin\beta(L-l_1) - B_2 cos\,\beta(L-l_1) +$$
$$C_2 sinh\,\beta(L-l_1) + D_2 cosh\,\beta(L-l_1) = 0 \tag{18}$$

The Eq. (17)-(18) can be written in the matrix form as given in

$$\begin{bmatrix}0\\0\end{bmatrix} = \begin{bmatrix}\sin\beta(L-l_1) & \cos\beta(L-l_1) & \sinh\beta(L-l_1) & \cosh\beta(L-l_1)\\ -\sin\beta(L-l_1) & -\cos\beta(L-l_1) & \sinh\beta(L-l_1) & \cosh\beta(L-l_1)\end{bmatrix}\begin{bmatrix}A_2\\B_2\\C_2\\D_2\end{bmatrix} \tag{19}$$

The constants of the second part A_2, B_2, C_2, and D_2 are related to the constants

$$A_1\,a1_s + B_1\,a1_c + C_1\,a1_{sh} + D_1\,a1_{ch} = A_2\,a2_s + B_2\,a2_c + C_2\,a2_{sh} + D_2\,a2_{ch}$$

$$-A_1 a1_s - B_1 a1_c + C_1 a1_{sh} + D_1 a1_{ch} = -A_2\,a2_s - B_2\,a2_c + C_2\,a2_{sh} + D_2 a2_{ch}$$

$$-A_1 a1_c + B_1 a1_s + C_1 a1_{ch} + D_1 a1_{sh} = -A_2\,a2_c + B_2\,a2_s + C_2\,a2_{ch} + D_2 a2_{sh}$$

$$A_2\,a2_c - B_2\,a2_s + C_2\,a2_{ch} + D_2 a2_{sh} - (A_1 a1_c - B_1 a1_s + C_1 a1_{ch} + D_1 a1_{sh})$$
$$= \left[\frac{(EIc)}{l_1}\right] l_1(-A_2\,a2_s - B_2\,a2_c + C_2 a2_{sh} + D_2\,a2_{ch}) \tag{20}$$

Whereas,

$$a1s = sin(\beta l_1),\quad a1_c = cos(\beta l_1),\quad a1_{sh} = sinh(\beta l_1),\quad a1_{ch} = cosh(\beta l_1),\quad a2_s = sin(\beta(l_2)),$$

$$a2_c = cos(\beta(l_2)),\quad a2_{sh} = sinh(\beta(l_2)),\quad a2_{ch} = cosh(\beta(l_2)) \tag{21}$$

The Eq. (20) can be written in matrix form as

$$\begin{bmatrix}A_2\\B_2\\C_2\\D_2\end{bmatrix} = \begin{bmatrix}a1_c - r*a1_s & -a1_s - r*a1_c & r*a1_s & r*a1_c\\ a1_s & a1_c & 0 & 0\\ -r*a1_s & -r*a1_c & a1_c + r*a1_s & a1_s + r*a1_c\\ 0 & 0 & a1_s & a1_c\end{bmatrix}\begin{bmatrix}A_1\\B_1\\C_1\\D_1\end{bmatrix} \tag{22}$$

$$\left(r = \frac{EIc\beta}{2}\right) \tag{23}$$

Inserting the Eq. (22) into (19)

$$\begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} v3 - r*v11 & v4 - r*v12 & r*v13 + v7 + v5 & r*v14 + v9 + v8 \\ -v3 - r*v11 & -v4 - r*v12 & v6 - r*v13 + v5 & v10 - r*v14 + v8 \end{bmatrix} \begin{bmatrix} A_1 \\ B_1 \\ C_1 \\ D_1 \end{bmatrix} \qquad (24)$$

Whereas,

$v1 = (a1_c - r*a1_s),, v2 = (a1_{ch} + r*a1_{sh}), v3 = a2_c * a1_s + a2_s * v1,$
$v4 = a2_c * a1_c + a2_s * (-r*a1_c - a1_s), v5 = a2_{sh} * v2, v6 = a2_{ch} * a1_{sh},$
$v7 = a2_{sh} * a1_{sh}, v8 = a2_{sh} * (r*a1_{ch} + a1_{sh}), v9 = a1_{ch} * a2_{sh}, v10 = a2_{ch} * a1_{ch}$
$v11 = a1_s * a2_{sh}, v12 = a1_c * a2_{sh}, v13 = a2_s * a1_{sh}, v14 = a1_{ch} * a2_s$

For the non-trivial solution of the beam

$$\begin{vmatrix} v3 - r*v11 & r*v13 + v7 + v5 \\ -v3 - r*v11 & v6 - r*v13 + v5 \end{vmatrix} = 0 \qquad (25)$$

The characteristic equation for simply supported beam can be obtained from the Eq. (25). From the characteristic equation, we calculated the non-dimensional frequency. Using this non-dimensional frequency, the constants for first and second segment of the beam can be obtained.

Now forced vibration solution of two parts of the beam can be written as

$$\omega_n^2 q_{ni}(t) + \ddot{q}_{ni}(t) = \frac{2}{ml} \left[MgY_{ni}(vt) - M\left(\ddot{q}_{mi}(t) \sum_{m=1}^{N} Y_{ni}(vt) Y_{mi}(vt) + 2v\dot{q}_{mi}(t) \sum_{m=1}^{N} Y'_{mi}(vt) Y_{ni}(vt) + v^2 q_{ni}(t) \sum_{m=1}^{N} Y''_{mi}(vt) Y_{ni}(vt) \right) \right] \qquad (26)$$

Where, n and m =mode number, i=1and 2 (1= first part of beam and 2=second part of beam).

The Eq. (26) is a system of simultaneous differential equations with time-dependent coefficients. The closed form solution is not possible. Therefore, approximate analytical methods or more often numerical methods are used. The deflection of first part of beam can be found out by inserting Eq. (14) into the Eq. (26) for length ($0 < x < l_1$). Similarly, deflection of second part of beam is obtained by inserting Eq. (15) into the Eq. (26) for length ($l_1 < x < L$) and the Equation is solved by fourth order Runge-Kutta numerical method.

III. Results and Discussions

The following example is considered for the calculation: $L=20m$, $m=312Kg/m$, $E= 2.06\times10^{11}$ N/m^2, and $\rho=7,800$ kg/m^3. The results are obtained by expanding the equation of motion for first three modes of vibration and the moving mass is 20% of the total beam mass. The crack of size 25% is taken at the middle of the beam. The critical velocity (Vcr=46.3 m/s) is calculated from the natural frequency of the damaged beam. The results are plotted to analyze the deflection and acceleration response of cracked beam under the moving mass with varying velocities. The effect of the crack on the dynamic response of the beam under the moving mass with varying speed for fixed accelerometer at midpoint and moving accelerometer has been studied. The deflection and acceleration responses of healthy beam and damaged beam under the moving mass are compared. The results obtained are in good agreement with the results presented in literature [15]. The first three accelerating modes of healthy and damaged beam are shown in Fig. (3). The eigenvalues of the healthy beam are differentiated with very small value from that of the damaged beam.

A. *The Effect of Crack with Change in Velocity*

It can be observed from the results shown in Fig. (4-7) that the maximum deflection occurs at different positions depending on the velocity of the moving mass and the maximum deflection position is shifting towards right with the increase in velocity. For the lower velocities, the maximum deflection position occurs near the midpoint of beam in the region of 40% to 60% length of beam, while for higher velocities the maximum deflection occurs when the mass crosses the centre of beam.

The profile of curve depends on the velocity; it changes to very small extent with the presence of crack. Obviously, the presence of crack increases the maximum deflection but this increment is very small compare to the deflection of healthy beam for this crack size. It is observed that at the critical velocity, the deflection response of healthy and damaged beam are nearly overlapped. So, we can say that for higher velocities greater than critical, the crack present at the middle of beam does not have major effect and the effect will be more for the velocities lower than critical. The mode shape and the dynamic response of beam contain a very small discontinuity at crack location but it is not visible.

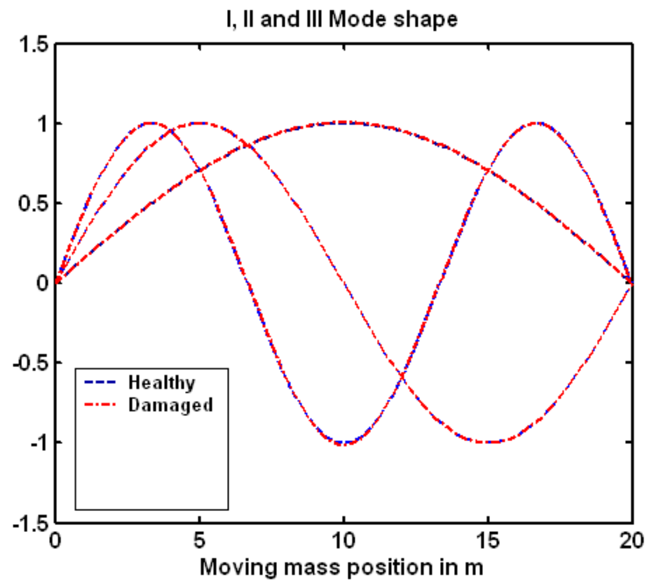


Fig. 3 First three mode shape of healthy and damaged beam

B. Comparison between the deflection of beam for moving sensor and fixed sensor

When compared between the deflection of beam under the moving mass graphs for moving accelerometer and fixed accelerometer, it is observed that the midpoint deflection is higher compare to the deflection found by moving sensor method for all positions of moving mass. Midpoint deflection of beam with increase in velocity is increasing until the critical velocity is reached while for moving accelerometer, the deflection with increase in velocity is increasing until the higher velocity (V/Vcr=0.432) is reached.

Also it is observed that the midpoint deflection curve is shifting the phase near the end of beam. This behavior of curve continues until the higher velocity is reached. At critical velocity, the midpoint deflection of the beam increases continuously even the moving mass reach to the end of beam. While the deflection curves with moving sensor approach are not shifting the phase at any velocity. The deflection curve at the critical velocity follows the reducing pattern after it attained the maximum.

The midpoint deflections are attained maximum earlier than the deflection with moving sensor at lower velocities. For higher velocities, the deflection with moving sensor is reached highest earlier and followed the decaying pattern while the midpoint deflection has taken longer time to reach highest.

The rate of increment of the midpoint deflection of beam is more than that for the moving sensor at respective positions under the moving mass. For higher velocities, the maximum midpoint deflection of beam has increased to almost double the value of maximum deflection with moving accelerometer.

C. Comparing acceleration response for undamaged and damaged

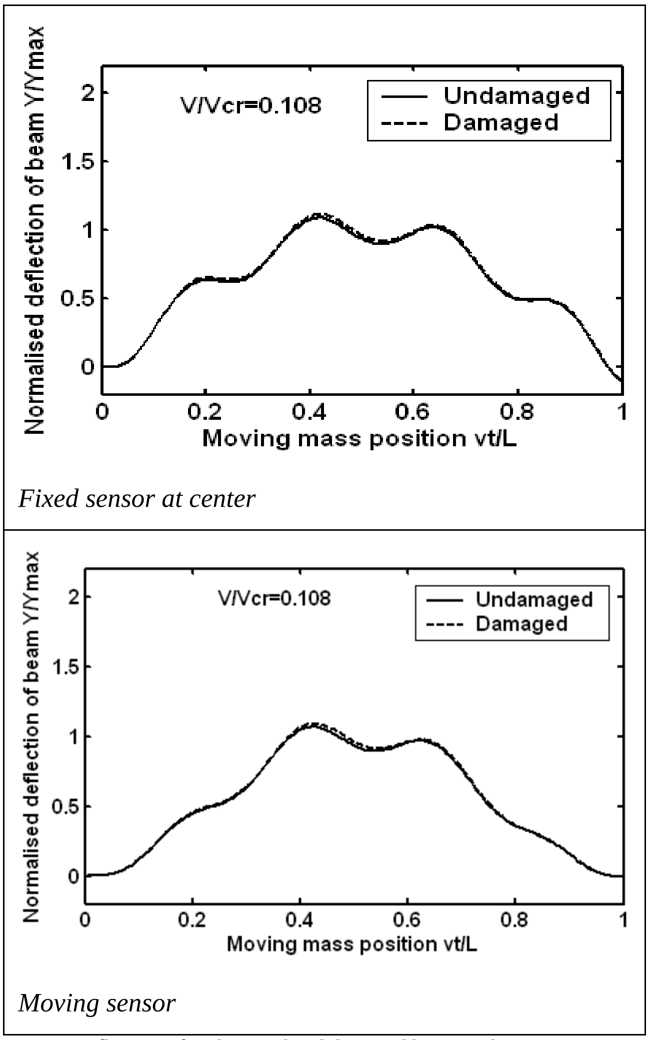
Acceleration graphs are studied for both undamaged and damaged beam under the moving mass with varying velocities. The graphs are also compared for moving sensor and fixed sensor at middle. The graphs are plotted for the same problem and analyzed to identify the crack presence and its location. It could be observed that discontinuity appears at the crack location whereas it is smooth curve for undamaged at same location. The graphs plotted with fixed sensor at center of the beam under the moving mass are shown in Figs. (8-11).

At lower velocity ratio (V/V_{cr}=0.108), discontinuity contained in signal is easily identifiable compare to other velocities. At all velocities, the discontinuity contained in signal is properly visible except for the velocity (V/V_{cr}=0.216).

At lower velocity ratio (V/V_{cr}=0.108), as moving mass reaches to the crack location midpoint acceleration of beam suddenly gets increased and again follows the decaying pattern. While for the velocity ratio (V/Vcr=0.216), this sudden increase in signal is not properly visible and this may be due to the fact that the frequency of the signal is high. For velocity ratios (V/V_{cr}=0.432) and (V/Vcr=1), the midpoint acceleration of beam increases when moving mass crosses the crack but this increment is small.

As such there is no change in acceleration values with the crack presence for lower velocity. At higher velocity, the acceleration is observed to be increased after the moving mass crossed the crack location. While for critical velocity, midpoint acceleration of cracked beam gets reduced when moving mass crosses the crack.

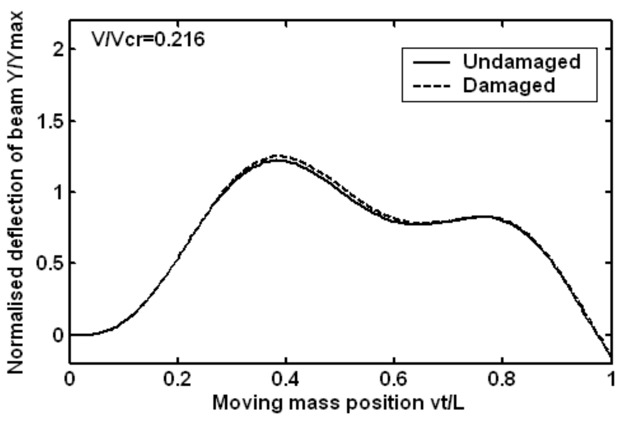
The acceleration signals obtained with the moving sensor approach are given in the Fig. (12-15).A discontinuity observed in the acceleration signals is quite high compare to the fixed sensor results. For velocity ratio (V/Vcr=0.216), again the discontinuity is very small, while it is relatively visible for higher velocities. The same changes in acceleration values for particular velocity are observed similar to the fixed sensor approach.



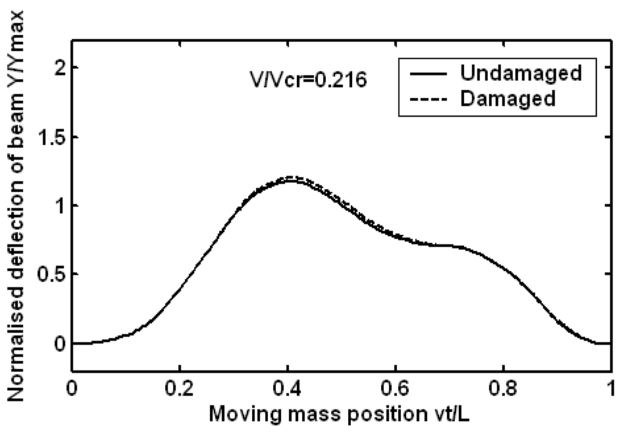
Fixed sensor at center

Moving sensor

Fig. 4 Deflection of undamaged and damaged beam under moving mass (α=0.25, l_1=10m, V/Vcr=0.108)

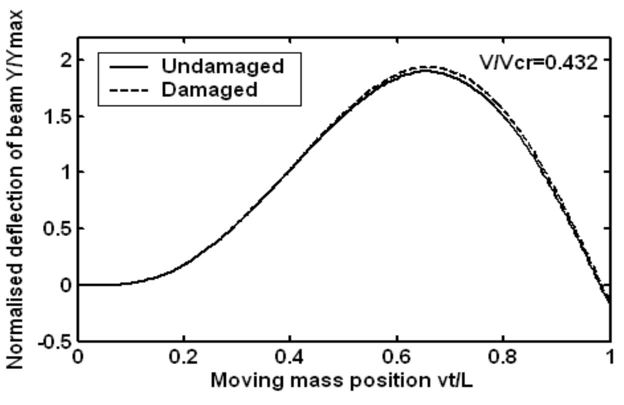


Fixed sensor at center

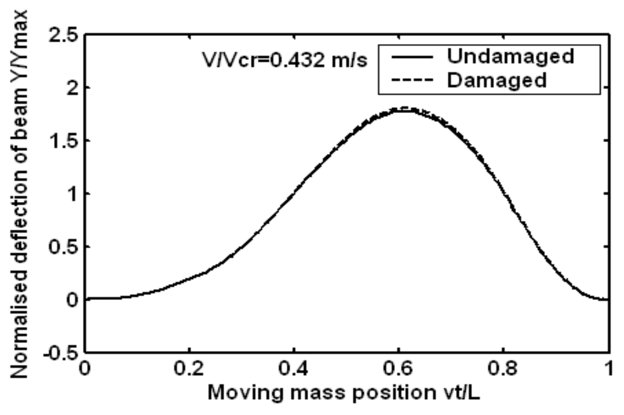


Moving sensor

Fig. 5 Deflection of undamaged and damaged beam under moving mass (α=0.25, l_1=10m, V/Vcr=0.216)

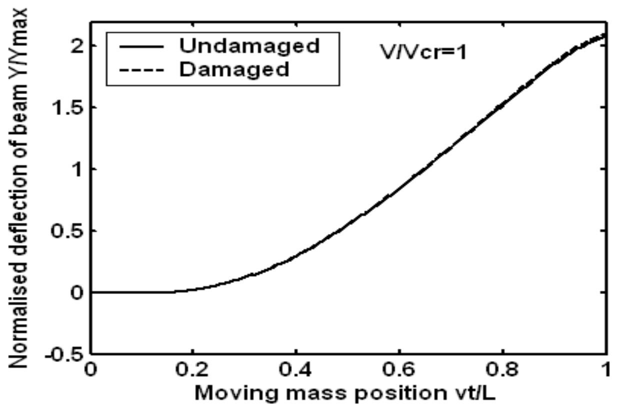


Fixed sensor at center

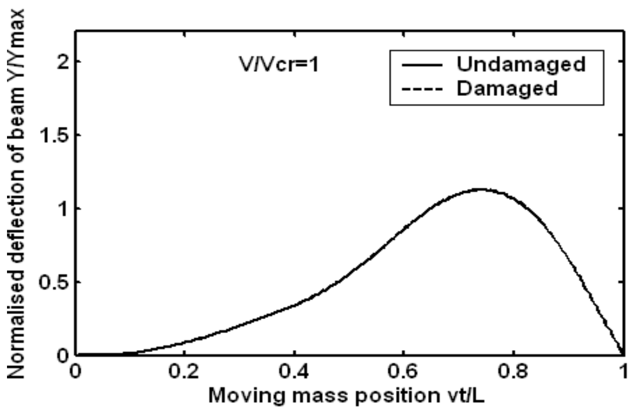


Moving sensor

Fig. 6 Deflection of undamaged and damaged beam under moving mass (α=0.25, l_1=10m, V/Vcr=0.432)

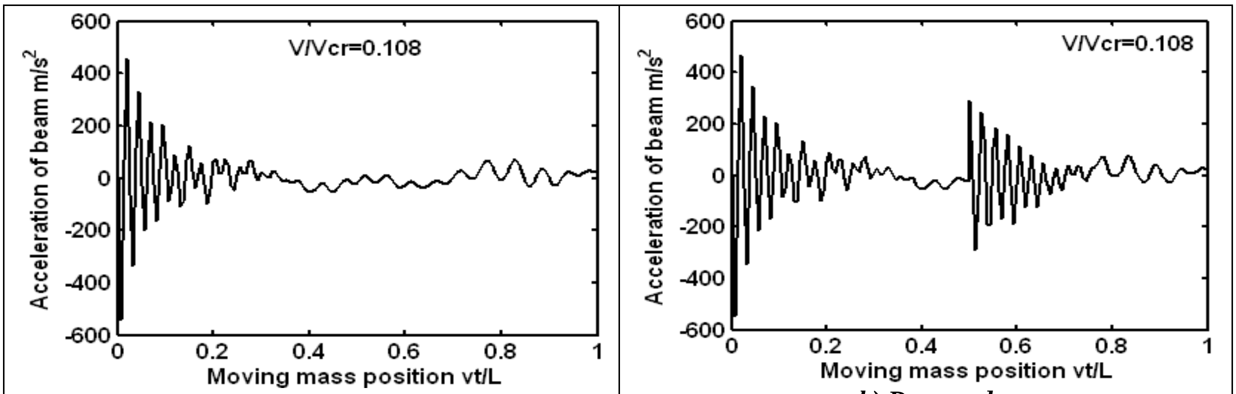


Fixed sensor at center



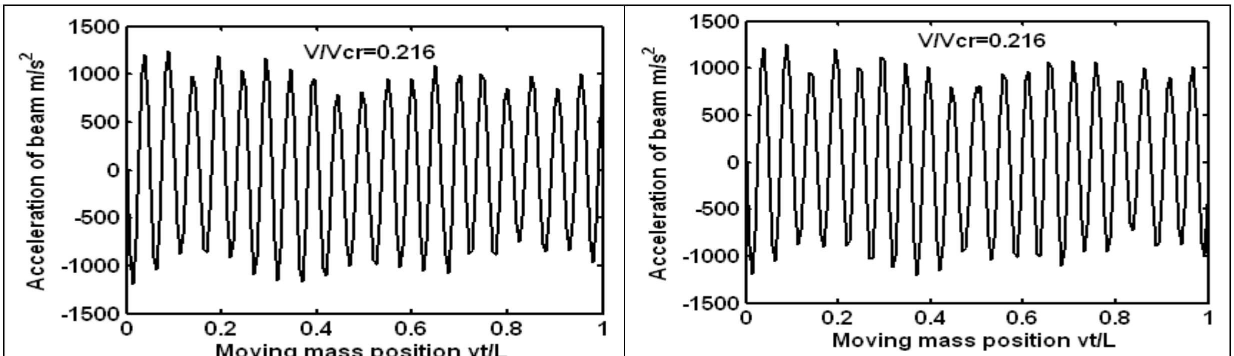
Moving sensor

Fig. 7 Deflection of undamaged and damaged beam under moving mass (α=0.25, l_1=10m, V/Vcr=1)



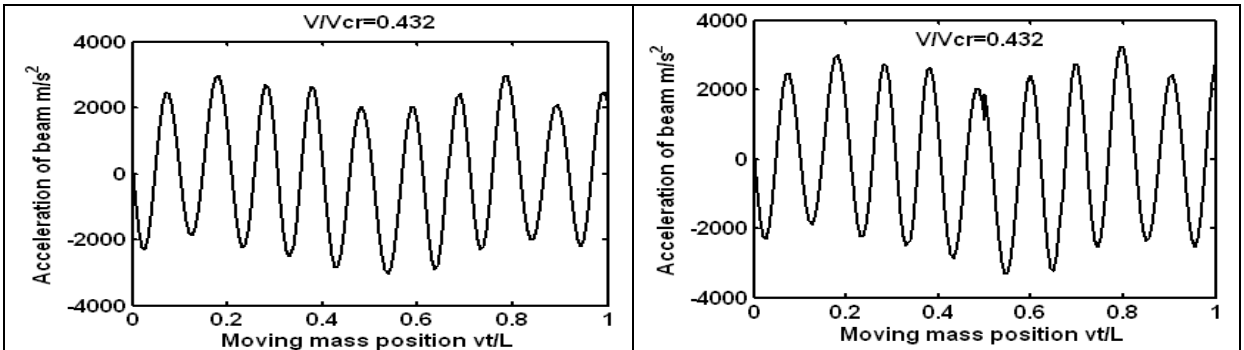
a) Undamaged *b) Damaged*

Fig. 8 Acceleration of undamaged and damaged beam with fixed sensor (α=0.25, l_1=10m, V/Vcr=0.108)



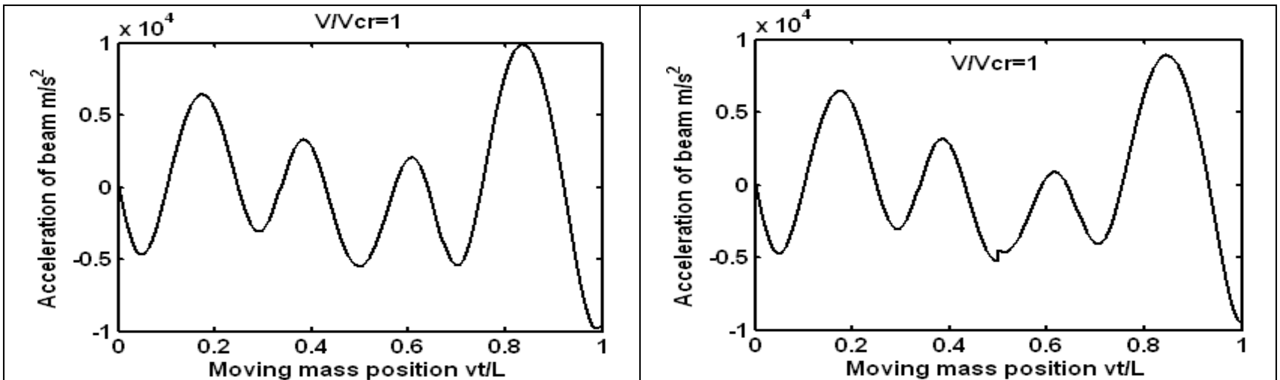
c) Undamaged *d) Damaged*

Fig. 9 Acceleration of undamaged and damaged beam with fixed sensor (α=0.25, l_1=10m, V/Vcr=0.216)



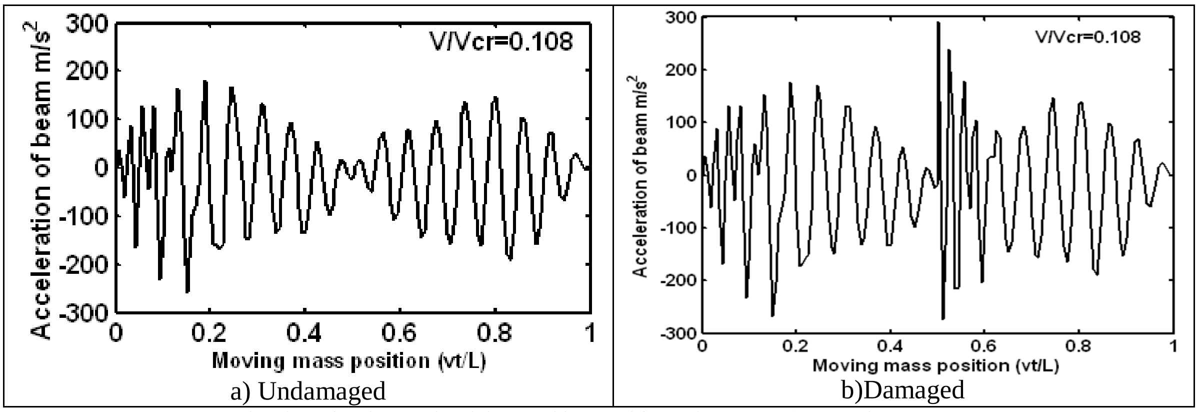
e) Undamaged *f) Damaged*

Fig. 10 Acceleration of undamaged and damaged beam with fixed sensor (α=0.25, l_1=10m, V/Vcr=0.432)



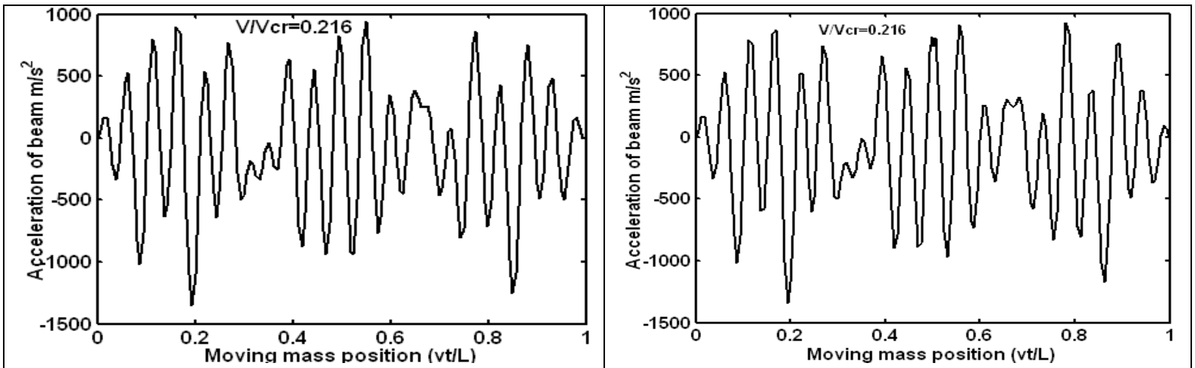
g)Undamaged *h)Damaged*

Fig. 11 Acceleration of undamaged and damaged beam with fixed sensor (α=0.25, l_1=10m, V/Vcr=1)



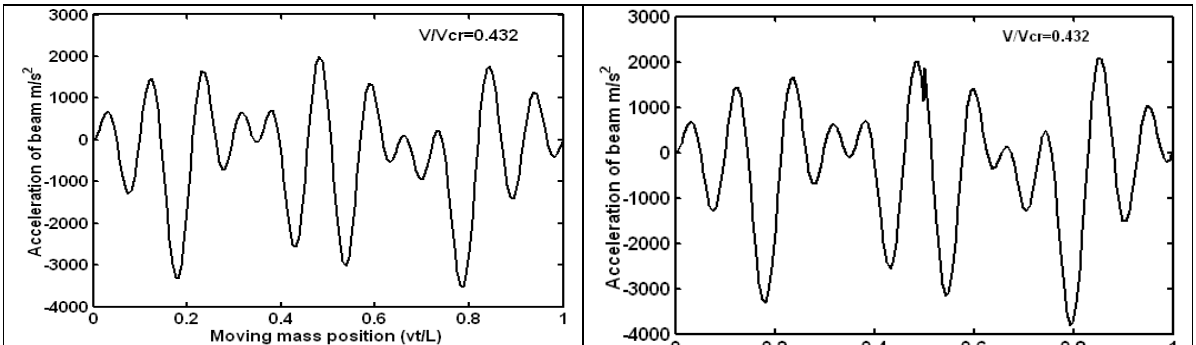
a) Undamaged b)Damaged

Fig. 12 Acceleration of undamaged and damaged beam with moving sensor (α=0.25, l_1=10m, V/Vcr=0.108)



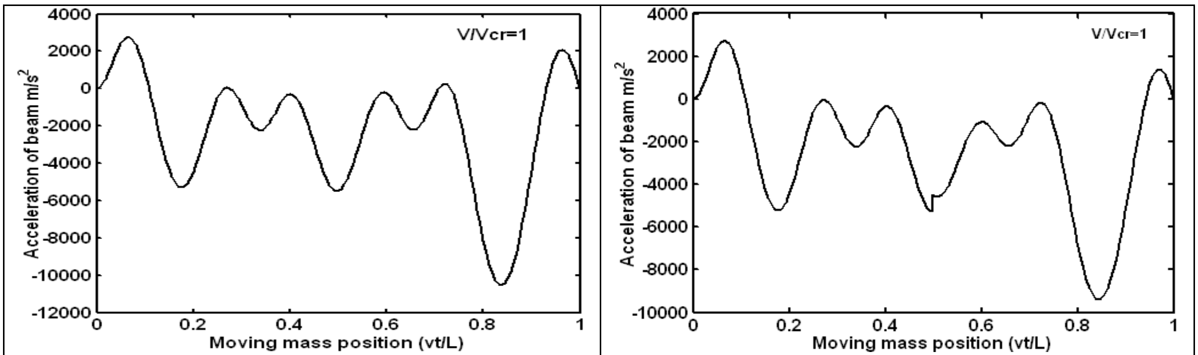
c) Undamaged d)Damaged

Fig. 13 Acceleration of undamaged and damaged beam with moving sensor (α=0.25, l_1=10m, V/Vcr=0.216)



e) Undamaged f) Damaged

Fig. 14 Acceleration of undamaged and damaged beam with moving sensor (α=0.25, l_1=10m, V/Vcr=0.432)



g)Undamaged h)Damaged

Fig. 15 Acceleration of undamaged and damaged beam with moving sensor (α=0.25, l_1=10m, V/Vcr=1)

IV. CONCLUSION

Dynamic deflection of the beam under the moving mass strongly gets affected with varying velocity of the moving mass. The dynamic response of beam under the moving mass is very complex, so attempt is made to understand the deflection and acceleration of beam with moving sensor and fixed sensor approach. With the fixed sensor approach, a large deflection is obtained at the critical velocity and with moving sensor approach the deflection is higher at higher velocity. The midpoint deflection is quite high compare to the deflection with moving sensor at all velocities. With the presence of crack, deflection of beam is evidently increasing but if the crack will be small, this increment of the deflection compare to the healthy beam deflection may not be visible. Acceleration graph at lower velocity (V/Vcr=0.108) is very helpful to identify the crack presence and its location. The change in acceleration values of the beam with the presence of crack at lower velocity is too small but for higher velocities, it is quite higher. The discontinuity contained in signals with moving sensor is evidently higher compare to that in signals with fixed sensor.

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