Edge-Odd Gracefulness of Middle Graphs and Total Graphs of Paths and Cycles

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Abstract

Graph theory is rapidly moving into the mainstream of mathematics mainly because of its applications in diverse fields which include biochemistry (genomics), electrical engineering (communication network and coding theory), computer science (algorithms and computations) and operations research (scheduling). The recent applications of graph theory are DNA sequencing (SNP assembly problem), computer network (worm propagation) and assignment of frequencies in GSM mobile networks [6]. *For a simple, finite, undirected and connected graph* $G(V,E)$, *an Edge Odd Graceful labeling* (*EOGL*) *is an injective function* $f: E(G) \rightarrow \{1,3,5,\ldots,2q-1\}$ *such that the induced function* $f^{+}: V(G) \rightarrow \{0,1,2,\ldots,2k-1\}$ *defined by* $f^{+}(x) = \sum_{xy \in E(G)} f(xy) \pmod{2k}$ *is injective where* $k=max\{p,q\}$. *A graph which admits an Edge Odd Graceful labeling is called an Edge Odd Graceful Graph* (*EOGG*) [2]. *In this paper, the Edge Odd Gracefulness of Middle Graphs and Total graphs of Paths and Cycles are obtained.*

Keywords: Graceful graph, Edge odd graceful labelling, Edge odd graceful graph, Middle graph, Total graph.
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1. Introduction

In Graph theory, labelled graphs are the most useful mathematical models in addressing and identification system on a communication network to a larger network obtained by introducing new user terminals and new communication links between the new terminals of the original network. Also labelled graphs serve as useful models for coding theory, X-Ray, radars, astronomy, circuit design, crystallography, network addressing, database management, secret sharing schemes, etc.,[7]. A.Solairaju and K.Chitra [2] introduced and obtained edge-odd graceful labelling of some graphs related to paths. The results on edge-odd graceful labelling are given in Gallian survey [5].

All the graphs considered in this paper are simple, undirected, finite and connected. The terms not defined here may be found in [4].

Definition 1.1: Graceful Graph: [1][5]

A (p,q) connected graph G is said to be a graceful graph if there exists an injective function f from the vertex set of G to the set $\{0, 1, 2, \ldots, q\}$ such that when each edge uv is assigned the label $|f(u)-f(v)|$ and the resulting edge labels are distinct.

Definition 1.2. Edge odd graceful graph: [2][3]

A (p,q) connected graph G is said to be an edge-odd graceful graph if there exists an injective function $f : E(G) \rightarrow \{1,3,5,\ldots,2q-1\}$ such that the induced function $f^{+} : V(G) \rightarrow \{0,1,2,3,\ldots,2k-1\}$, defined by $f^{+}(x) \equiv \sum_{xy \in E(G)} f(xy) \pmod{2k}$ where $k = \max\{p,q\}$ is also injective.

Definition 1.3. Middle graph: [9]

Let G be a graph with vertex set $V(G)$ and edge set $E(G)$. The middle graph of G, denoted by $M(G)$ is defined as follows. The vertex set of $M(G)$ is $V(G)$ U $E(G)$. Two vertices x, y in the vertex set of $M(G)$ are adjacent in $M(G)$ in case one of the following holds: (i) x, y are in $E(G)$ and x ,y are adjacent in G. (ii) x is in $V(G)$, y is in $E(G)$ and x, y are incident in G.

Definition 1.4. Total Graph: [9]

Let G be a graph with vertex set $V(G)$ and edge set $E(G)$. The total graph of G, denoted by $T(G)$ is defined as follows. The vertex set of $T(G)$ is $V(G)$ U $E(G)$. Two vertices x, y in the vertex set of $T(G)$ are adjacent in $T(G)$ in case one of the following holds: (i) x ,y are in $V(G)$ and x is adjacent to y in G. *(ii)* x, y are in $E(G)$ and x, y are adjacent in G. (iii) x is in $V(G)$, y is in $E(G)$ and x, y are incident in G.

2. Results on Middle Graphs of Paths and Cycles:

Theorem 2.1.The Middle Graph of path P_n for n = 3, 4, 5, 6, 7, 8 and $n \equiv 2 \pmod 4$ is edge odd graceful.

Proof: From the diagram, the middle graph of the path P_3 is EOG.

Under (mod 10)

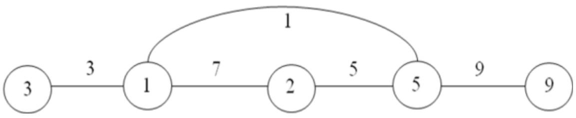

Figure 1. Middle graph of P_3

The total number of edges in $M(P_n)$ is $(3n - 4)$.
For $n > 3$ and $n \equiv 2 \pmod 4$, define a function
$f : E(M(P_n)): \rightarrow \{1,3,...,6n-9\}$ such that the edges are labeled as
$f(v_iu_i) \equiv (6i-5) \bmod (6n-8)$ for $1 \le i \le n-1$,
$f(u_iv_{i+1}) \equiv (6i-1) \bmod (6n-8)$ for $1 \le i \le n-2$,
$f(u_{n-1}v_n) \equiv (6n-9) \bmod (6n-8)$ and
$f(u_iu_{i+1}) \equiv (6i-3) \bmod (6n-8)$ for $1 \le i \le n-2$.

Define $f^+ : V(M(P_n)) \rightarrow \{0,1,2,3,...6n-9\}$ such that the vertices are labeled as
$f^+(v_1) = 1$, $f^+(v_n) \equiv (6n-9) \bmod (6n-8)$,
$f^+(v_i) \equiv 6i \bmod (6n-8)$ for $2 \le i \le n-1$,
$f^+(u_i) \equiv (24i-18) \bmod (6n-8)$ for $2 \le i \le n-2$,
$f^+(u_1) = 9$ and $f^+(u_{n-1}) \equiv (6n-19)$ for $n \ge 4$
Hence the induced map f^+ provides the distinct labels for vertices. Also edge labels are distinct.
Thus $M(P_n)$ for n = 3,4,5,6,7,8 and $n \equiv 2 \pmod 4$ is Edge Odd Graceful.

Theorem2.2.For all $n \ge 3$, the middle graph of cycle C_n for odd n is Edge Odd Graceful.

Proof: From the diagram, the middle graph of C_3 is EOG.

Under (mod 18)

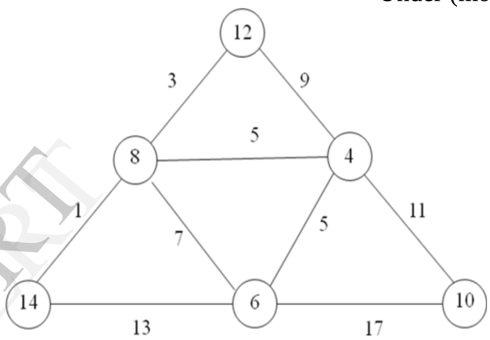

Figure 2. Middle graph of C_3

The Total number of edges in $M(C_n) = 3n$.

For all odd $n > 3$, define a function
$f : E(M(C_n)): \rightarrow \{1,3,...,6n-1\}$ such that the edges are labeled as
$f(u_nv_1) = 1$,
$f(u_iv_{i+1}) \equiv (6i+1) \bmod 6n$ for $1 \le i \le (n-1)$,
$f(v_iu_i) \equiv (6i-3) \bmod 6n$ for $1 \le i \le n$,
$f(u_iu_{i+1}) \equiv (6i+5) \bmod 6n$ for $1 \le i \le (n-1)$ and
$f(u_nu_1) = 5$.

Define $f^+ : V(M(C_n)) \rightarrow \{0,1,2,3,...6n-1\}$ such that the vertices are labeled as
$f^+(v_i) \equiv (12i-8) \bmod 6n$ for $1 \le i \le n$,
$f^+(u_i) \equiv (24i+2) \bmod 6n$ for $1 \le i \le n$.
Hence the induced map f^+ provides the distinct labels for vertices. Also edge labels are distinct.
Thus for all odd n, $M(C_n)$ is Edge Odd graceful.

3. Results on Total graphs of Paths and Cycles:

Theorem 3.1.For all $n \geq 3$, the total graph of a path P_n is Edge Odd Graceful.

Proof:
The Total number of edges in $T(P_n) = 4n\text{-}5$.
For $n \geq 3$, define a function
$f : E(T(P_n)): \rightarrow \{1,3,...,8n\text{-}11\}$ such that the edges are labeled as
$f(v_i\ u_i) \equiv (8i\text{-}7) \bmod (8n\text{-}10)$ for $1 \leq i \leq (n\text{-}1)$,
$f(u_i v_{i+1}) \equiv (8i\text{-}3) \bmod (8n\text{-}10)$ for $1 \leq i \leq (n\text{-}1)$,
$f(v_i v_{i+1}) \equiv (8i\text{-}5) \bmod (8n\text{-}10)$ for $1 \leq i \leq (n\text{-}1)$ and
$f(u_i u_{i+1}) \equiv (8i\text{-}1) \bmod (8n\text{-}10)$ for $1 \leq i \leq (n\text{-}2)$

Define $f^+ : V(T(P_n)) \rightarrow \{0,1,2,3,...8n\text{-}11\}$ such that the vertices are labeled as
$f^+(v_1)=4$, $f^+(v_n) \equiv (8n\text{-}14) \bmod (8n\text{-}10)$,
$f^+(v_i) \equiv (32i\text{-}36) \bmod (8n\text{-}10)$ for $2 \leq i \leq (n\text{-}1)$,
$f^+(u_i) \equiv (32i\text{-}20) \bmod (8n\text{-}10)$ for $2 \leq i \leq (n\text{-}2)$,
$f^+(u_{n-1}) \equiv (8n\text{-}23) \bmod (8n\text{-}10)$ for $n \geq 3$ and
$f^+(u_1)=13$.
Hence the induced map f^+ provides the distinct labels for vertices. Also edge labels are distinct.
Thus for all $n \geq 3$, the total graph of P_n, $T(P_n)$ is Edge Odd Graceful.

Example 3.2.
The total graph of a path P_4 is EOG.

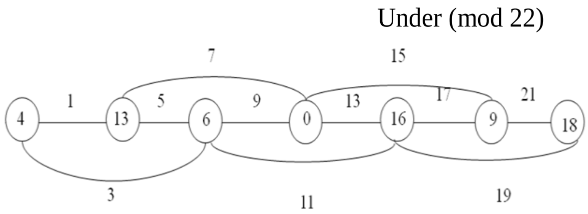

Figure 3. Total graph of P_4

Theorem 3.3.For a given odd integer $n \geq 3$, the total graph of cycle C_n is Edge Odd Graceful.

Proof:
The Total number of edges in $T(C_n)$ is $4n$.
For $n \geq 3$, define a function $f : E(T(C_n)): \rightarrow \{1,3,...,8n\text{-}1\}$ such that the edges are labeled as
$f(u_n v_1) = 1$, $f(v_n v_1) \equiv 8n\text{-}1$, $f(u_n u_1) = 5$,
$f(v_i\ v_{i+1}) \equiv (8i\text{-}1) \bmod 8n$ for $1 \leq i \leq (n\text{-}1)$,
$f(u_i v_{i+1}) \equiv (8i+1) \bmod 8n$ for $1 \leq i \leq (n\text{-}1)$,
$f(v_i\ u_i) \equiv (8i\text{-}5) \bmod 8n$ for $1 \leq i \leq n$ and
$f(u_i u_{i+1}) \equiv (8i+5) \bmod 8n$ for $1 \leq i \leq (n\text{-}1)$

Define $f^+ : V(T(C_n)) \rightarrow \{0,1,2,3,...8n\text{-}1\}$ such that the vertices are labeled as
$f^+(v_i) \equiv (32i\text{-}22) \bmod 8n$ for $1 \leq i \leq n$,
$f^+(u_i) \equiv (32i\text{-}2) \bmod 8n$ for $2 \leq i \leq (n\text{-}1)$ and
$f^+(u_n) \equiv (8n\text{-}2) \bmod 8n$.
Hence the induced map f^+ provides the distinct labels for vertices. Also edge labels are distinct.
Thus for all odd n, the total graph of C_n, $T(C_n)$ is Edge Odd Graceful.

Example 3.4.

The total graph of a cycle C_7 is EOG.

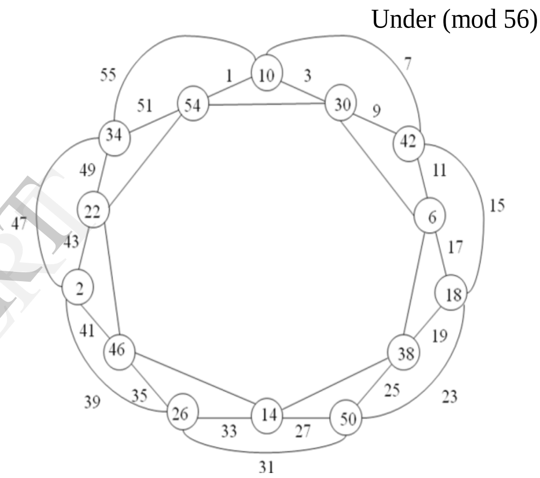

Figure 4. Total graph of C_7

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References

[1] A.Solairaju and A.Sathik Batcha, Edge odd gracefulness of Halin Graphs of types H (4,n) and H(5, n) by International Journal of open problems in Mathematics andApplications,Vol.1,No.1,March 2011.
[2] A.Solairaju and K.Chitra "Edge-odd graceful labeling of some graphs "Electronics Notes in Discrete Mathematics,Volume 33,April 2009, Pages 15 – 20.
[3] A.Solairaju, A.Sasikala, C.Vimala Edge-odd Gracefulness of a spanning tree of Cartesian product of P_2 and C_n, Pacific-Asian journal of Mathmatics, Vol .3, No. 1-2, (Jan-Dec. 2009) ,Pages:39 – 42.
[4] Frank Harary, Graph Theory, Narosa Publishing House, Tenth Reprint, 2000.

[5] Joseph A. Gallian, A Dynamic Survey of Graph Labeling, the electronic journal of combinatorics 17 (2010), #DS6, Thirteenth edition, November 17, 2011.

[6] R.Uma, N.Murugesan, Asian Journal of Current Engineering and Maths 1: 6 Nov–Dec (2012) 367 - 370.

[7] Shariefuddin Pirzada and Ashay Dharwadker, Applications of Graph Theory, Journal of the korean society for industrial and applied mathematics, Vol.11, No. 4, 2007.

[8] S Padmapriya, V.Sriram, B.Sooryanarayana, Edge odd gracefulness of the Wheel Graph, International Journal of Engineering Research and Technology Vol.2 (06), 2013,987-989

[9] Vernold Vivin J, Venkatachalam M, Akbar Ali M.M.,Achromatic Coloring on Double Star Graph Families, International Journal of Mathematical Combinatorics Vol.3 (2009), 71-81