Effect of Sectional Modulus on Universal and Hollow Steel Columns Subjected To Flexure

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ABSTRACT

The sectional modulus of rolled universal and circular hollow steel section columns in BS 5950(2000) was investigated in order to determine the safety of the available section modulus when subjected to flexure. The BS 5950 (2000) was evaluated in the light of Load Resistance Factor Design (LRFD) (1999) of the American Institute of Steel Constructions (AISC) due to their similarities. Results indicate that the safety levels of UC and CHS steel columns varies with the amount of sectional modulus available in flexure while the safety values to be used which depend extensively on column sections are predicted in each column type.

KEYWORDS: Sectional Modulus, Universal Steel Column, Hollow Steel Column, Flexure.

1.0 INTRODUCTION

In all engineering designs, the principal aim is to design against failure. Failure, from structural engineering point of view, has occurred when the structure or any of its element or part fails to satisfy the purpose of its construction. Failure is implied in the sense of exceeding a certain limit state corresponding to a measure of instability or unserviceability. The two types of limit states of particular interest here are: ultimate and serviceability limit states.

Ultimate limit states are those associated with collapse or with other forms of structural failure including loss of equilibrium, excessive deformation(s), rupture, etc. While exceedance of the ultimate limit state can have immediate adverse effects, the serviceability limit state affects the effective use of the structure which can be checked and repaired; this include vibration, cracking, fire resistance, etc.

In the design of steel columns, the first step is to determine the governing slenderness ratio, which should not exceed 200 (AISC 1999). From this ratio the threshold compressive stress is determined and hence the critical load. By applying the appropriate resistance factor the design capacity of the column can be determined. However, sometimes a column or compressive member may be subjected to flexural loads. It is therefore structurally wise to ensure that such columns satisfy their respective criteria in both compression and flexure at the ultimate and serviceability limit states.

Columns such as Universal Columns (UC) and Circular Hollow Sections (CHS) are members often used to sustain compressive and flexural loads in a structural system. The essence of this work is to verify the effect of sectional modulus of UC and CHS steel columns subjected to flexure and to determine the extent at which the section modulus actually influences the stability or failure of these steel columns.

Therefore, the effect of section modulus and cross sectional area on both the design capacities and on the following types of failure becomes relevant: Yielding (elestic and plastic), Overall column buckling (Flexural buckling about principal axis, Torsion buckling or twisting about shear axis and Torsional flexural buckling (simultaneous bending and twisting)) and Local and composite bulking of individual element as in spaced columns.

When columns are subjected to flexure, failure due to deflection or buckling under load may occur. The degree of flexural bending or deflection will highly depend on the available cross-sectional area and section modulus of the steel material.

Beam-columns are structural members that are subjected simultaneously to axial forces and bending moments. Thus, their behaviour falls somewhere between that of pure, axially loaded columns and that of a beam with only moment applied. To understand the behaviors of beam-columns, it is common practice to look at the response as predicated through an interaction equation between axial loads and moments (Dogan, 2005). For steel beam-columns, AISC (1999) uses two straight lines to model the interaction of flexure and compression.

The required strength of steel columns is determined by structural analysis for the appropriate factored load combinations. Design by either elastic or plastic analysis is permitted (AISC 1999), except that design by plastic analysis is permitted only for steel with specified minimum yield stresses not exceeding 450 N/mm^2 (AISC 1999). Generally, the properties of sections are determined using full cross section, except in computation of the elastic section modulus of flexural members, the effective width of uniformly compressed stiffened elements is used in determining the effective cross-sectional properties (AISC 1999).

A stiffened UC and CHS will fail in yielding if its web/thickness (w/t) ratio is relatively small. It may fail in local buckling at a stress level of less than the yield point if its w/t ratio is relatively large (AISC 1999).

A slender axially loaded column may fail by overall flexural buckling if the cross section of the column is a doubly, symmetric shape (I-section), closed shape (square or rectangular) tube cylindrical shape, or point symmetric shape (Z shape or cruciform). If a column has a cross section other than the above discussed shapes but is connected to other parts of the structure such as wall sheathing material, the material can fail by flexural buckling (Frederick and Jonathan, 2001).

In the analysis of flexural column buckling in the inelastic range, two concept have been used in the past. These concepts are the tangent modulus and reduced modulus methods (Frederick and Jonathan, 2001; Yamaguchi, 1999). It was later concluded that (Frederick and Jonathan, 2001):

i. The tangent modulus concept gives the maximum load up to which an initially straight column remains straight.

ii. The actual maximum load exceeds the tangent modulus load, but it cannot reach the reduced modulus load.

Local buckling is the buckling of a compression element which may precipitate the failure of the whole member.

2.0 MATERIALS AND METHOD

Structural reliability is the probability that a structure will not attain a specified limit state (ultimate or serviceability) during a specified reference period. The idea of a `reference period' is because the majority of structural loads vary with time in an uncertain manner. Hence the probability that any selected load intensity will be exceeded in a fixed interval of time is a function of the length of that interval. Thus, in general, structural reliability is dependent on the time of exposure to the loading environment.

Therefore, if we assume that R and S are random variables whose statistical distributions are known very precisely as a result of a very long series of measurements; and R is a variable representing the variations in strength between nominally identical structures, whereas S represents the maximum load effects in successive T-yr periods. Then, the probability that the structure will collapse during any reference period of duration T-years is given by:

$$P_{f=}P(R-S\leq 0)=\int_{-\infty}^{\infty}F_R\,(x)f_s(x)dx$$

Where, F_R is the probability distribution function of R and f_s the probability density function of S. Note that R and S are statistically independent and must necessarily have the same dimensions.

The reliability of the structure is the probability that it will survive when the load is applied, given by:

$$\Re = 1 - \ P_f \ = 1 - \int_{-\infty}^{\infty} F_R\,(x)f_s(x)dx$$

2.1 DETERMINATION OF THE RELIABILITY INDEX

For the estimation of the probability of failure, the method employed involves approximate iterative calculation procedures. In this method, two important measures are used:

$$(a)Expectations\colon\ \mu_i = E[X_i], i = 1, \dots \dots, n$$

$$(b)Covariances\colon\ C_{ij} \ = Cov\left[X_i, X_j\right], i, j, = 1,2, \dots \dots .. n$$

The “safety margin” is the random variable M = g(x) (also called the `state function’). Non-normal variables are transformed into independent standard normal variables, by locating the most likely failure point, β-point (called the reliability index), through an optimization procedure. This is also done by linearizing the limit state function in that point and by estimating the failure probability using the standard normal integral.

The reliability index, β, is then defined (Hasofer and Lind, 1974) by:

$$\beta = \frac{\mu_m}{\sigma_m}$$

Where μ_m = mean of M

And σ_m = Standard deviation of M

If R and S are uncorrelated and with M = R-S, then

$$\mu_m = \mu_R - \mu_S \quad \text{and} \quad \sigma_m^2 = \sigma_R^2 + \sigma_S^2$$

Therefore,

$$\beta = \frac{\mu_R - \mu_s}{\left(\sigma_R{}^2 + \sigma_s{}^2\right)^{1/2}}$$

A relationship can be drawn between the probability of failure, P_f, and the reliability index, β. It, however, holds true only when the safety margin, M, is linear in the basic variables, and these variables are normally distributed. This relationship is stated below:

$$P_F = -\Phi(-\beta) \qquad \text{and} \qquad \beta = -\Phi^{-1}\left(P_f\right)$$

where Φ is the standardized normal distribution function.

$$P_f = P\{(R - S) \le 0\} = P(M \le 0) = \varphi\left\{\frac{0 - \left(\mu_R - \mu_s\right)}{\sqrt{\sigma_R{}^2 + \sigma_s{}^2}}\right\} = \Phi\left(-\beta\right)$$

Table 1: Basic variables and their statistical characters for UC

Variables	Unit	Expectations E(x)	Coeff. Of Var. (COV)	Standard Dev. S_x	Basic Variables
P_n	N	21400000	0.045	963000	X1
M_n	N-mm	1955250000	0.15	293290000	X2
M_u	N-mm	15000000	0.05	750000	X3
P_u	N	200000	0.05	10000	X4

Table 2: Basic variables and their statistical characters for CHS

Variables	Unit	Expectations E(x)	Coeff. Of Var. (COV)	Standard Dev. S_x	Basic Variables
P_n	N	63000	0.045	2835	X1
M_n	N-mm	497750	0.15	746625	X2
M_u	N-mm	15000000	0.05	750000	X3
P_u	N	200000	0.05	10000	X4

Table 3: UNIVERSAL COLUMNS

Dimensions and properties, BS 5950 (2000)

Section Designation	**Radius of Gyration**	**Plastic Modulus**	**Area of Section**
	r (cm)	**Z (cm^3)**	**A (cm^2)**
356 x 406 x 634	11.0	7110	808
356 x368 x202	9.6	1920	257
305 x 305 x 283	8.27	2340	360
254 x 254 x 167	6.81	1140	213
203 x 203 x 86	5.34	456	110
152 x 152 x 37	3.87	140	47.1
152 x 152 x 30	6.76	112	38.3
152 x152 x23	6.54	80.2	29.2

Table 4: HOT-FINISHED CIRCULAR HOLLOW SECTIONS

Dimensions and properties, BS 5950 (2000)

Section Designation		Radius of Gyration	Plastic Modulus	Area of Section
Outside Diameter	Thickness			
D (mm)	t (mm)	r (cm)	S (cm^3)	A (cm^2)
26.9	3.2	0.846	1.81	2.38
48.3	3.2	1.6	6.52	4.53
60.3	5.0	1.96	15.3	27.7
114.3	6.3	3.82	73.6	21.4
139.7	10.0	4.60	169	40.7
168.3	10.0	5.61	251	49.7
193.7	10.0	6.5	338	57.7
219.1	12.5	7.32	534	81.1
273.0	12.5	9.22	849	102
323.9	16.0	10.9	1520	155
406.4	16.0	13.8	2440	196
508.0	20.0	17.3	4770	307

3.0 RESULTS AND DISCUSSION

The stochastic models generated in Tables 1, 2, 3 and 4 are analyzed using the First Order Reliability Method to give values of safety index (β) and probability of failure (P_f) for some selected sections of both UC and CHS sections in BS5950(2000). An algorithm developed into FORTRAN module was designed for the different failure modes in all sections of the UC and CHS steel columns. The column slenderness parameter λ_c was varied for all sections of both UC and CHS for their corresponding values of sectional modulus (Plastic) for λ_c values of 0.3, 0.6, 0.9, 1.2 and 1.5. Results obtained are given below.

From figures 1 to 8 for UC, it is observed that the higher the sectional (plastic) modulus and area of section, the safer the stability of the section and the more it become independent of the column slenderness details even when high section modulus and cross-sectional area used are above the recommended design margin.

For the UC sections, section modulus of ≥ 450 and section area of ≥ 110 are safe but not economical. The most economical section should have section modulus range of 200 – 100cm^3 and cross-sectional area of a range 50 – 35cm^2. Failure may occur with sections below this range as indicated in figure 8.

For CHS sections (figures 9 to 20), failure is prone to occur with the use of smaller outside diameters, thickness, cross-sectional area and sectional modulus (plastic).

From figures 9 to 12, it is obvious that the first four sections in Table 4 when subjected to an axial load of about 150KN will fail. The most economical and safe sections are the fifth, sixth and seventh sections in Table 4 (figures 13 to15) other sections may be safe but not economical.

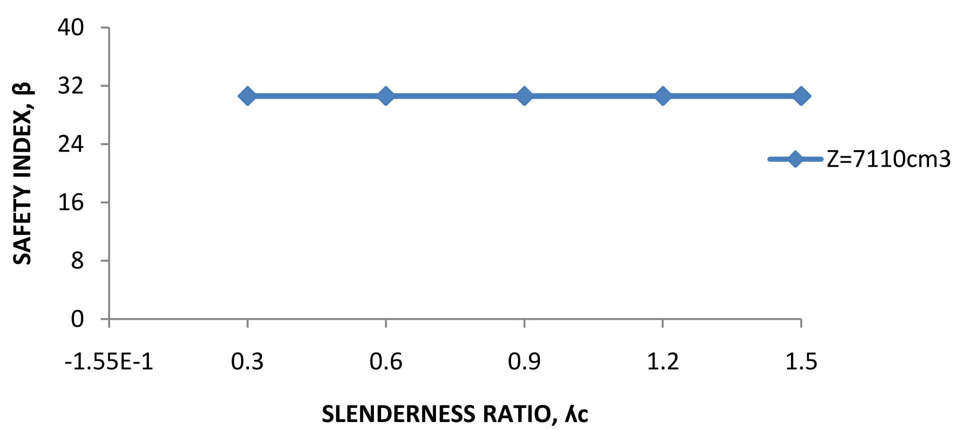

Figure 1: UC section 356mm x 406mm x 634kg/m

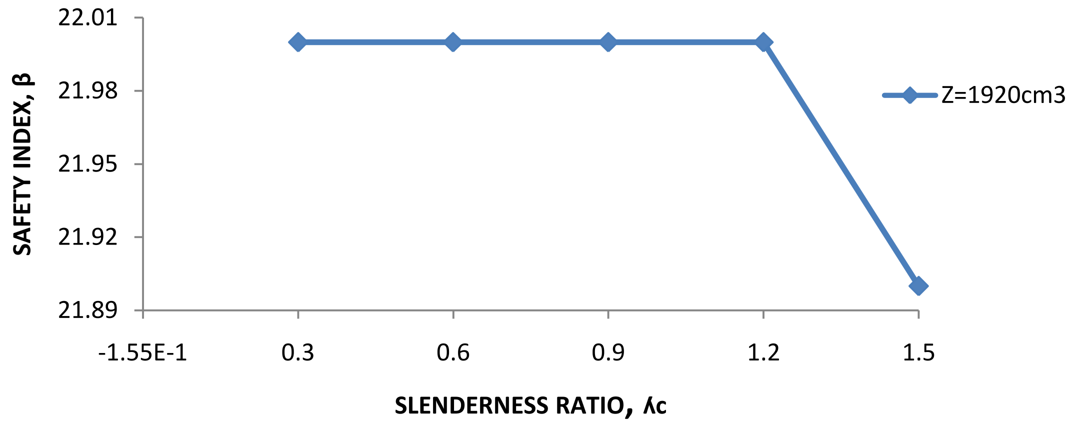

Figure 2: UC section 356mm x 368mm x 202kg/m

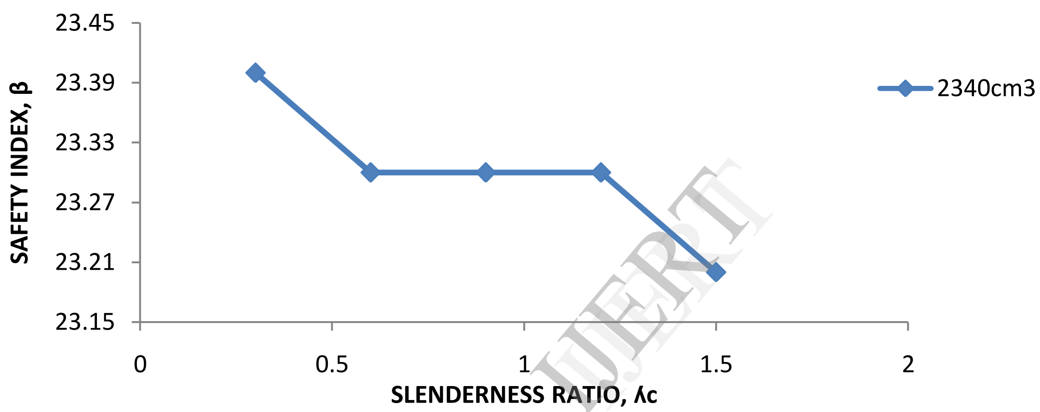

Figure 3: UC section 305mm x 305mm x 283kg/m

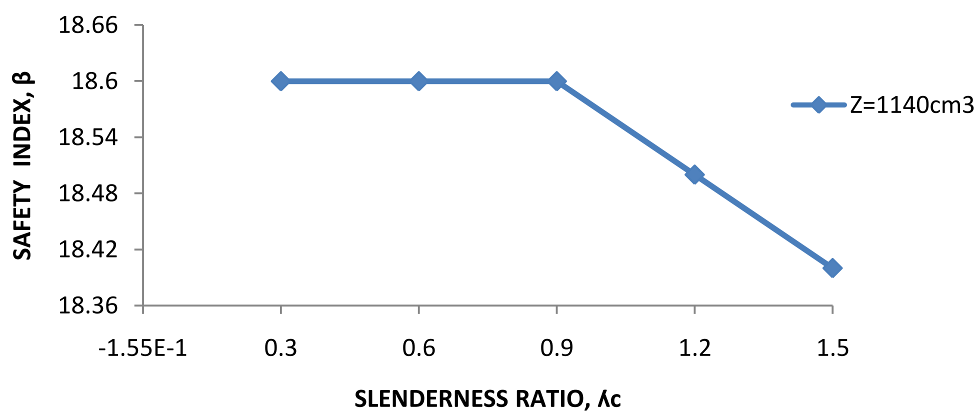

Figure 4: UC section 254mm x 254mm x 167kg/m

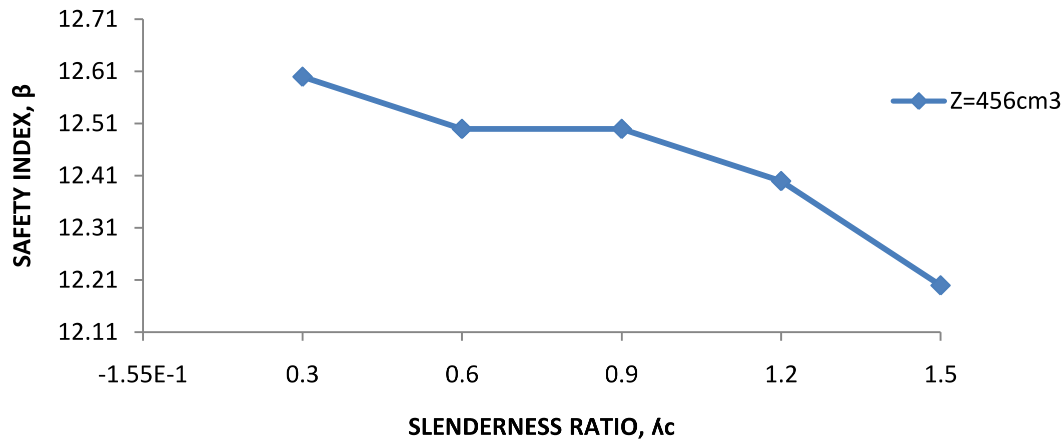

Figure 5: UC section 203mm x 203mm x 86kg/m

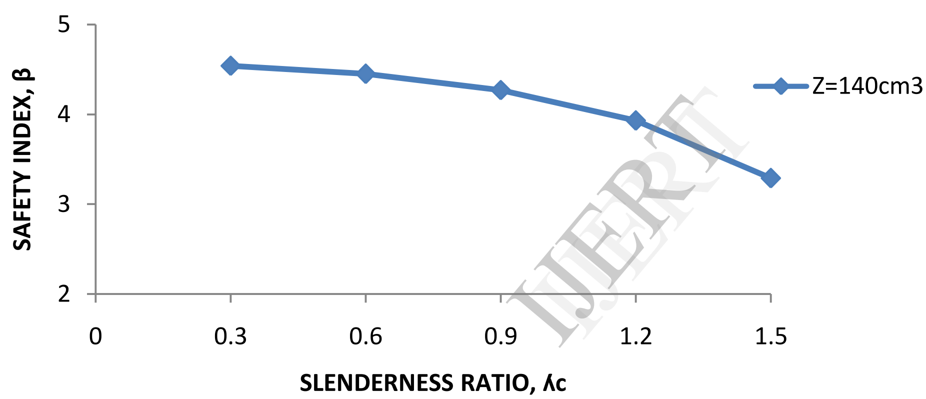

Figure 6: UC section 152mm x 152mm x 37kg/m

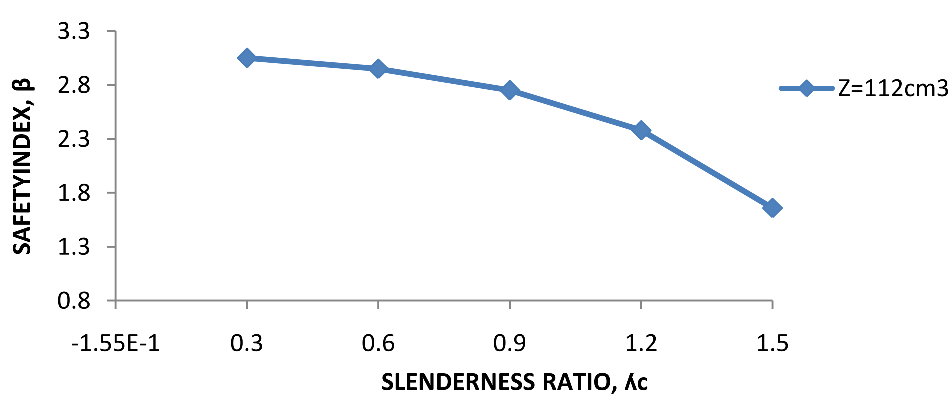

Figure 7: UC section 152mm x 152mm x 30kg/m

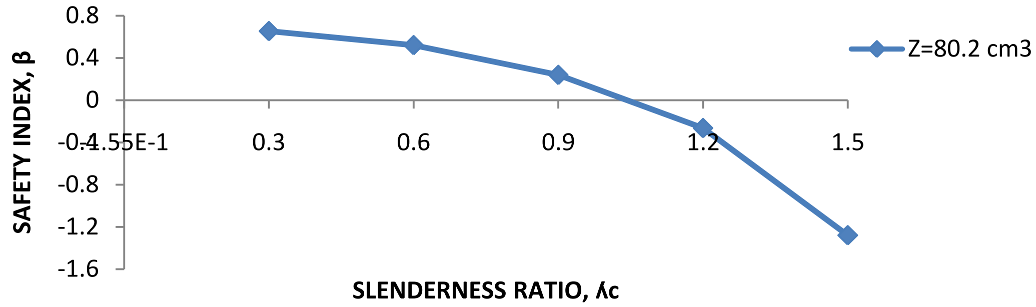

Figure 8: UC section 152mm x 152mm x 23kg/m

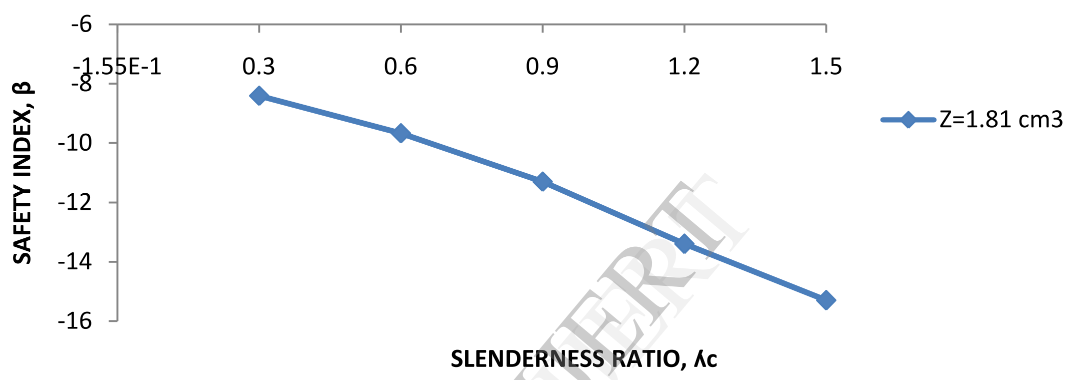

Figure 9: CHS section 26.9mm x 3.2mm

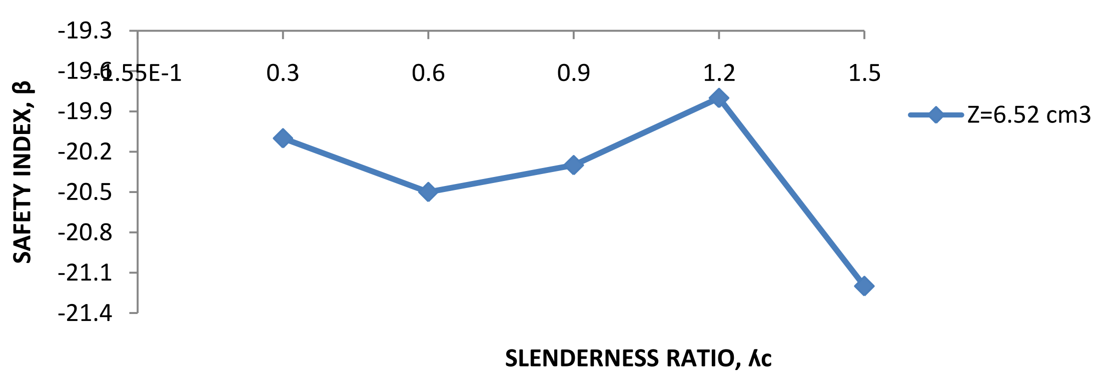

Figure 10: CHS section 48.3mm x 3.2mm

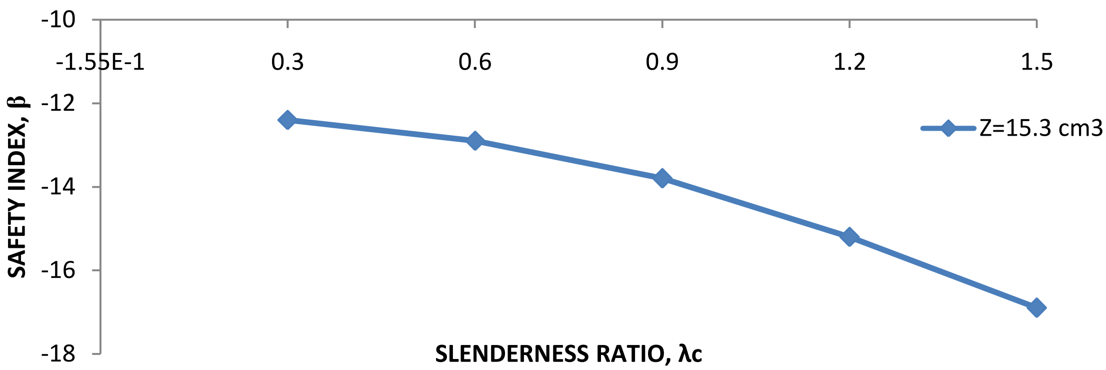

Figure 11: CHS section 60.3mm x 5.0mm

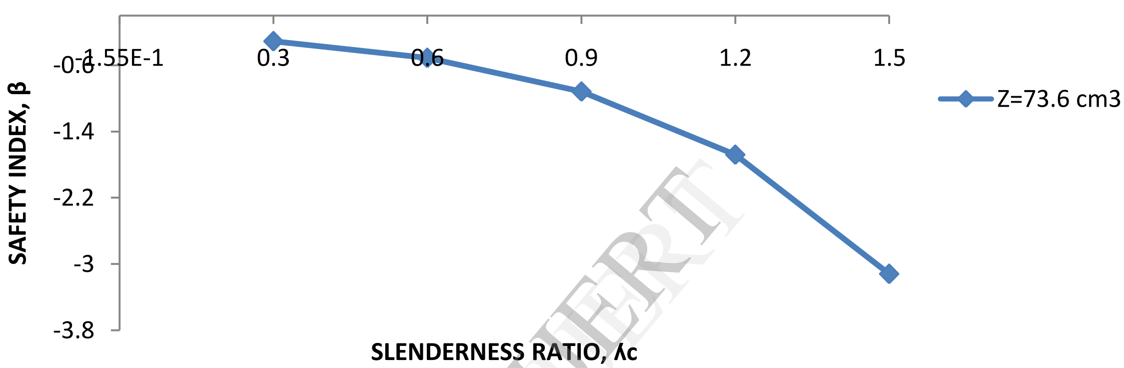

Figure12: CHS section 114.3mm x 6.3mm

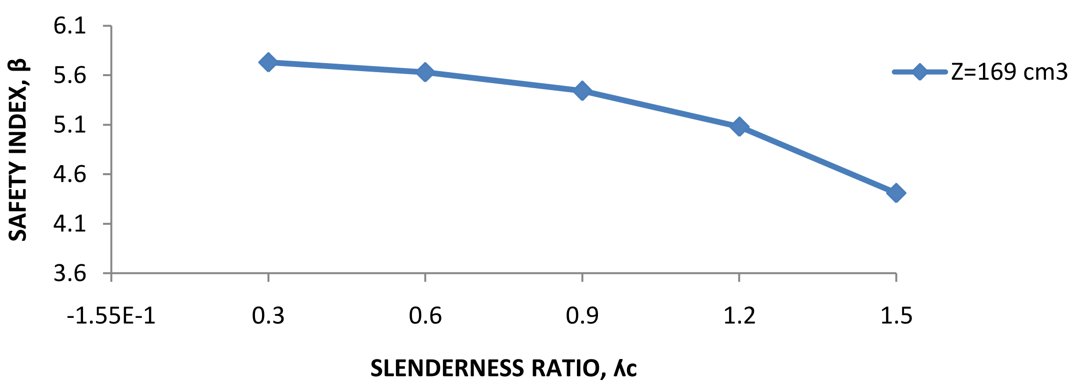

Figure 13: CHS section 139.7mm x 10.0mm

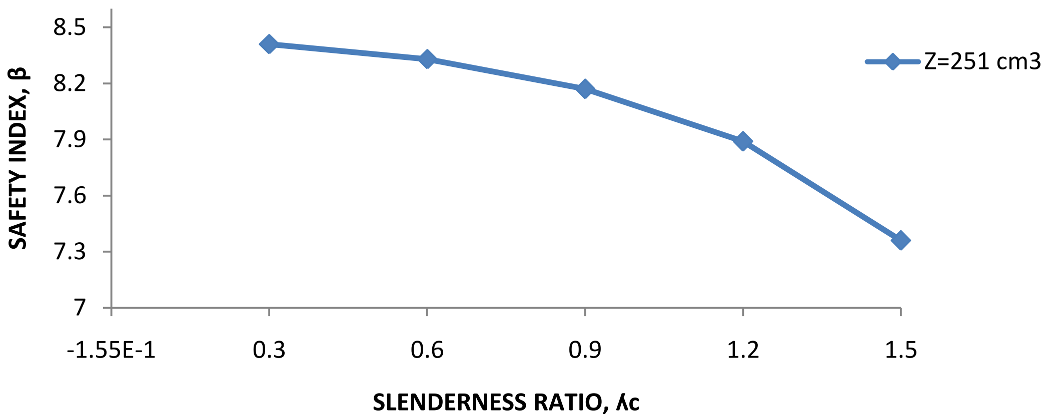

Figure 14: CHS section 168.3mm x 10.0mm

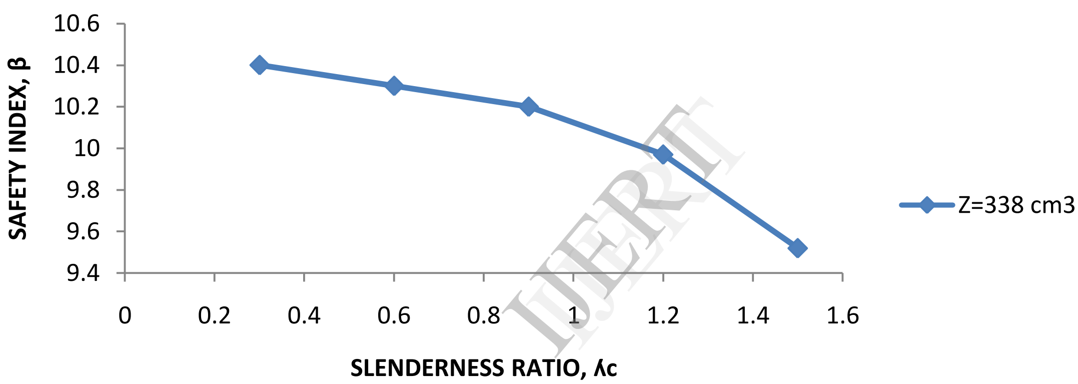

Figure 15: CHS section 193.7mm x 10.0mm

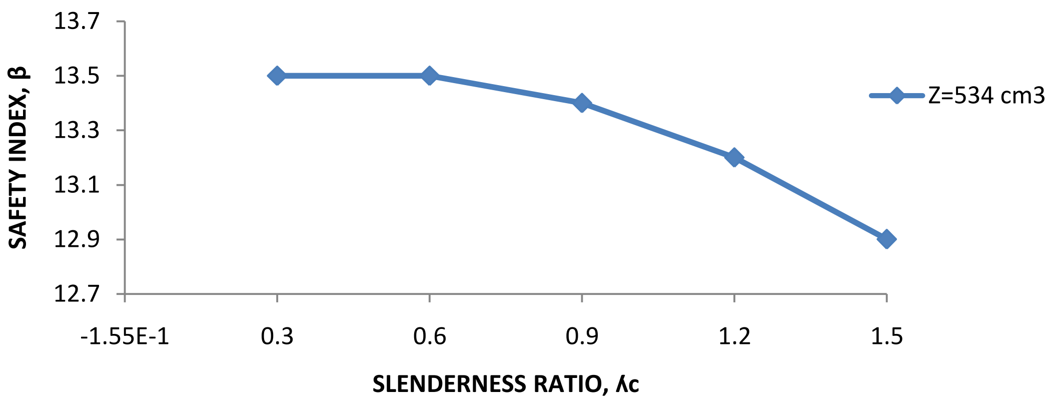

Figure 16: CHS section 219.1mm x 12.5mm

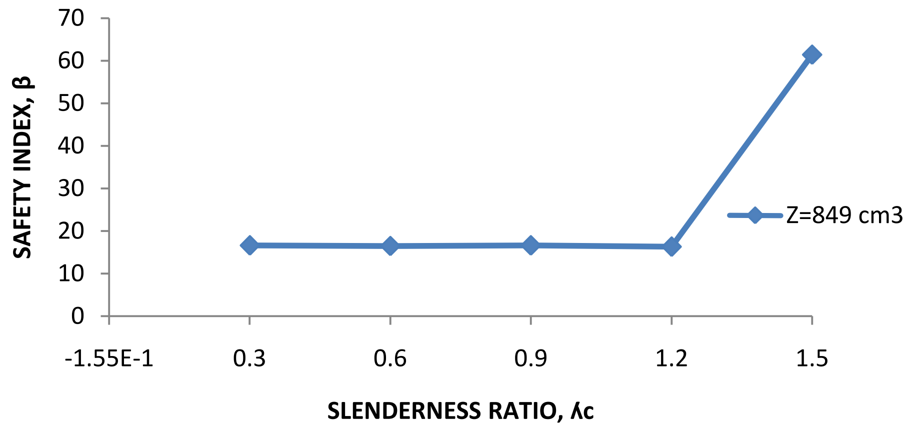

Figure 17: CHS section 273.0mm x 12.5mm

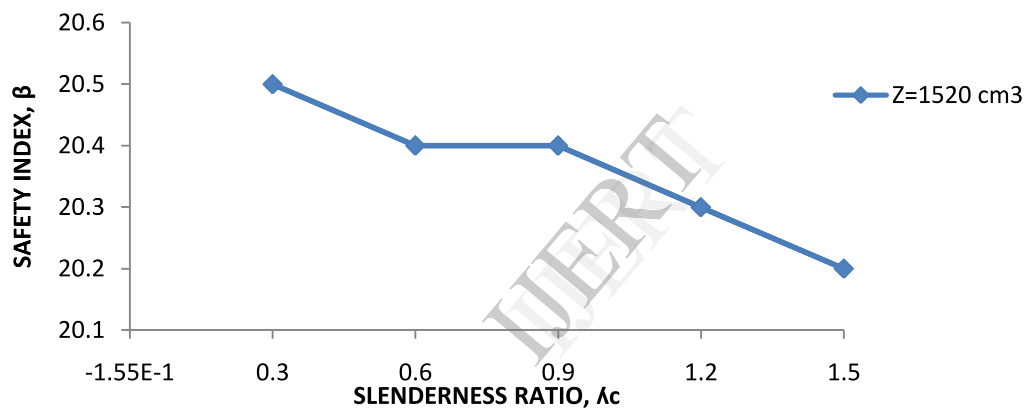

Figure 18: CHS section 323.9mm x 12.5mm

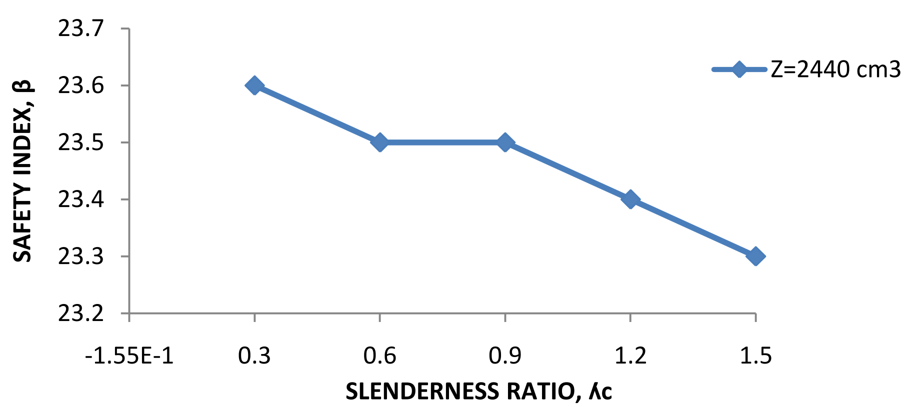

Figure 19: CHS section 406.4mm x 16.0mm

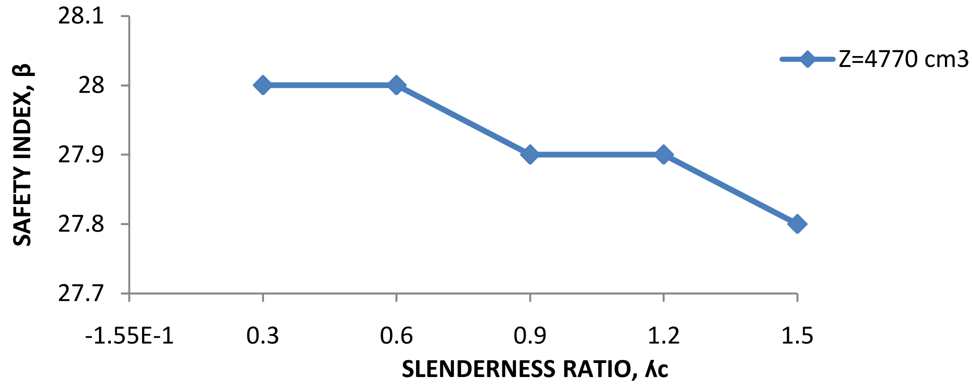

Figure 20: CHS section 508.0mm x 20.0mm

4.0 CONCLUSION

From the recommendations of BS5950(2000), steel sections and as formulated in AISC (1999), suggests that universal columns subjected to high axial load and moment of say about 150KN and 15KN-m should have a safety index (β) of 3.5 which corresponds to 200-1000cm^3 plastic section modulus. Higher values of section modulus (Z) will also be safe but may not be economical while lower values will cause failure.

Also, for higher values of Z, the performance of the column tends to be independent of the slenderness parameter λ_c for compact or rolled UC and CHS.

Thus, for all sections of UC, the column slenderness parameter λ_c and safety index (β) can be predicated when faced with challenges on site as results also indicate.

It was observed that when CHS columns are subjected to the same axial load and moment with UC, they will perform better at safety indices (β) of 2 - 6 corresponding to Z values of 150-300cm^3.

5.0 REFERENCES

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