

# Effect of Slips and Radiation on Aligned MHD Boundary Layer Flow of Williamson Nanofluid Through Porous Medium

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**Abstract:** The present study explores the effect of velocity, thermal and solute slip conditions with aligned magnetic field of Williamson Nano fluid along a stretching sheet through porous medium in the presence of thermal radiation. The object of this study is to examine the phenomena of heat and mass transport caused by thermophoresis and Brownian motion. A set of governing nonlinear partial differential equations for the modelling of boundary layer flow are transformed to ordinary differential equations by using suitable similarity transformations. The resulting coupled non-linear ordinary differential equations are solved numerically with the help of MATLAB. The effect of the flow variable parameters on relevant profiles is calculated and measured qualitatively to get the complete understanding of the present study. To confirm the present analysis, the numerical results are compared with previously published data and they found to be in good agreement. The findings reveal that the velocity profile decreases, the temperature and concentration profiles are increased with increase in inclined angle. Moreover velocity slip diminishes the velocity profile of the fluid and will shows the opposite result in temperature, concentration graphs and both thermal and solutal jumps/slips shows the decreasing nature in temperature and concentration profiles of the fluid as the values of the parameters are increased.

**Keywords:** Velocity, thermal and solutal slips - Aligned MHD – Williamson nanofluid- Thermal radiation

## INTRODUCTION:

Magnetohydrodynamics, often abbreviated as MHD, is a multidisciplinary field of physics and engineering that combines principles of magnetism and fluid dynamics. It focuses on the study of the behavior of electrically conducting fluids (plasmas, liquid metals and electrolytes) in the presence of magnetic fields. In MHD, the fundamental equations of fluid dynamics are extended to incorporate the effects of electromagnetic forces. This coupling of magnetic and fluids properties has wide-ranging applications in various fields, including astrophysics, fusion energy research, engineering and space exploration. MHD plays a crucial role in understanding various astrophysical phenomena and MHD is integral to the study of controlled nuclear fusion, a potential source of clean and limitless energy. Magnetic confinement techniques like tokamaks and stellarators rely on MHD principles to contain and stabilize high-temperature plasma. MHD technology is employed in aerospace applications and used in the design of electromagnetic pumps and flow control systems, which find applications in metallurgy, cooling of electrical equipment and industrial processes also MHD is employed in the casting and shaping of metals, particularly in the steel industry to control the flow and solidification of molten metals. In the fields of metallurgy and polymer technology, it appears to be frequently utilized. Magnetic drug targeting, astrophysical sensing, and engineering may all benefit from MHD applications [1, 2]. Based on all these significant benefits, analysts and researchers monitor MHD flows on a continuous basis. Metri et al. [3] explore the impact of heat sink/source over MHD forced convective visco-elastic fluid flow subjected to dissipation with porous medium. Akbar et al. [4] numerically studied the influence of MHD heat transport nanofluid flow over a stretched surface for the newheat flux concept. Numerous authors Rashidi et al. [5], Rashidi and Erfani [6], Khedr et al. [7], Fang and Zhang [8], Magyari and Chamkha [9], Ishak et al. [10], Yasin et

al. [11], Falodun and Omowaye [12], Madhusudan et al. [13] have recently examined the problems on MHD impact of magnetic fields on stretching sheet and various aspects of flow issues. Magnetic effect is said to have a key impact in heat control applications.

A nanofluid is a fluid that contains tiny volumetric amounts of nanometer-sized particles (0–100 nm) called nano-particles. Nanofluids are a class of engineered fluids created by dispersing nanometer-sized solid particles (typically nanofluid) into a base fluid, which can be a liquid or even a gas. These nanoparticles are usually chosen for their unique properties, such as high thermal conductivity or optical characteristics. The addition of nanoparticles to the base fluid results in a nanoscale colloidal suspension with enhanced properties that can be tailored for specific applications. It has many applications such as to improve the cooling efficiency of electronic devices, such as computer chips and LEDs by enhancing their conductivity and heat dissipation properties. In industrial processes and HVAC system, nanofluids are employed to increase the efficiency of heat exchangers, leading to reduced energy consumption and improved performance. Nanofluids are continually finding new applications across various industries due to their ability to enhance the thermal, electrical, mechanical and optical properties of fluids. Their tunable properties of fluids. Their tunable properties and versatility make them a valuable tool for improving existing technologies and enabling innovative solutions in fields ranging from energy production to healthcare. The dispersion, amorphous movement, thermophoresis effects, Brownian motion, and thermophoresis impacts are a few of these physical properties. The examination of nanofluid flow has piqued the interest of researchers in recent years, due to a rise in applications in many sectors of science & technology, biomechanics, nuclear, and chemical sectors. For instance, food materials (mayonnaise, chocolate, ketchup, alcoholic beverages, yoghurt, milk and apple sauce in liquefied form), chemical materials (paints, cosmetics, pharmaceutical chemicals, toothpastes, oil, reservoirs, shampoos, grease, etc.), and biological materials (synovial fluid, syrups, vaccines, blood, etc.). The term nanofluid was first coined by Choi [14]. The thermal conductivity of copper nanoparticles embedded in a nanofluid of ethylene glycol was explored by Eastmann et al. [15]. Since then, thermophoresis and Brownian diffusion demonstrations have fueled the advancement of mathematical modeling of nanofluids. The free convective flow of a nanofluid was investigated by Kuznetsov and Nield [16] using an analytical approach. The stagnation-point flow of a nanofluid toward a non-linear stretching sheet was explored with Anwar et al. [17]. Azimi and Riazi [18] discussed heat transmission between two parallel disks of Go-water Nanofluid. Bhargava et al. [19] described an effective hybrid method for modeling MHD nanofluid flow through a permeable stretched sheet. [20] Raza et al. scrutinized MHD Flow of nano-Williamson liquid caused via multiple slips on a stretched plate. Khan [21] investigated the stability of magneto convective flow of SiO<sub>2</sub>–MoS<sub>2</sub>/C<sub>2</sub>H<sub>6</sub>O<sub>2</sub> hybrid nanofluids using a shrinking/stretching wedge. Chamkka et al. [22] investigated the laminar MHD mixed convection flow of a nanofluid over a stretched permeable area with heat production or absorption. Turkyilmazoglu [23] investigated an accurate analytical solution for heat and mass transport in nanofluids with MHD slip flow. Wubshet and Shankar [24] studied the flow and heat transfer of a nanofluid via a permeable stretched sheet under a variety of boundary conditions, including velocity, temperature, and solutal slip. Nagendra et al. [25] investigated the numerical simulation of hydromagnetic heat and mass transport in a nano-Williamson liquid passing through a vertical plate through thermal and momentum slip effects. Garoosi et al. [26] studied numerical modeling of natural convection in heat exchangers utilizing the Buongiorno model. Konda et al. [27] analyzed the performance of a non-uniform heat sink/source on the MHD boundary layer flow and melting heat transport of a Williamson nanofluid in a porous media. Yahaya et al. [28] explored nanofluid hydromagnetic slip flow with thermal stratification and convective heating. Acharya et al. [29] investigated the impact of numerous slips and chemical reactions on the radiative MHD Williamson nanofluid flow in porous media: A computational approach. Mishra and Mathur [30] used a semi-analytical technique to investigate Williamson nanofluid flow in porous media with melting heat transport boundary conditions. Asogwa et al. [31] investigated the comparative analysis of water-based Al<sub>2</sub>O<sub>3</sub> nanoparticles via H<sub>2</sub>O-based CuO nanoparticles through heat transport toward an exponentially accelerated radiative Riga plate area. Warke et al. [32] investigated nonlinear radiation through magneto micro polar stagnation point flow over a heated stretching sheet using a numerical investigation.

Thermal radiation, which is the emission of electromagnetic radiation from a body due to its temperature, has several important applications in scientific research and it encompasses electric power, food, solar cell panels, gas turbines, different propulsion mechanisms for aero planes, and the medical sector. The impact of radiation and mixed convection on nanofluid flow over a vertical plate with melting heat transport was investigated by Mahanthesh et al. [33]. Fazole Mabood et al. [34] investigated the radiation impact on MHD Williamson nanofluid flow with a heated surface. On a shrinking/stretching porous sheet, thermo-diffusion and radiation effects on Williamson nanofluid were investigated by Bhatti and Rashidi [35]. The influence of radiation on nanofluid flow across a stretched surface was inspected by Krishnamurthy et al. [36], Dulal Pal et al. [37], Ghadikolaei et al. [38], and Almakki et al. [39]. This work is based on several recent articles as [40–47].

Slip conditions typically refer to the behavior of the fluid at the fluid-solid interface. Slip boundary conditions are classified into no-slip conditions (where the fluid velocity at the interface is zero) and slip conditions (where the fluid velocity at the interface is not zero). Understanding and how slip conditions affect the flow of nanofluid through a porous medium involves considering factors like fluid dynamic, heat transfer and slip conditions can significantly impact the velocity profile of the fluid within the porous medium. Slip condition generally imply a non-zero velocity at the fluid-porous medium interface. This affects the flow pattern and can lead to changes in flow rate and pressure distribution.

Inspired by above reputed researchers and importance of slips on nanofluids, the present work discusses the effect of slips on Williamson nanofluid flowing via a porous media in the presence of aligned magnetic field with a constant stretching sheet. According to the aforementioned literatures, no study has been conducted on aligned MHD nanofluid via a stretching sheet when thermal radiation and slip conditions are present. The solution of coupled nonlinear equations is obtained by numerical computing and for numerous values of physical restrictions, the consequence of dimensionless parameters has been evoked. The originality of this observation is to consider the behavior of the radiation, velocity, thermal and solutal slips on aligned MHD boundary layer flow through heat and mass transport in a Williamson nanofluid with porous media.

#### FORMULATION OF THE PROBLEM:

Assume that the steady-state two dimensional incompressible MHD Williamson nanofluid flow toward a horizontal stretching sheet under the influence of aligned magnetic field through the porous medium and with the effect of thermal radiation and slips. From the physical model of the problem which demonstrated in Figure 1 the plate is extended along the direction of  $X$  axis

and  $Y$  axis is the direction normal to the stretching sheet. Along with these the fluid is permitted by an aligned magnetic field. Plate velocity is

$U_w(x) = bx$ , ( $b > 0$ ), the wall temperature

$T_w$  and the nanoparticle fraction  $C_w$  are assumed constant at the stretching surface. When  $y$  tends to infinity, the ambient values of temperature and nanoparticle fraction are denoted by  $T_\infty$  and  $C_\infty$ , respectively. The boundary layer equations [48] for the flow of two-dimensional under the above constraints and occurrence of aligned magnetic field and thermal radiation are

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (1)$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \nu \frac{\partial^2 u}{\partial y^2} + \sqrt{2} \gamma \Gamma \frac{\partial u}{\partial y} \frac{\partial^2 u}{\partial y^2} - \frac{\sigma B_0^2}{\rho} \sin^2 \gamma u - \frac{\gamma}{k} u \quad (2)$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2} + \tau \left\{ D_B \frac{\partial C}{\partial y} \frac{\partial T}{\partial y} + \frac{D_T}{T_\infty} \left( \frac{\partial T}{\partial y} \right)^2 \right\} - \frac{1}{(\rho C)_{nf}} \frac{\partial q_r}{\partial y} \quad (3)$$

$$u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = D_B \frac{\partial^2 C}{\partial y^2} + \frac{D_T}{T_\infty} \frac{\partial^2 T}{\partial y^2} \quad (4)$$

The following are subject to the appropriate conditions at the boundary

$$u = u_w + A^* \mu \left( \frac{\partial u}{\partial y} \right), \quad v = 0, \quad T = T_w + B^* \left( \frac{\partial T}{\partial y} \right), \quad C = C_w + C^* \left( \frac{\partial C}{\partial y} \right) \quad \text{at } y = 0$$

$$u \rightarrow 0, \quad T = T_\infty, \quad C = C_\infty, \quad \text{as } y \rightarrow \infty \quad (5)$$

Where  $A^*$ ,  $B^*$  and  $C^*$  are velocity, thermal and solutal slip factors

Using the Rosseland [30] approximation the radiative heat flux is simplified as:

$$q_r = - \frac{4\sigma_i}{3K^*} \frac{\partial T^4}{\partial y} \dots\dots\dots (6)$$

We assume that the temperature differences within the flow region, namely, the term  $T^4$  can be expressed as a linear function of temperature. The best linear approximation of  $T^4$  is obtained by expanding it in a Taylor series about  $T_\infty$  is

$$T^4 = T_\infty^4 + 4T_\infty^3(T - T_\infty) + 6T_\infty^2(T - T_\infty)^2 + \dots\dots\dots (7)$$

Neglecting higher order terms of  $(T - T_\infty)$  from equation (7) and attained as

$$T^4 \cong 4TT_\infty^3 - 3T_\infty^3 \dots\dots\dots (8)$$

Using equation (9) into equation (7) we get

$$q_r = - \frac{16\sigma_i T_\infty^3}{3K^*} \frac{\partial T}{\partial y} \dots\dots\dots (9)$$

The modified equation of (3) by the assistance of equation (9) is

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2} + \tau \left\{ D_B \frac{\partial C}{\partial y} \frac{\partial T}{\partial y} + \frac{D_T}{T_\infty} \left( \frac{\partial T}{\partial y} \right)^2 \right\} + \left( \frac{16\sigma_i T_\infty^3}{3(\rho C_p)_f K^*} \frac{\partial^2 T}{\partial y^2} \right) \quad (10)$$

Employing the following similarity Transformations:

$$\eta = y \sqrt{\frac{b}{\gamma}}, \quad u = bx f'(\eta), \quad v = -\sqrt{b\gamma} f \cdot \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}, \quad \phi(\eta) = \frac{C - C_\infty}{C_w - C_\infty} \quad (11)$$

Using transformations (11) in equations (2), (4) and (8) subject to the constraints in (5) will be in the following form:

$$f''' + \lambda f'' f''' + f f'' - (f')^2 - (\text{Sin}^2 \gamma M + k) f' = 0 \quad (12)$$

$$\left( \frac{3+4R}{3} \right) \theta'' + Pr \cdot f \theta' + \frac{Nc}{(Le)(Nbt)} (\theta')^2 - 2Pr \cdot f' \theta + \left( \frac{Nc}{Le} \right) \theta' \phi' = 0 \quad (13)$$

$$\phi'' + Sc f \phi' + \left( \frac{1}{Nbt} \right) \theta'' = 0 \quad (14)$$

The governing parameters are

$$\lambda = \Gamma_X \sqrt{\frac{2b^3}{\gamma}}, \quad Le = \frac{\alpha}{D_B}, \quad M = \frac{\sigma B_0^2}{b\rho}, \quad Pr = \frac{\gamma\rho C_p}{\kappa}, \quad R = \frac{4\sigma^* T_\infty^3}{\kappa\kappa}, \quad k = \frac{\gamma}{bk}, \quad Nc = \frac{(\rho c)_p}{(\rho c)_f} (C_w - C_\infty), \quad Nbt = \frac{T_\infty D_B (C_w - C_\infty)}{D_T (T_w - T_\infty)}, \quad Sc = \frac{\gamma}{D_B}$$

Where  $f$ ,  $\theta$  and  $\phi$  are functions of  $\eta$  and prime labels differentiation w.r.to  $\eta$ . The equivalent transformed boundary conditions are

$$f(0) = 0, \quad f'(0) = 1 + Af''(0), \quad \theta(0) = 1 + B\theta'(0), \quad \phi(0) = 1 + C\phi'(0) \quad \text{at } \eta = 0$$

$$f'(\infty) = 0, \quad \theta(\infty) = 0, \quad \phi(\infty) = 0 \quad (15)$$

The quantities of physical interest are described as the coefficient of friction  $C_f$ , the local Nusselt number  $Nu_x$  and the Sherwood number  $Sh_x$  are expressed as follows

$$C_f = \frac{\tau_w}{\rho U_w^2}, \quad Nu_x = \frac{xq_w}{\kappa(T_f - T_\infty)} \quad \text{and} \quad Sh_x = \frac{q_{m,x}}{D_B(C_w - C_\infty)} \quad (16)$$

Where  $\tau_w$  is the shear stress,  $q_w$  is the heat flux and  $q_m$  is the mass flux volume fraction of the nanoparticle at the plate surface, which are specified by

$$\tau_w = \mu \left[ \frac{\partial u}{\partial y} + \frac{\Gamma}{\sqrt{2}} \left( \frac{\partial u}{\partial y} \right)^2 \right], \quad q_w = -k \left( \frac{\partial T}{\partial y} \right)_{y=0} \quad \text{and} \quad q_m = -k \left( \frac{\partial C}{\partial y} \right)_{y=0} \quad (17)$$

Finally using equations (17) and (11), equation (16) becomes

$$Re_x^{\frac{1}{2}} C_f = f''(0) + \frac{\lambda}{2} f(0)''^2, \quad Re_x^{\frac{-1}{2}} Nu_x = -\theta'(0) \quad \text{and} \quad Re_x^{\frac{-1}{2}} Sh_x = -\phi'(0) \quad (18)$$

#### METHOD OF SOLUTION:

The non-linear ordinary differential equations (12) to (14) together with boundary conditions (15) are solved numerically by an implicit finite difference scheme namely the Keller Box method and the following steps are involved in this method.

- Reduce the ordinary differential equations to a system of first order equations.
- Write the difference equations for ordinary differential equations using central differences.
- Linearize the algebraic equations by Newtons method, and write them in matrix vector form.
- Solve the linear system by the block tri-diagonal elimination technique. The accuracy of the method is depends on the appropriate initial guesses. We made an initial guesses are as follows.

$$f(\eta) = \left( \frac{1}{1+A} \right) (1 - e^{-\eta}), \quad \theta(\eta) = \left( \frac{1}{1+B} \right) e^{-\eta}, \quad \phi(\eta) = \left( \frac{1}{1+C} \right) e^{-\eta}$$

The choices of the above initial guesses depend on the convergence criteria and the transformed boundary conditions of equations (15). The step size 0.01 is used to obtain the numerical solution with six decimal place accuracy as the criterion of convergence.

#### RESULTS AND DISCUSSION:

We used the Kellor-Box method to get a physical understanding of the present investigation of aligned MHD heat and mass transport Williamson nanofluid flow with thermal radiation, porous medium and slips. The findings are authorizes the effect of various non-dimensional factors on the velocity, temperature and concentration profiles in addition to coefficient of friction factor, heat transport and mass transfer using tables and diagrams. In this investigation the non-dimensional parameter values  $\lambda = 0.5, M = 0.3, k = 0.2, \beta = 1.0, Pr = 7.0, Nc = 2.5, Nbt = 2.0, Le = 10, Sc = 5.0, R = 0.5, \gamma = \frac{\pi}{2}, A = 0.25, B = 0.5, C =$

0.5 were used. To validate the current numerical outcomes, a comparison with Y. Dharmendar Reddy et al was performed and found in good agreement, as shown in Table I, II and III.

Table I: Comparison of skin friction ( $-f''(0)$ ) values for various values of  $\delta, \lambda, M, k$  when

$$\beta = 1.0, Pr = 7.0, Nc = 2.5, Nbt = 2.0, Le = 10, Sc = 5.0, R = 0.5, \gamma = \frac{\Pi}{2}, B = 0, C = 0$$

$\delta$	$\lambda$	$M$	$k$	Y. Dharmendar Reddy et al	Present study
0.25	0.5	0.3	0.2	-1.1551	-1.155109
0.75				-0.6689	-0.668911
1.25				-0.4694	-0.469408
1.75				-0.3598	-0.359812
	0.0			-0.7201	-0.720121
	0.3			-0.7903	-0.790365
	0.5			-0.8461	-0.846174
	0.7			-0.9630	-0.963097
		0.0		0.7062	0.706204
		0.2		0.8004	0.800456
		0.4		0.8911	0.891132
		0.8		1.0684	1.068461
			0.1	0.8004	0.800455
			0.3	0.8911	0.891189
			0.5	0.9799	0.979941
			0.7	1.0684	1.068477

Table II: Comparison of  $-\theta'(0)$  and  $-\phi'(0)$  for various values of  $\delta, \beta, \lambda, Pr, Nc, Nbt$  when  $Le = 10, Sc = 5, M = 0.5, R = 0.5, k = 0.2$

$\delta$	$\beta$	$\lambda$	$Pr$	$Nc$	$Nbt$	Y. Dharmendar Reddy et al $-\theta'(0)$	Present study	Y. Dharmendar Reddy et al $-\phi'(0)$	Present study
0.25	1.0	0.5	7	2.5	2	1.3357	1.335709	0.8086	0.808633
0.75						1.0444	1.044465	0.7169	0.716909
1.25						0.9168	0.91689	0.6239	0.623902
1.75						0.8244	0.824424	0.5558	0.555834
	0.2					1.8752	1.875287	0.5016	0.501683
	0.4					1.6095	1.609550	0.5999	0.599929

	0.6					1.4098	1.409829	0.6739	0.673931
	0.8					1.2541	1.254128	0.7315	0.731522
		0.0				1.1461	1.146148	0.8014	0.801449
		0.3				1.1386	1.138693	0.7874	0.787431
		0.5				1.1294	1.129475	0.7777	0.777785
		0.7				1.1180	1.118008	0.7633	0.763344
			4			0.8074	0.807481	0.9135	0.913519
			6			1.0313	1.031336	0.8198	0.819811
			8			1.2208	1.220807	0.7379	0.737962
			10			1.3882	1.388211	0.6639	0.663907
				5		1.0819	1.081939	0.7988	0.798881
				10		0.9932	0.993299	0.8377	0.837777
				15		0.9129	0.912962	0.8726	0.872638
				20		0.8404	0.840404	0.9038	0.903842
					0.3	1.0939	1.093977	-1.4932	-1.493254
					0.4	1.1041	1.104111	-0.8427	-0.842766
					0.6	1.1145	1.114508	-0.1780	-0.178031
					0.9	1.1216	1.121639	0.2734	0.273419

Table III: Comparison of  $-\theta'(0)$  and  $-\phi'(0)$  for different values of  $Le, Sc, M, R, k$

$Le$	$Sc$	$M$	$R$	$k$	Y. Dharmendar Reddy et al $-\theta'(0)$	Present study	Y. Dharmendar Reddy et al $-\phi'(0)$	Present study
1	5	0.3	0.5	0.2	0.7752	0.775211	0.9315	0.931511
1.5					0.8879	0.887905	0.8834	0.883408
2.0					0.9520	0.952026	0.8556	0.855695
2.5					0.9932	0.9932	0.8377	0.837729
	2				1.1487	1.148738	0.1866	0.186633
	4				1.1345	1.134547	0.6090	0.609008
	6				1.1251	1.125105	0.9294	0.929461
	8				1.1180	1.118094	1.1970	1.197039
		0.0			1.1526	1.152639	0.8160	0.816009
		0.2			1.1369	1.136929	0.7899	0.789922
		0.4			1.1221	1.122122	0.7666	0.766686
		0.8			1.0944	1.094475	0.7228	0.722822
			0.1		1.3874	1.387408	0.6645	0.664571
			0.3		1.2406	1.240623	0.7293	0.729388



			0.5		1.1294	1.129463	0.7777	0.777731
			0.9		0.9695	0.969544	0.8459	0.845905
				0.1	1.1369	1.136911	0.7899	0.789911
				0.3	1.1221	1.122180	0.7660	0.766099
				0.5	1.1080	1.108088	0.7437	0.743739
				0.7	1.0944	1.094431	0.7228	0.722845

Figures 2, 3 and 4 disclose the consequences of the Williamson parameter  $\lambda$  on the profiles of velocity, temperature and concentration. It is noticed that growing values of  $\lambda$  diminishes the fluid velocity and increases the temperature and concentration distributions. Effect of magnetic parameter  $M$  on the velocity profile is displayed in Figure 5 by keeping the other parameters fixed. It is observed that an increase in magnetic field parameter decreases the velocity due to increasing magnetic field parameter improves the opposite force to the flow of nanofluid direction called 'Lorentz force' and this force opposes the motion of the nanofluid which results in decreases the velocity of the fluid. Figure 6 shows the influence of magnetic field parameter  $M$  on the thermal field. Transverse magnetic field has increased the thermal boundary layer thickness, so it causes the temperature increment in the boundary layer when it increases. It can be seen from Figure 7 the increase of magnetic parameter  $M$  increases the magnitude of concentration profiles. Figures 8, 9 and 10 are plotted to observe the effects of angle of inclination on velocity, temperature and concentration profiles respectively. It is clear from figures that the temperature and concentration profiles are increased with increase in inclined angle, this enhancement is due to the raise in the magnetic field parameter. The velocity profile decreases with the increase of aligned angle. It may happen with the increase in angle of inclination, causes to strengthen the applied magnetic field. Because of enhanced magnetic field an opposite force is produced to the flow, called Lorentz force. Consequently the velocity profile decreases. It is also seen that for the  $\gamma = 0$  the magnetic field is not effected on the velocity profile while maximum resistance is offered for the fluid particles when  $\gamma = \frac{\pi}{2}$ . The porosity parameter  $k$  is the dimensionless number which defines the level of porosity within a fluid. Figure 11, 12 and 13 illustrates the momentum, heat and mass profile for different values of porosity parameter  $k$ . With increasing permeability parameter, the resistance to the fluid motion increases and hence velocity decreases. For increased values of the porosity parameter  $k$ , temperature and concentration profiles increase, and the opposite behavior is seen in the velocity profile. Porosity parameter  $k$  grows a resistance force (due to the increase in the pores in the fluid) that works conversely to the flow field and enhances the thermal and solutal boundary layer thickness. Figure 14 demonstrates the variation of temperature with respect to radiation parameter  $R$ . When the Rosseland radiative absorptivity  $k^*$  decreases, the divergence of the radiative heat flux  $\partial q_r / \partial y$  increases, which leads to increases the rate of radiative heat transfer to the fluid at the surface, i.e., it provides more heat to the fluid which causes the fluid temperature to increase. Hence, the temperature profile as well as thermal boundary layer thickness increases as the value of thermal radiation increases. The variation in Prandtl number  $Pr$  on temperature is shown in Figure 15. The temperature field decreases when  $Pr$  increases. It is obvious that, an increase in the values of  $Pr$  reduces the thermal diffusivity, therefore thermal boundary layer thickness is a decreasing function of  $Pr$ . Figure 16 is demonstrated to examine the effect of  $Nc$  on the temperature profile. When the increment is done in the parametric value of heat capacity ratio, the curves of temperature profile gave an increasing behavior. So, temperature was increased with increasing the heat capacity ratio. In Figure 17, diffusivity ratio parameter  $Nbt$  is depicted to show its behaviour on concentration profile. A significant downfall in the curve of concentration profile is obtained when gradual increase is done in diffusivity ratio parameter  $Nbt$ .



Figure 18 is drawn to summarize the consequence of Lewis number  $Le$  on the temperature profile. The curve pattern shows that the increasing Lewis number markedly decreased the temperature profile due to a higher value of Lewis number represents a lower nanoparticle diffusivity and a higher thermal diffusivity. If  $Le > 1$ , the thermal diffusion rate exceeds the Brownian diffusion rate. Lower Brownian diffusion leads to less mass transfer rate, as a result, the nanoparticle volume fraction graph and the concentration boundary layer thickness decrease. Behavior of concentration profile regarding Schmidt number  $Sc$  is depicted in Figure 19. When parametrical values of  $Sc$  are boosted up, a significant decrease is resulted in concentration profile. The effects of velocity slip parameter  $A$  on the dimensionless velocity, temperature and concentration is displayed in Figures 20, 21 and 22 respectively. From the figures velocity decreases with increase in the values of slip parameter, velocity distribution is found to decrease along the boundary layer, but the effect reverse in the case of temperature and concentration profiles. Physically, in the presence of slip the slipping fluid shows a decrease in the surface skin friction between the stretching sheet and the fluid because not all the pulling force of the stretching surface can be transferred to the fluid. So the flow velocity decreases when the value of  $A$  increases. As the slip parameter increases in magnitude the friction force is occurred which allows more fluid to slip past the sheet, the flow slows down for distances close to the sheet and the temperature and concentration enhances due to the occurrence of the force. The influence of thermal jump parameter  $B$  on the temperature and concentration profiles is shown in Figures 23, 24 which describes that both the fluid temperature and concentration decreases on increasing thermal jump parameter in the boundary layer region. As the effect increases more fluid will be transferred to the thermal boundary layer. Hence, rate of heat and mass transfer will increase and as a result of this, the thermal and solutal boundary layer decreases. From Figure 25 we can observe the variation of concentration with respect to solutal slip parameter  $C$ . As it can be seen from the graph, increasing in the concentration slip parameter  $C$ , the concentration profile is decreasing.

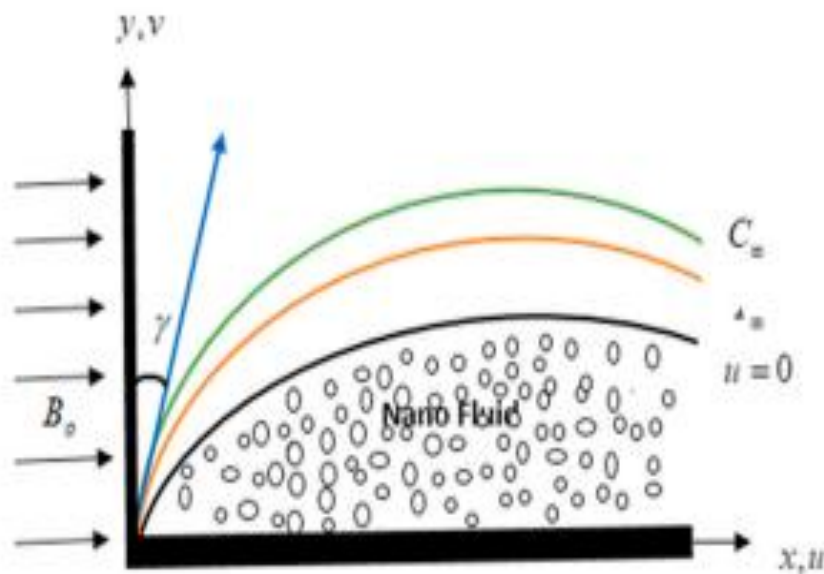


Fig.1. Physical model of the problem

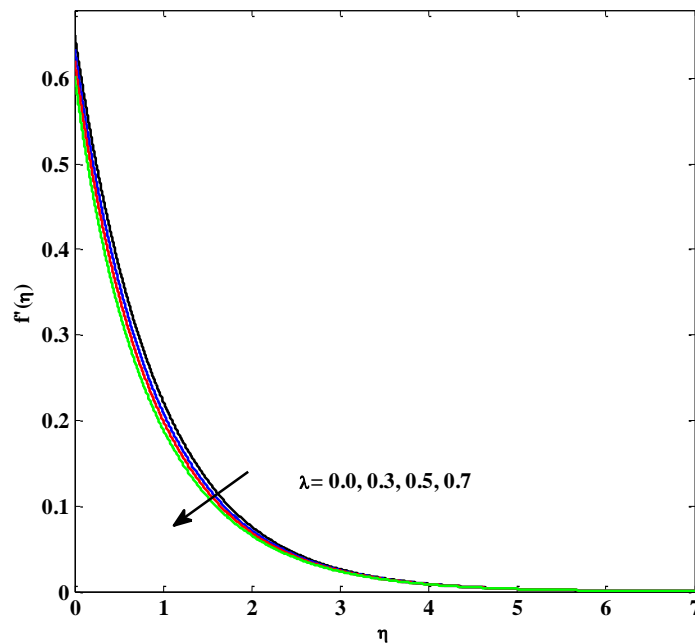


Fig.2.  $f'(\eta)$  for picked values of  $\lambda$

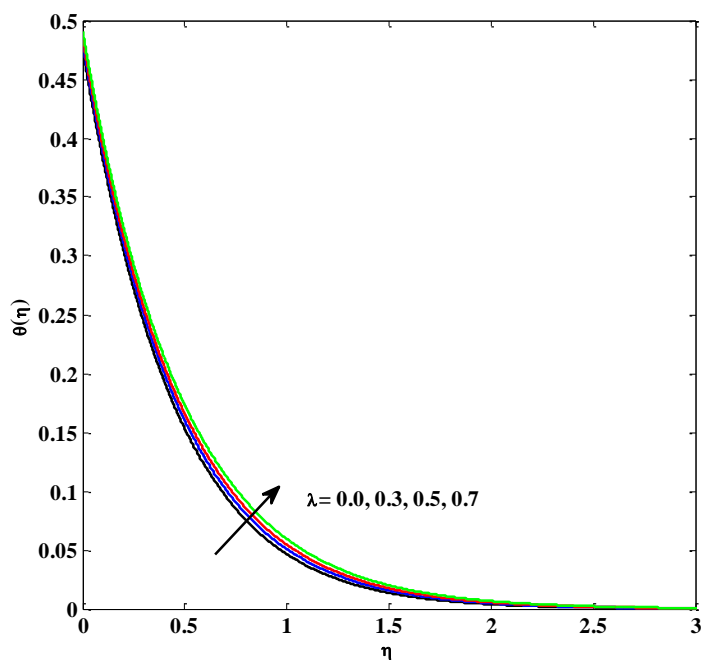


Fig. 3.  $\theta(\eta)$  for picked values of  $\lambda$

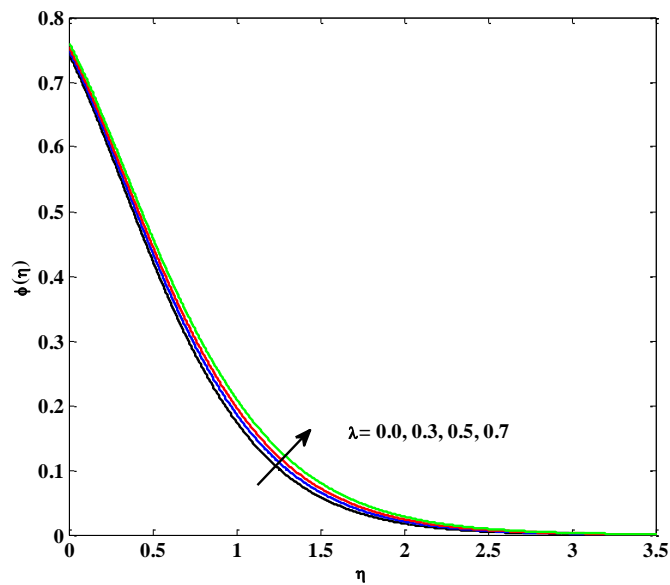


Fig. 4.  $\phi(\eta)$  for picked values of  $\lambda$

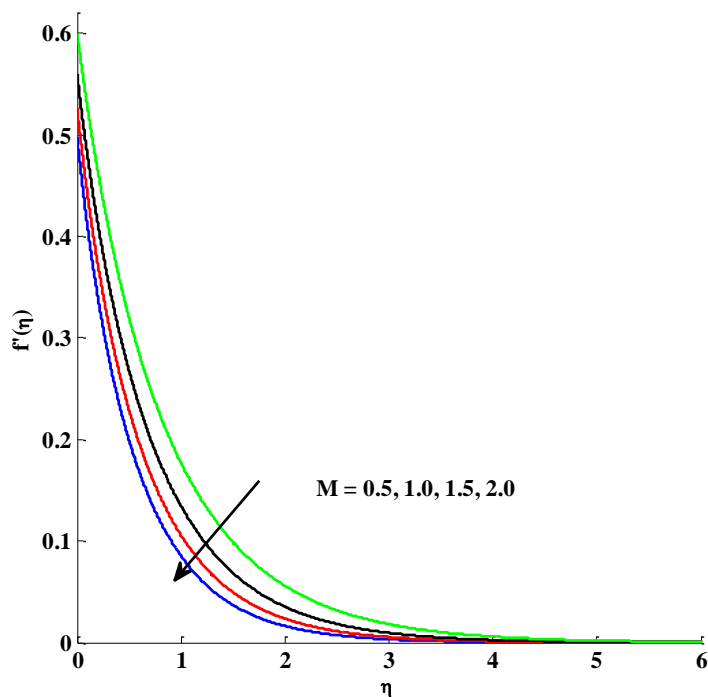


Fig. 5.  $f'(\eta)$  for picked values of  $M$

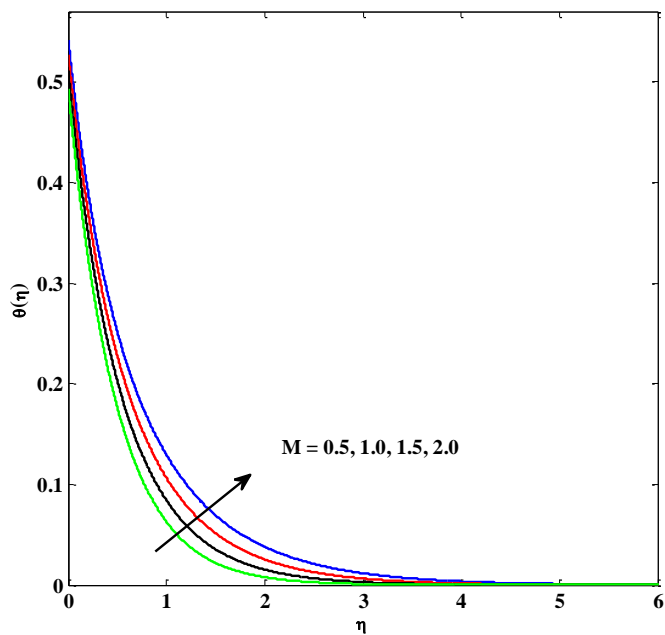


Fig. 6.  $\theta(\eta)$  for picked values of  $M$

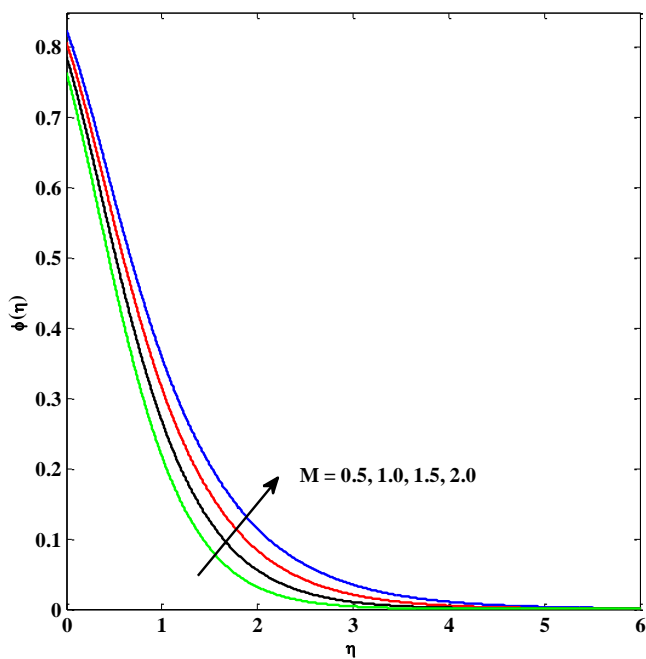


Fig. 7.  $\phi(\eta)$  for picked values of  $M$

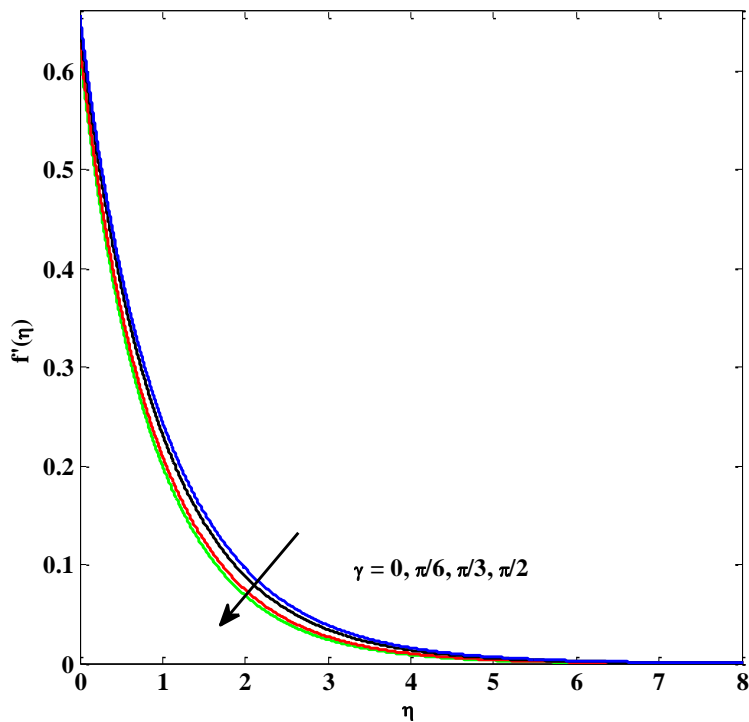


Fig. 8.  $f'(\eta)$  for picked values of  $\gamma$

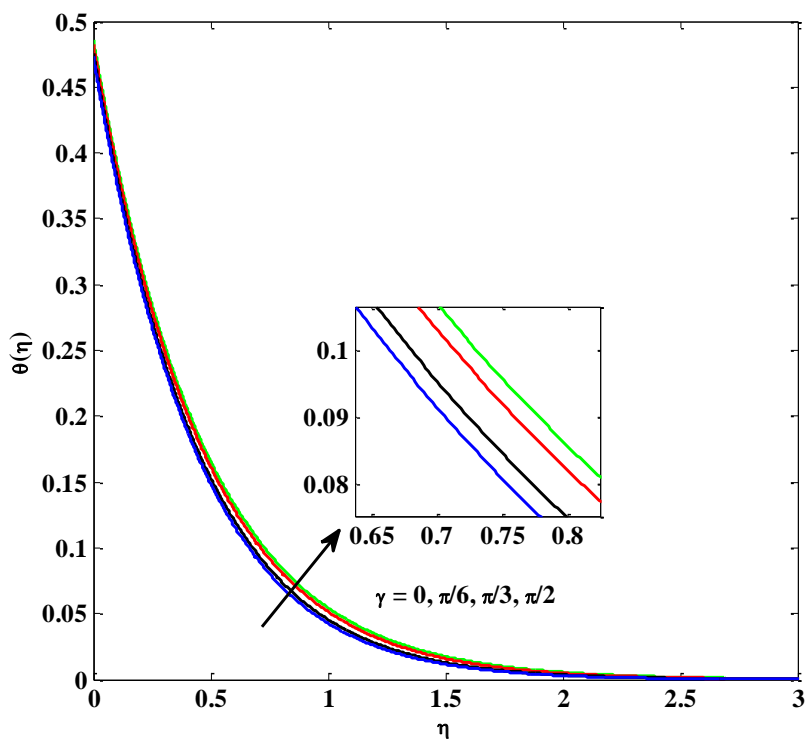


Fig. 9.  $\theta(\eta)$  for picked values of  $\gamma$

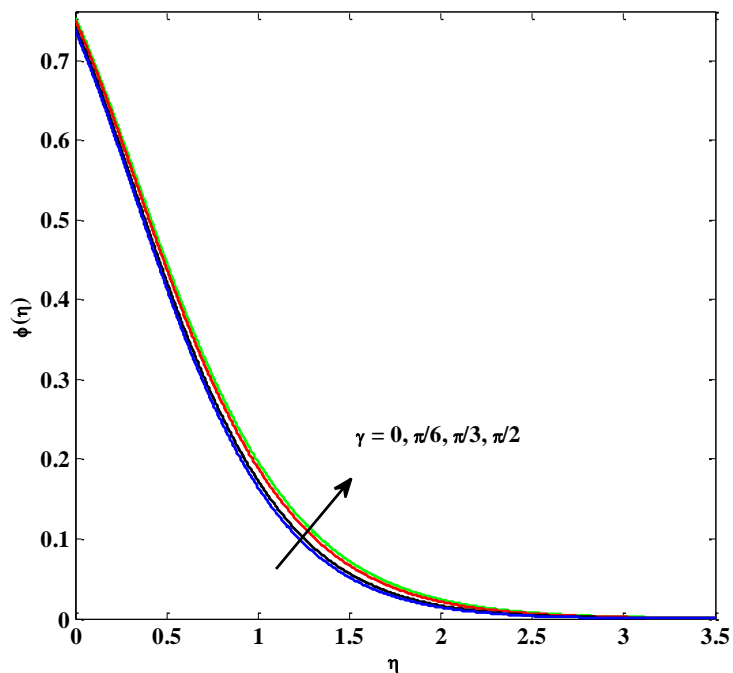


Fig. 10.  $\phi(\eta)$  for picked values of  $\gamma$

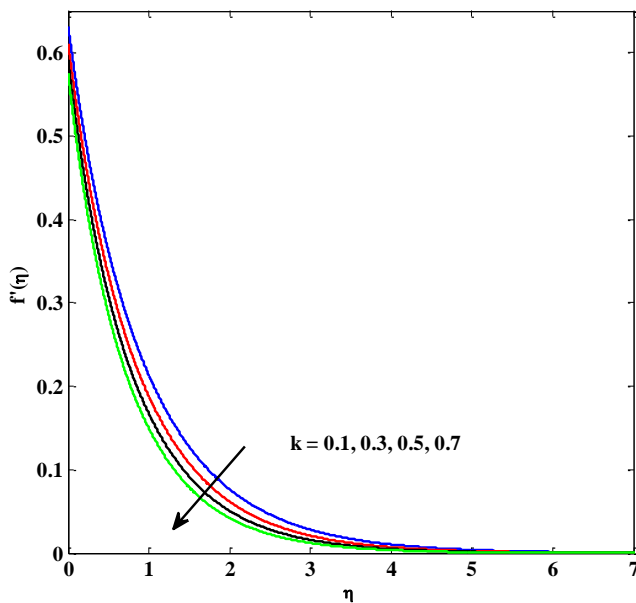


Fig. 11.  $f'(\eta)$  for picked values of  $k$

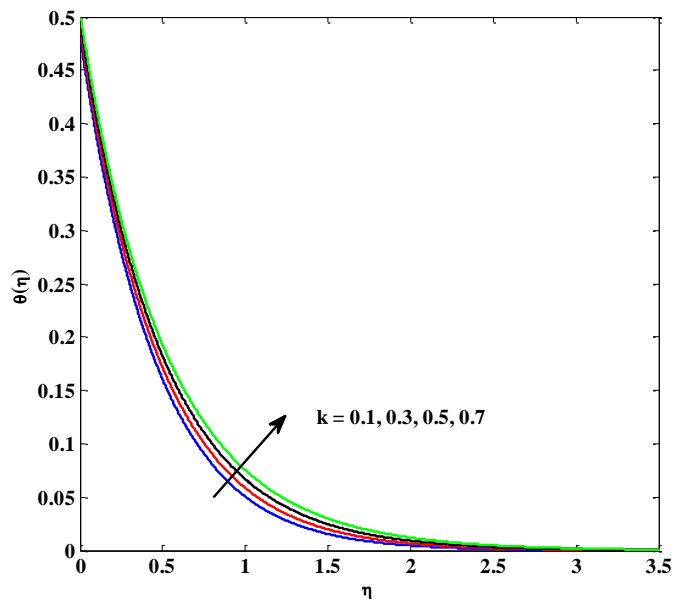


Fig. 12.  $\theta(\eta)$  for picked values of  $k$

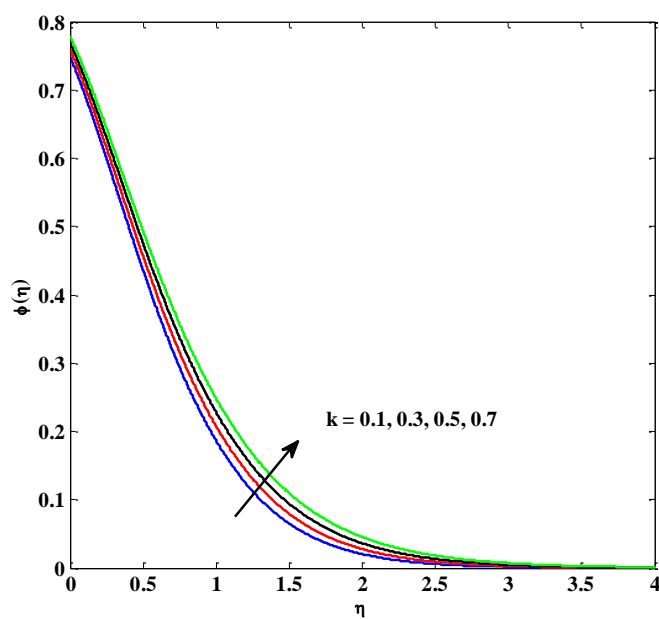


Fig. 13.  $\phi(\eta)$  for picked values of  $k$



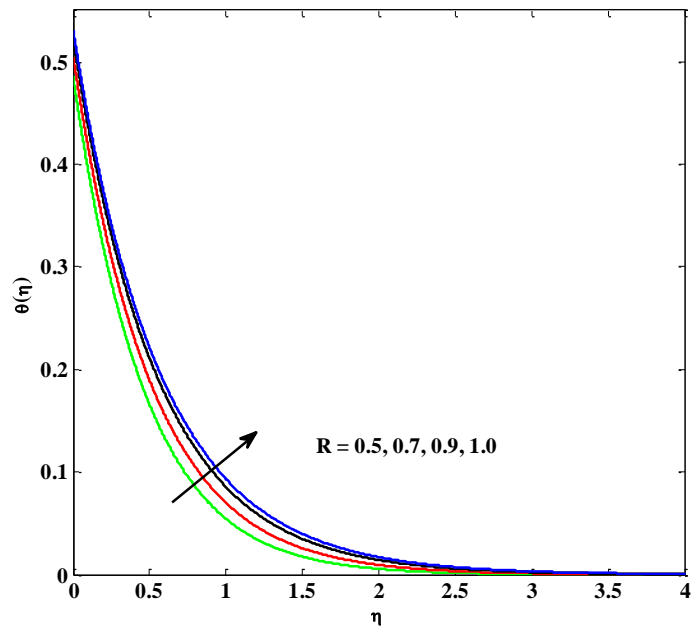


Fig. 14.  $\theta(\eta)$  for picked values of  $R$

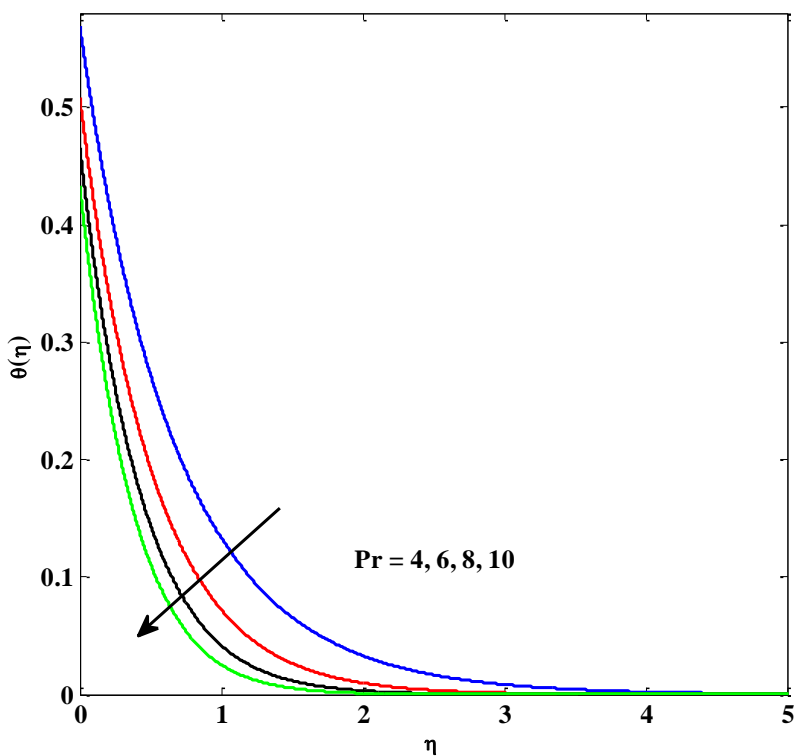


Fig. 15.  $\theta(\eta)$  for picked values of  $Pr$

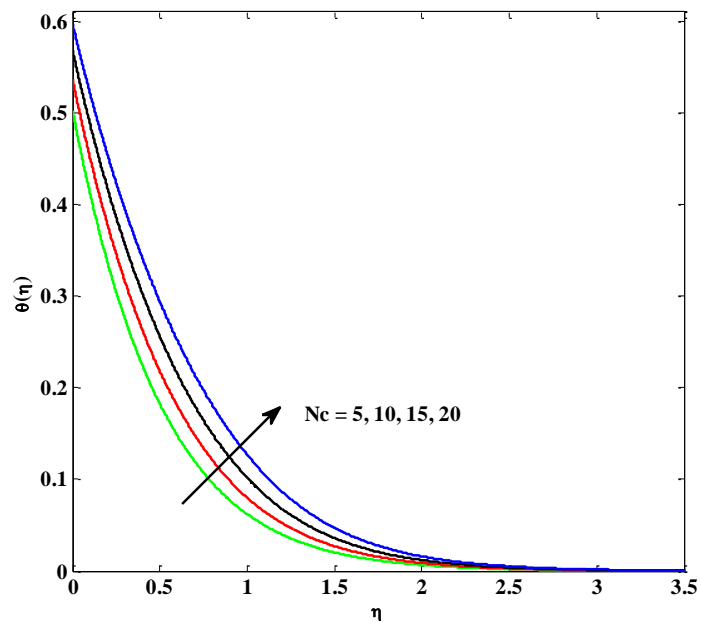


Fig. 16.  $\theta(\eta)$  for picked values of  $N_c$

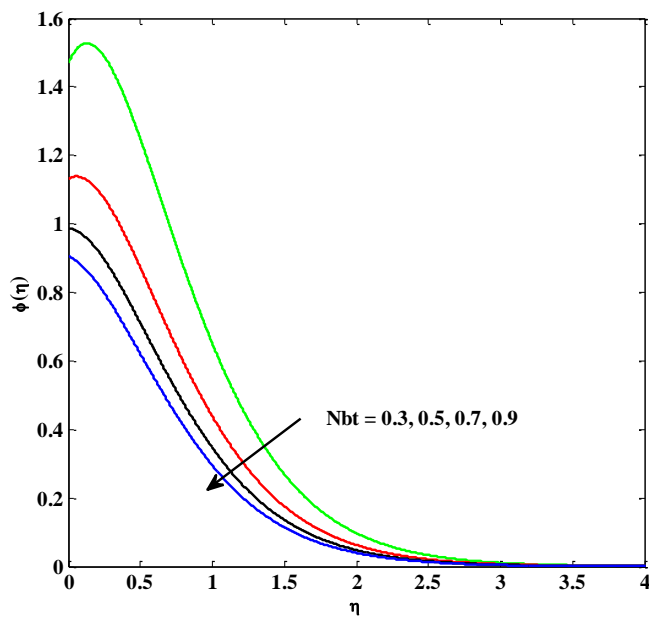


Fig. 17.  $\phi(\eta)$  for picked values of  $N_{bt}$

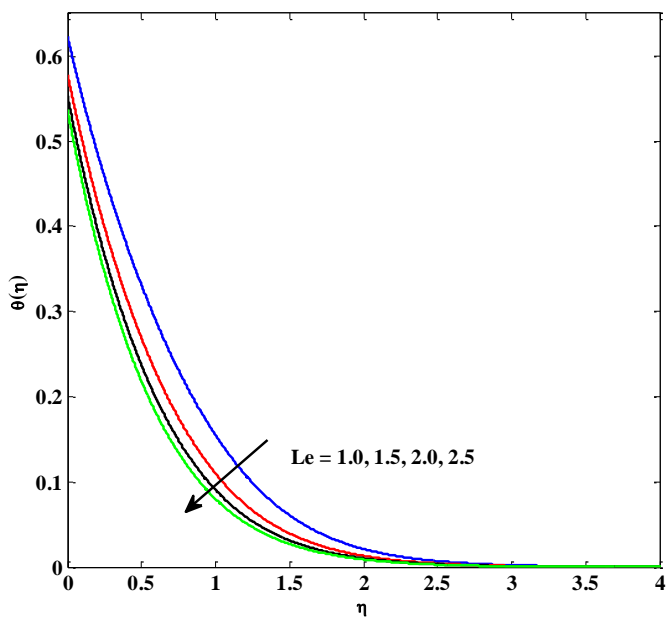


Fig. 18.  $\varphi(\eta)$  for picked values of  $Le$

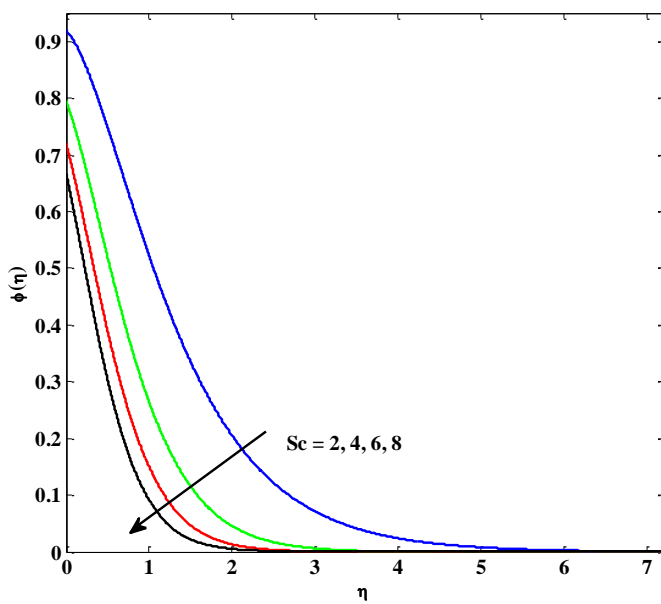


Fig. 19.  $\varphi(\eta)$  for picked values of  $Sc$

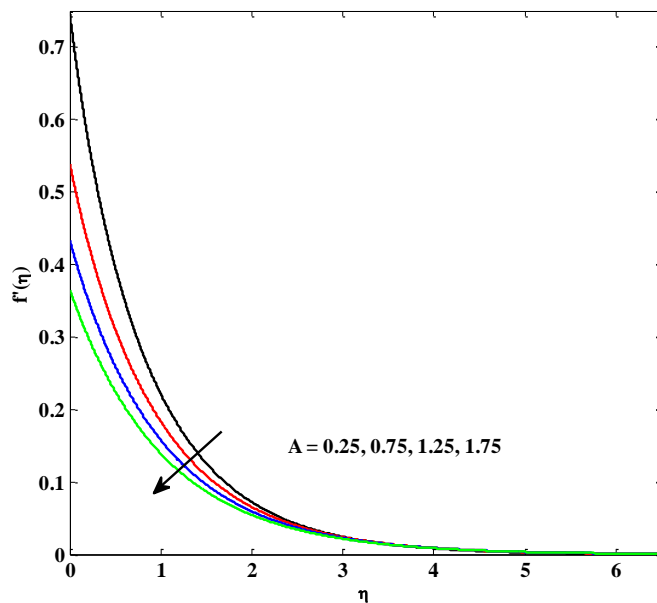


Fig. 20.  $f'(\eta)$  for picked values of  $A$

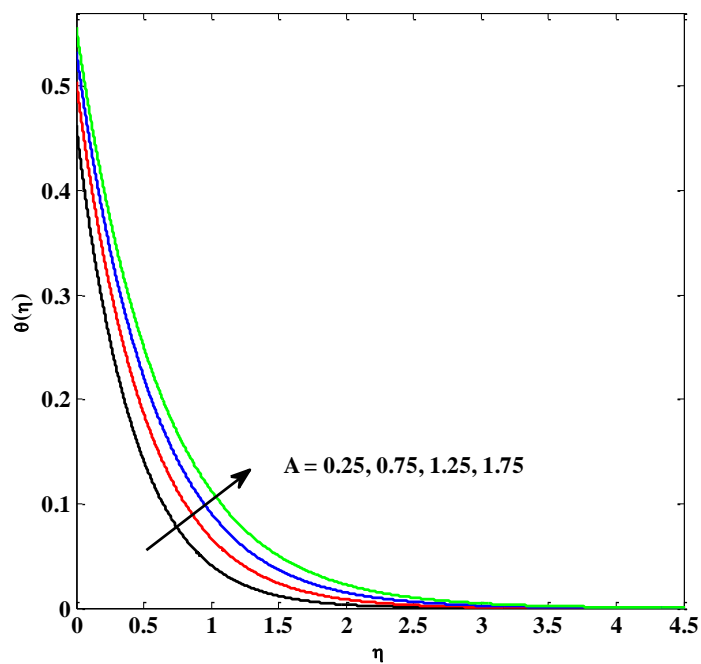


Fig. 21.  $\theta(\eta)$  for picked values of  $A$

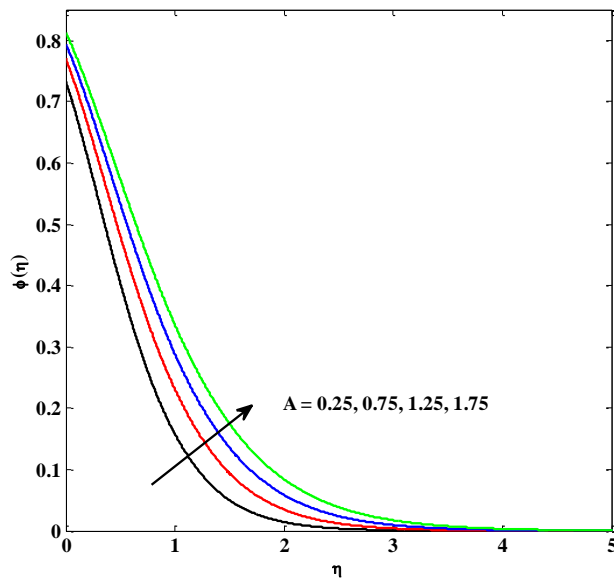


Fig. 22.  $\phi(\eta)$  for picked values of  $A$

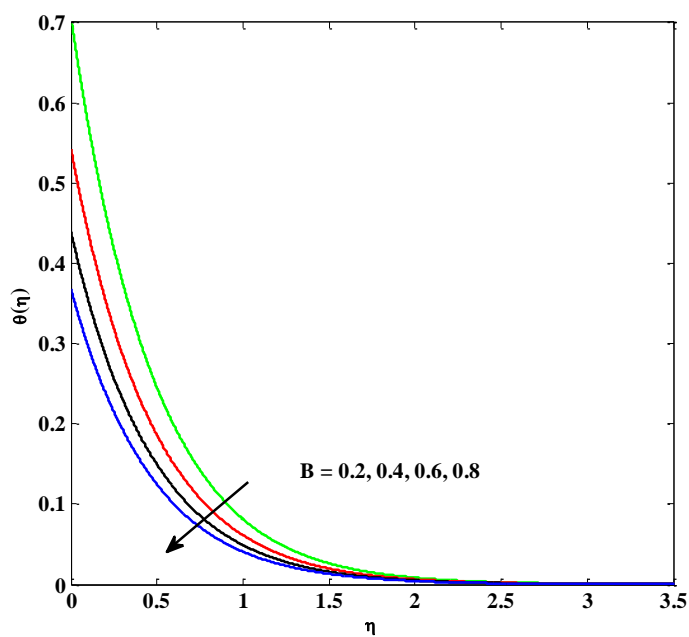


Fig. 23.  $\theta(\eta)$  for picked values of  $B$

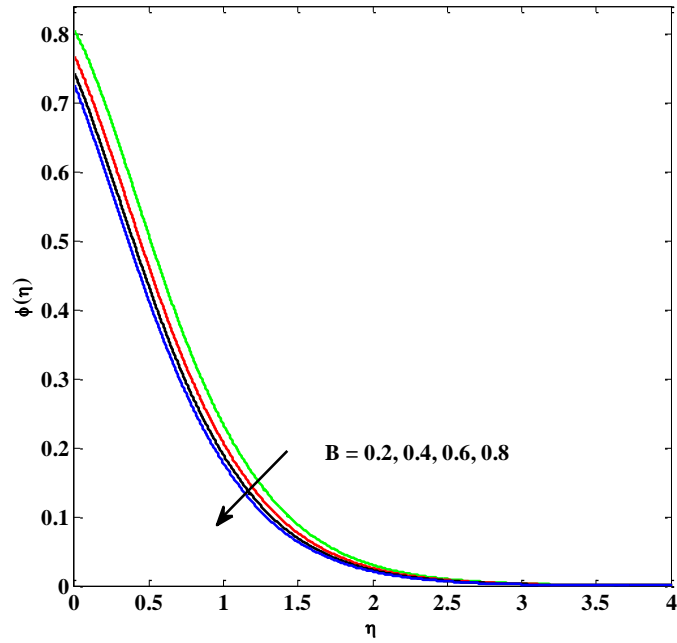


Fig. 24.  $\phi(\eta)$  for picked values of  $B$

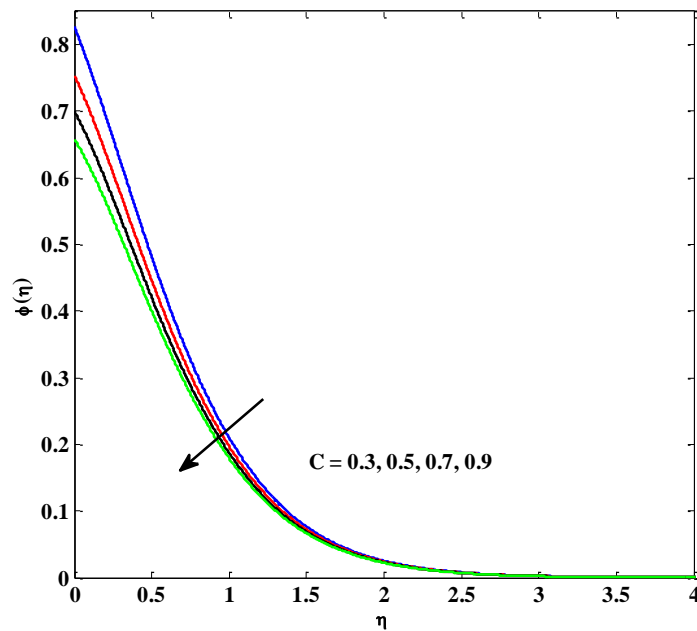


Fig. 25.  $\phi(\eta)$  for picked values of  $C$

FINAL REMARKS:

A numerical study of Williamson nanofluid with the effect of aligned MHD passing through a stretched surface and porous medium is performed in addition to the effects of heat radiation and slips. By Kellor-Box method the influence of several flow parameter on three fluid flow profiles is represented through graphically and the major findings are as follows:

- The velocity profile decreases and the temperature, concentration profiles are increased with increase in inclined angle  $\gamma$ .
- Both the fluid temperature and concentration decreases on increasing thermal jump parameter  $B$  and increasing in the concentration slip parameter  $C$ , the concentration profile is decreasing.
- While an increase in the value of  $A$  reduces the velocity but opposite result reflect in heat and mass transfer gradients.
- Velocity profile would make smaller while an increase in the values of  $M, k$ .
- Enhancing the values of Williamson parameter  $\lambda$ , a drop in velocity field is seen along with upgradation in temperature and concentration.
- The fluid temperature gradients diminish as enhanced values of  $Le, Pr$  and reverse trend in upsurge values of  $Nc, R$ .
- The fluid concentration gradient moderate as enhanced values of  $Sc, Nbt$ .

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