

# Effect of Source Voltage and Core Losses on Ferro-Resonance in Transformer

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**Abstract**—The applied source voltage and core losses of transformer play an important role in formation of ferro-resonance. A detailed analysis of many simulation results demonstrates that the possibility of ferro-resonance increases as the applied voltage increases or core losses of transformer decreases. The effect of source voltage and core losses on chaotic solution of the system has been studied. The resulting ferro-resonance over-voltage, over-current and highly distorted waveform with different mode of ferro-resonance are presented and critical value of supply voltage and core losses also calculated. With detailed analysis of many simulation results, to simulate ferro-resonance and to obtain Poincare maps, phase plane diagrams and bifurcation diagrams the Electromagnetic Transient Program (EMTP) and Matlab is used and results are found to be desired.

**Keywords**—Ferro-resonance, Bifurcation diagram, Phase plane, Poincare map, Single-pole switching, Transformer, power frequency spectrum, Non-linearly, saturation curve.

## Introduction

Ferroresonance is a type of nonlinear resonance characterized by overvoltage and over-current whose occurrence leads to highly distorted waveforms which can cause power quality problem and equipment damage in power distribution and transmission system. Also in simple words ferroresonance is an LC “resonance” involving a nonlinear inductance and a capacitance. For ferroresonance to occur in power networks, the system must have a voltage supply (generally sinusoidal), capacitance (due to lines), a nonlinear transformer or inductance coil (including saturable ferromagnetic materials), and low losses [3], [4]. Because of the nonlinearity in these systems, more than one mode of ferroresonance can occur. Ferroresonant voltages depend on the line capacitance, core losses and magnitude of the source voltage. At the same time, the nonlinear inductance characteristic which is represented by the transformer excitation curve can also affect the ferroresonant voltage. In most instances, ferroresonance occurs when one or two of the source phases are disconnect while the transformer is lightly loaded or unloaded. One of the fundamental properties of ferroresonance is the fact that several stable solutions can exist under steady-state conditions for a given circuit. For example, residual magnetization of the core, voltage magnitude at the time of

switching and the amount of charge present on the capacitance are all initial condition which determines the system’s steady state response. With a small difference in these initial conditions, the ferroresonance overvoltage can have very different waveforms.

In nonlinear dynamical systems, due to the nonlinearities inherent in the system, the behavior may not be predictable. Depending on the magnitude of the forcing function and on the system parameters, the system’s output waveform may be either periodic or non-periodic (chaotic). It can be shown that the transitions between periodic and non-periodic modes commonly occur due to small changes in circuit parameters or initial conditions. Therefore, newly-developed techniques for analysis of nonlinear dynamical system and chaos should now be evaluated for use with ferroresonance. Power system is characterized by nonlinear differential equations, and unusual and unexpected behavior has been observed in both simple and complex networks. Now a day, ferroresonance is a widely studied phenomenon in power systems involving saturable inductors, capacitors and low losses.

I. BASIC FERR-RESONANT CIRCUIT

Ferroresonance is a resonance consists of nonlinear inductance of transformer core, so the inductive reactance depends on frequency and on the magnetic flux density of an iron core coil (e.g. transformer iron core) as well. The inductive reactance of transformer is represented by the saturation curve of a magnetic iron core. Theoretically, this nonlinear inductance could be represented by two inductive reactance's, which is depend on whether the transformer working in normal zone or saturation zone according to the situation on the saturation curve.

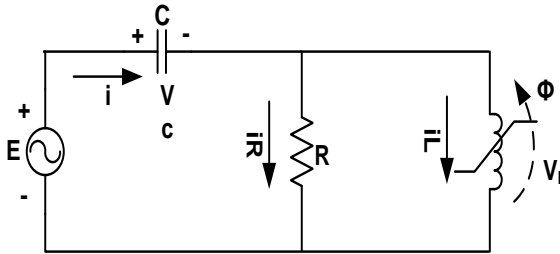


Fig.I Series Ferroresonance circuit

II. SYSTEM DESCRIPTION AND MODELING

Most distribution systems make wide use of grounded-wye to grounded-wye transformers to serve three phase loads. In modelling these systems, normally distribution line is represented by its RLC π-equivalent circuit, gang-operated switches and the three-phase circuit breakers are used at the starting point of substation where distribution lines originate. The distribution lines connecting with transformers to the system source through underground cables overhead lines [5]. However, overhead lines have less capacitance relatively large shunt capacitance of cables and for this reason ferroresonance mostly occur when system involves underground cables. At the end of a distribution line or at any point along the line three-phase or single-phase transformers can appear. Fig.2 shows a simplified schematic of such a system.

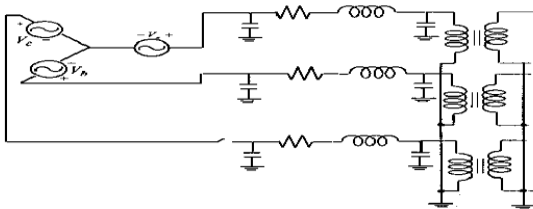


Fig. 2 Distribution system supplying a three phase load Through a grounded-star to grounded-star transformer

In this case, if only two phases of the transformer were energized and one of the three switches was open, it would leads to a voltage induced in the "open" phase. This induced voltage will "back feed" the distribution line, back to the open switch. if two phases open in a three-phase line because of single-pole switching and only one pole is closed; branch loop forms through phase capacitance and other phases transformers where source have the grounding as shown in Fig.2. The capacitance of cable between the open conductor

location and the transformer may have critical enough to cause ferroresonance at the operating voltage when transformers at lightly loaded or no-load and total losses presented in the circuit are low[1]. The network studied here consists of an unloaded power transformer with one of the supply conductors being interrupted with a three phase source feeding in Fig.2. The transformer supply power through the capacitive coupling with the other phases which is remaining connected. Fig.3 shows the circuits feed power the disconnected coil through the capacitive coupling and two phases one and two.

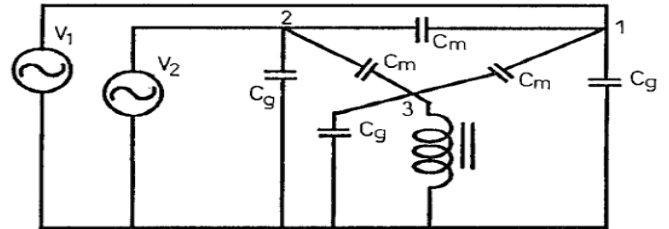


Fig. 3 The circuit feeds the disconnected phase

Thevenin's theorem is used to obtain an equivalent circuit to derive the mathematical equation of the above circuit. The equivalent capacitance can be calculated by shorting the two remaining phases 1 and 2. In doing this, both ground capacitance and transformer windings are shorted and which can be neglected. Because phases 1 and 2 having the same potential, so the mutual capacitance not carry any current connecting the nodes 1 and 2, and it can also be neglected as well. Therefore, the remaining equivalent circuit consists of one ground capacitance and two mutual capacitance which all connect between ground potential and node 3. Thus, the equivalent capacitance can be found as:

$$c = c_g + 2c_m$$

(1)

Next, the equivalent source voltage is derived. Assuming steady-state conditions before interrupting phase 3, the current  $i_3$  is supplied from the voltage source in the third phase. The amount of current which arrives at the transformer winding to contribute to the flux can be found by subtracting the fraction of the current that is lost due to the ground capacitance and the mutual capacitance, such that:

$$i_L = i_3 - i_g - i_1 - i_2$$

$$i_1 = j\omega c_m (v_3 - v_1)$$

$$i_2 = j\omega c_m (v_3 - v_2)$$

$$i_g = j\omega c_g v_3$$

$$i_L = i_3 - j\omega c_g V_3 - j\omega c_m (2V_3 - V_2 - V_1)$$

(2)

In equations 2,  $i_L$  stands for the current in phase 3, arriving at the transformer end of the line. Since the voltages  $V_1, V_2$  and  $V_3$  are of identical magnitude, each delayed by a  $120^\circ$  phase shift, the phasor addition of  $V_1, V_2$  and  $V_3$  is zero. Therefore, equation (2) can be written as:

$$i_L = i_3 - j\omega (c_g + 3c_m) V_3$$

(3)

The voltages V1, V2 and V3 differ only in their phase shift and they can be rotated by 120° without loss of generality. After the switch opening in phase 3, the source current goes to zero and equation (3) can be written as:

$$i_L = -j\omega(c_g + 3c_m)V_1$$

(4)

This current has to be equal to the current in the equivalent circuit of Fig.4

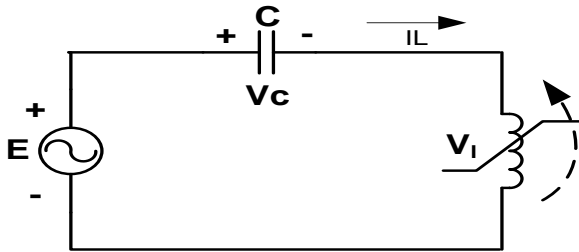


Fig. 4 Preliminary equivalent circuit

$$i_L = j\omega c(E - V_L)$$

(5)

Where,

$$c = c_g + 2c_m$$

Relating equation (4) and (5), the equivalent source E can be calculated as:

$$E = \frac{c_m}{(c_g + 2c_m)} V_1$$

(6)

During loaded conditions, the current flow in the secondary winding is immediately compensating the flux induced in the primary transformer winding so that the transformer core can be neglected. Normally, in the case of power transformer which is very lightly loaded or an unloaded, current can develop in the secondary side and this current causes the flux to originate from the transformer leg and flow in the iron core. So the transformer core losses increase, therefore losses can no longer be omitted. Fig.5 shows the finally reduced equivalent circuit of the transformer with the involving of R which represents the transformer core losses presented.

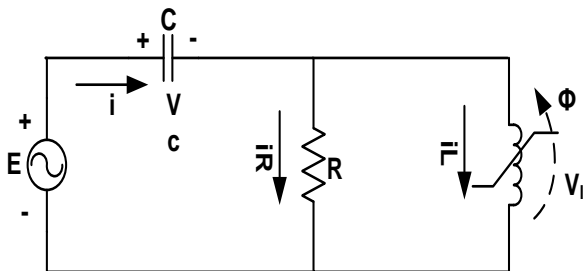


Fig. 5 Reduced equivalent circuit

In the peak current range for normal operation, the flux-current characteristic can be

Represented by a linear graph such as:

$$i = a\phi$$

(7)

Where the coefficient 'a' in the equation (7) corresponds to the reciprocal of the inductance of transformer core (a=1/L). However, the flux-current characteristic becomes highly non-linear for very high currents the iron core might be driven into saturation. In this study the flux-current  $\phi-i$  characteristic of the transformer is modeled by an eleventh-order polynomial indicated by red color in Fig.6 given as:

$$i = a\phi + b\phi^{11}$$

(8)

Where

$$a = 2.8 \times 10^{-3}$$

$$b = 7.2 \times 10^{-3}$$

'I' represent the current in Pu value; and

' $\phi$ ', represent the flux in the transformer core in Pu value.

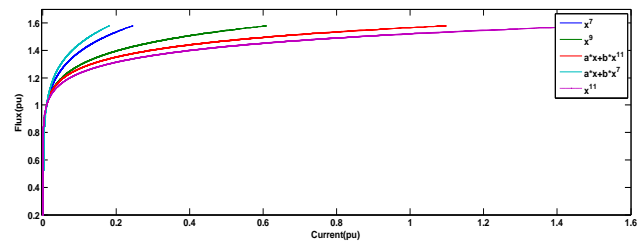


Fig. 6 Approximation of the saturation region of the test transformer

In the eleven order polynomial, coefficient 'b' of equation (8) along with only two terms of polynomial are chosen so that the saturation region can plot effectively or best fit. Fig.6 shows the different approximations of the saturation region and the true magnetization characteristic that was developed by Dick and Watson. Generally less than eleven orders is used to represent the magnetization curve for small capacity transformers, but for higher capacity transformer more than eleven order polynomial because to satisfy the magnetization characteristic of modern high-capacity transformers it does not bend sharply enough at the knee point. During ferroresonance with the operating point of the transformer located in the saturated region, a single valued curve can be used for the representation of the magnetization curve. For the core materials used in modern high voltage power transformers, hysteresis loops are not significant. The voltage across the transformer winding of Fig.4 can be defined as

$$V_L = \frac{d\phi}{dt} \quad \& \quad i_R = \frac{V_L}{R}$$

Knowing all the branch current along with  $\phi-i$  characteristic of the transformer, one can write the differential equation for the circuit of Fig.5 as:

$$\frac{d^2\phi}{dt^2} + \frac{1}{RC} \frac{d\phi}{dt} + \frac{1}{C} (a\phi + b\phi^{11}) = \sqrt{2}E\omega \cos(\omega t)$$

(9)

Where,

$\omega$  Is the frequency, 1pu; and

E is the RMS value of the Thevenin voltage applied to the equivalent circuit of Fig.4

A. Abbreviations and Acronyms

- 1, 2, 3 number of phase
- 'a' coefficient for linear part of magnetizing curve
- 'b' coefficient for no- linear part of magnetizing curve
- C linear capacitance (total capacitance)
- Cm mutual capacitance
- Cg ground capacitance
- E instantaneous value of driving source
- L non-linear magnetizing inductance of transformer
- R core loss resistance
- W angular frequency of driving force

B. Parameters considered for simulation

Parameter	Actual value	per unit value
Sbase	25MVA	-----
Vbase	63.5Kv	-----
Ibase	131A	-----
Rbase (1%loss)	484ohm	-----
R	48.4Kohm	100p.u
C	777nF	0.14177p.u
W	377(rad/sec)	1p.u

III. VARYING THE SOURCE VOLTAGE (E)

For the first set of simulations, considering the effect of varying the magnitude of the equivalent source voltage on the behavior of equation (9) was considered. The length of the transmission line was fixed at 100km, the transformer core losses were assuming(fixed) at 1% of the rated transformer capacity and the initial conditions were kept constant at  $\phi$

$\phi(0) = 0.0$  and  $\dot{\phi}(0) = \sqrt{2}$  pu. This initial condition was chosen (maximum voltage and zero flux) because it is the one that most frequently occurs in a circuit breaker when the current extinguishes. If the current goes through zero (flux zero) and the circuit load is inductive, the voltage is at a maximum, and for a 1pu. Voltage, the peak value is at  $\sqrt{2}$  Pu. A bifurcation diagram is used to predict which modes of ferroresonance may occur during a wide range of the magnitude of the source voltage,. The bifurcation diagram is a plot of the magnitude taken from a family of Poincare sections vs. the system parameter that is being varied [6]. Fig.7 shows the bifurcation diagram when magnitude of the source voltage (E) is varying. [11]

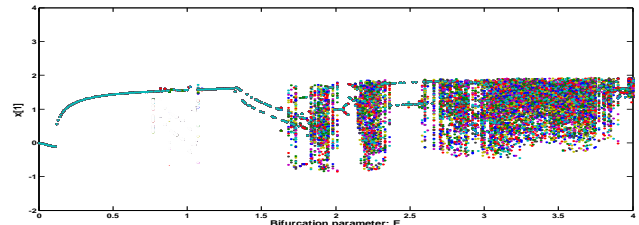


Fig. 7 Bifurcation diagram with varying E

The above bifurcation diagram shows for a given value of the source voltage possible modes of ferroresonance. In order to find the critical values of E where the type of solution changes, E was varied from 0.15pu. to 4pu. Up to a voltage of 1.3pu, the response was a period-one motion at the fundamental frequency [7], [8]. Fig.8 shows phase plane diagram and Fig.9 indicates Poincare map, Fig.10 indicate the frequency-power spectrum having fundamental frequency component and its odd harmonics for this period-one region.[8] In these diagrams x[1] denotes the flux and x[2] indicates the voltage . For the frequency-power spectrum, with  $(w) = 1.0pu$ .

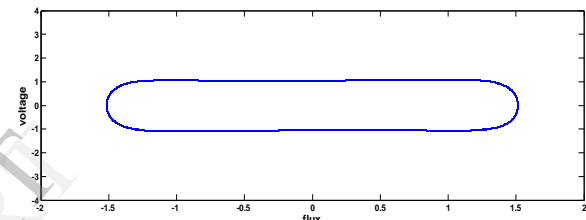


Fig. 8 phase-plane diagram for period-one motion

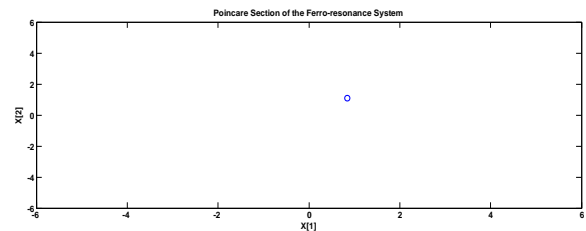


Fig. 9 Poincare map of period one motion

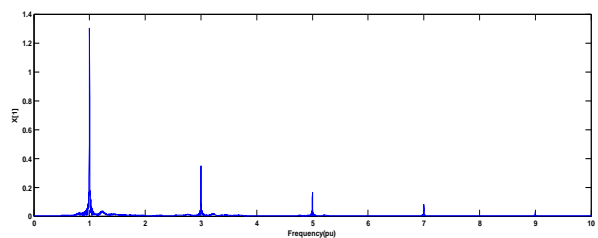


Fig. 10 Frequency power spectrum for period one motion

At the point when E was increased to 1.31 Pu. A bifurcation occurs and a period two waveform was observed. Fig.11, Fig.12 and Fig.13 shows output voltage, the Poincare map and the phase plane trajectory respectively for this period doubling, in which the two points in the Poincare map represent two distinct frequencies in the output waveform.

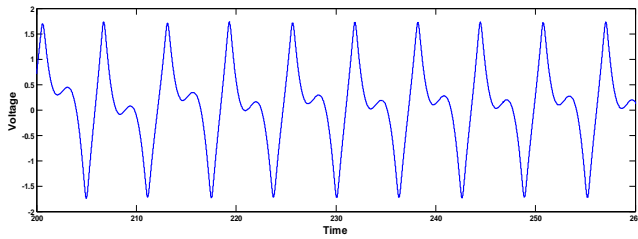


Fig. 11 Output voltage with double frequency component

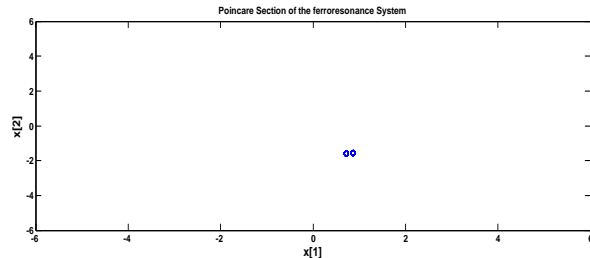


Fig. 12 Poincaré map of period two- motion

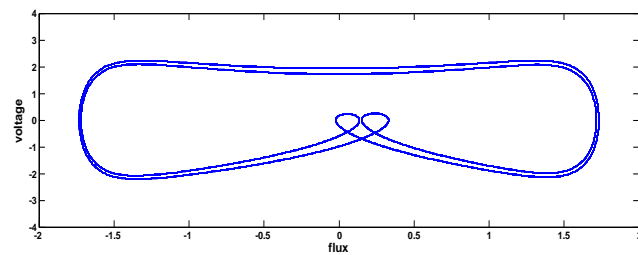


Fig. 13 Phase plane for period two motion

At the point when E was increased to 1.87 pu. a second bifurcation occurs and a period four waveform was observed. Fig.14 and Fig.15 shows Poincaré map and phase plane trajectory respectively for this period four waveform.

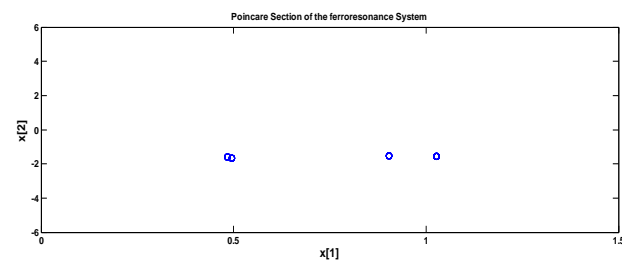


Fig. 14 Poincaré map of period four- motion

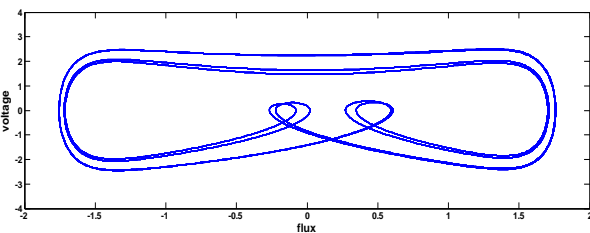


Fig. 15 Phase plane for Period four motion

Further small increments in the forcing voltage lead to a decaying bifurcation curve that reaches the first chaotic region at E = 1.9 Pu. Fig.16 shows output voltage with high magnitude and sustain level of distortion leads to power

quality problem. Fig.17 and Fig.18 shows the Poincaré map and phase plane diagram respectively for this chaotic region.

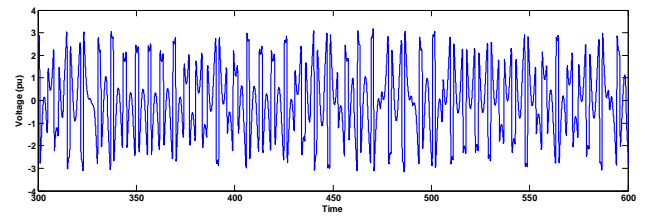


Fig.16 Output voltage with irregular frequency components

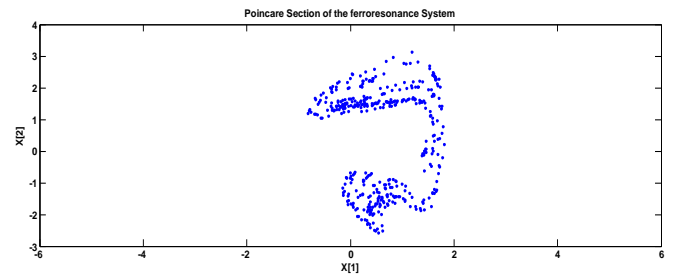


Fig.17 Poincaré map of chaotic solution

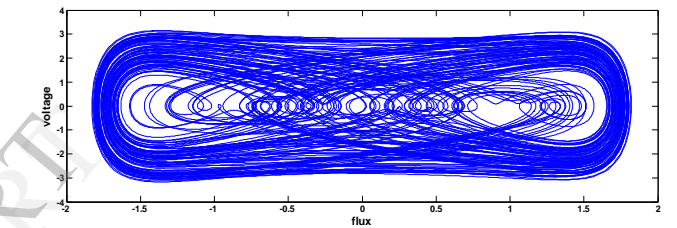


Fig.18 Phase plane for chaotic motion

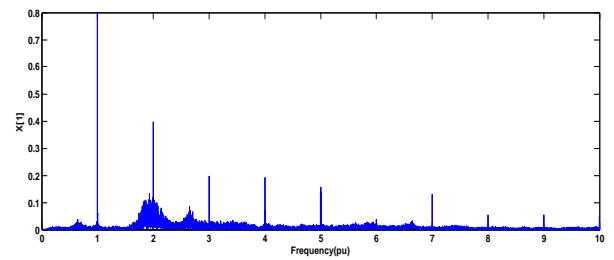


Fig.19 Frequency power spectrum for chaotic motion

During ferro-resonance output voltage has odd and even harmonics with high magnitude which cannot be ignored is shown in Fig.19

#### IV. VARYING THE TRANSFORMER CORE LOSSES(R)

For the second set of simulations, the behavior of equation (9) was studied by varying the transformer core losses R and the effect of core loss on ferro-resonance mode. In this case, the transmission line was taken as 100km and the equivalent source voltage was fixed at E=0.15pu. (Corresponding to V=1.0pu at the supply voltage). The initial conditions were kept

constant at  $\phi(0) = 0.0$  and  $\dot{\phi}(0) = \sqrt{2} .Pu$ . Ferroresonant steady-state response was of period one with the transformer core losses at 1% (R =48.4) Fig.20 and Fig.21 shows phase plane and Poincaré



map respectively for period one motion. As the transformer losses were decreased the ferroresonant responses of period two and three were observed. The first period-tripling occurred at  $R=0.005\%$ . Fig.22 and Fig.23 shows the phase-plane and Poincare map diagram of this period-three response when the transformer losses are at  $0.005\%$  ( $R = 9.68M\Omega$ ).

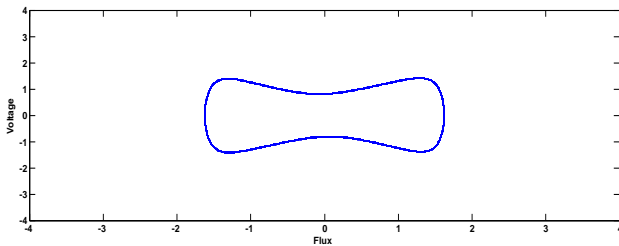


Fig. 20 Phase plane for period one motion  $R=1\%$

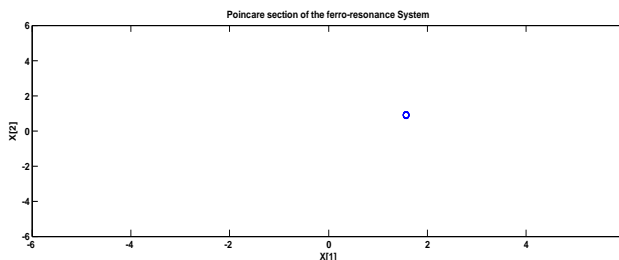


Fig. 21 Poincare map for period one

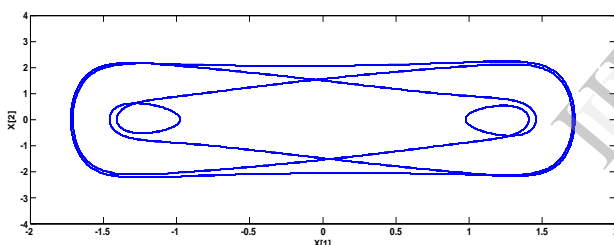


Fig. 22 Phase plane for period three motion  $R=0.005\%$

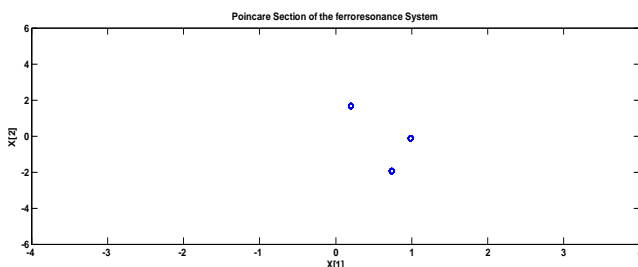


Fig. 23 Poincare map for period three motion

Reducing the transformer core losses below  $0.001\%$  ( $R = 48.4M\Omega$ ) introduced chaotic behavior. Any further decreasing core loss the response reached into the stable chaotic region. Fig. 24 and Fig. 25 shows the phase plane and Poincare map of the chaotic region respectively when the losses are at  $0.001\%$  and the value of the driving voltage  $E=0.15pu$ .

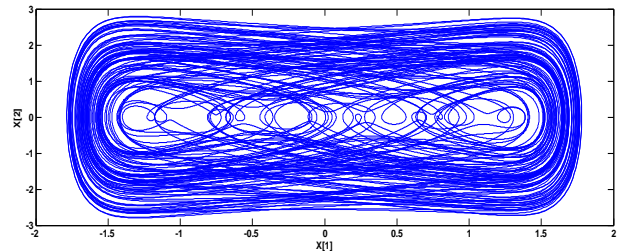


Fig. 24 Phase plane for chaotic solution

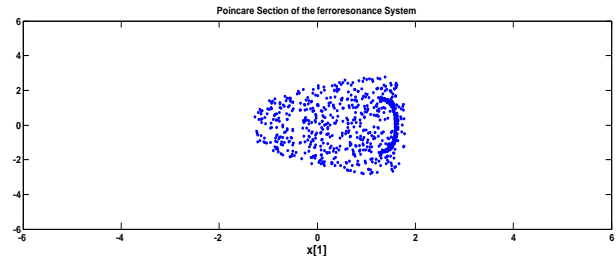


Fig. 25 Poincare map for chaotic motion

As the value of the transformer core losses was decreased, a smaller source voltage was needed to drive the system into the chaotic region. The above result indicates the importance of the insertion of the transformer core losses in ferroresonance studies. Even small change in the transformer core losses can make the difference between the chaotic behavior and ordinary ferro-resonance or periodic operation. Therefore, the recent trend of decreasing the core losses of power transformers increases the possibility of chaotic behavior in transformer [9],[10].

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#### CONCLUSION

Ferroresonance generally refers to nonlinear oscillations in power system involving the nonlinear inductance with a linear capacitance in series. Ferroresonant voltage depends on the magnitude of the source voltage, length of the transmission line, losses, initial conditions and nonlinear inductance characteristics. The aim of paper is to investigate ferro-resonance from the standpoint of nonlinear dynamics and chaotic system and chaos are evaluated for means for studying ferroresonance to determine whether the potential for chaos exists in power networks. The equivalent circuit of a typical ferroresonance circuit in a power system presented along with the set of differential equation that describes it. The parameter of the ferroresonant model and their values all calculated. Different modes of behavior for the ferroresonant circuit were found by varying the value of the source voltage ( $E$ ) and core loss ( $R$ ). The critical values of the source voltage and core loss which drive the ferroresonant circuit into

chaotic region found. Phase plane diagrams, used to categorize ferroresonant behavior. As the applied voltage is gradually increased, or core loss gradually decrease the response goes into a chaotic region.

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