Effect of Specimen Dimensions on Flexural Modulus in a 3-Point Bending Test

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Abstract

Flexural modulus is one of the important properties of a material which is a measure of stiffness of the material. Several standards are available in literature for the specimen dimensions and loading for the estimation of flexural modulus of isotropic and reinforced composites. A range of values are specified for length, width and thickness of three-point bend specimen in the available standards. Present investigation aims in finding the effect of the specimen dimensions on flexural modulus of isotropic and FRP composite materials using three-dimensional finite element analysis. The problem is modelled in ANSYS software and the flexural modulus evaluated from the transverse deflection is compared with the actual material property given as input to solve the problem. The effect of physical dimensions of the specimen on percentage error of Young's modulus is discussed.

1. Introduction

Two different test methods viz. 3-point bend and 4-point bend are suggested in most of the literature for prediction of flexural modulus of isotropic and fiber reinforced composites. Out of these, 3-point bend is simple to design but ambiguous in the result as the flexural formula used to find the modulus is derived from mechanics of materials theory that is based on several assumptions such as plane cross section remains plane after deformation, structure is under pure bending etc.

Some of the standards (ASTM D790M-93[1], CRAG[2], and ISO-14125[3]) specifying the specimen dimensions are listed in Tables 1 and 2. The ASTM specifications allow a wide freedom of choice in terms of specimen dimensions, as long as the cross-section is rectangular and specific span to thickness ratio (s/h) ratios (16:1, 32:1, 40:1 and 60:1) in both three-point and four point bending.

CRAG three-point bend loading arrangement requires a particular laminate

thickness of 2mm and specifies (s/h) ratios dependent on layup and type of fiber used [4].

Table.1Dimensional possibilities for flexurespecimens in several specifications

Specification	Thickness	Width	Length
-	(mm)	(mm)	(mm)
ASTM D790 M	1-25	10-25	50-1800
CRAG	2	10	100

ISO-14125 appears to be a combination of above two and Table 2 gives specimen dimensional possibilities for 3-point bending tests in this standard.

Table.2 Recommended specimen dimensions for different material types for three-point flexure in ISO-14125

Material	Length	Span	Width	Thickness
	(mm)	(mm)	(mm)	(mm)
Class I	80	64	10	4
Class II	80	64	15	4
Class III	60	40	15	2
Class IV	100	80	15	2

In Table 2, Class I: discontinuous fiber-reinforced thermoplastics. Class II: mat, continuous mat, fabric and mixed format reinforced plastic. DMC (dough moulding compound), BMC (bulk moulding compound) and SMC (sheet moulding compound). Class III: transverse (90) unidirectional composites. Unidirectional (0) and multi directional composites with $5 < E_{11}/G_{13} \le 15$ (for example, glass fibres systems). Class IV: unidirectional (0) and multidirectional composites with $15 < E_{11}/G_{13} \le 50$ (for example, carbon-fibre systems).

Bending properties (strength and modulus) of woven composites were determined experimentally using 3-point and 4-point bend tests according to JIS K 7055 [5] The dimensions recommended are length 80mm, width 26mm, depth 3.5mm. Tensile and bending properties of $[0/\pm 45/90]$ s woven FRP composites are investigated by Khashaba and Seif [6].

In the present work, 3-point bend problem of isotropic and unidirectional FRP composite materials is simulated using three-dimensional finite element method in ANSYS 12 software for different geometries to assess the effect of geometry in accurate prediction of flexural modulus.

2. Problem Modelling

2.1 Geometric Modelling

Case (i): The width and depth are taken as 25mm, 3 mm respectively and span of the model varies as 70, 100, 150, 200, 300, 400mm.

Case (ii): The span of the specimen is taken as 70mm and the depth equal to 3 mm. The width is varied as 25, 20, 16, 12.5, 5mm.

Case (iii): The length, width, depth are taken as 70mm, 25mm, 3mm respectively for different values of E_1/E_2 ratio. E_1/E_2 values are taken as 1, 2, 4, 8, 10, 15, 20, 25, and 30.

Figure 1 shows a sample FE model for 3-point bend test.



Figure 1 Geometry and FE mesh for a 3-D model

2.2 Finite Element Modelling

SOLID 20 node 95 element of ANSYS software [7] is used. This element has three degrees of freedom at each node.

2.3 Boundary conditions and Loading

Simply supported beam conditions are taken for beam. A load of 1N is applied in negative z direction of the beam at the centre.

2.4 Material Properties

The following material properties are used.

E = 200 GPa and v = 0.25 for isotropic material. Carbon FRP with 63% volume fraction possessing following properties is considered for orthotropic case [8].

$$E_1 = 147 \text{ GPa}; E_2 = E_3 = 10.3 \text{ GPa}$$

 $v_{12} = v_{13} = 0.27; v_{23} = 0.54$
 $G_{12} = G_{12} = 7.0 \text{ GPa}; G_{22} = 3.7 \text{ GPa}$

In case of composite material with variable E_1/E_2 , E_1 is varied as per the assumed ratio by keeping other properties same.

3. Discussion of Results

The results are obtained by changing the dimensions of the beam. The transverse deflection, Uz is evaluated from the successful execution of the ANSYS software after conducting several convergence tests. From the Uz value obtained, the Young's modulus (E) is calculated using Euler-Bernoulli beam equation. The obtained value and the original value of Young's modulus are compared and the error is plotted for both isotropic and composite materials.

3.1 Convergence & Validation of FE model

A beam of length 70mm, width 25mm, and depth 3 mm is taken. Mesh refinement is done by changing the number of divisions and the deflection Uz is calculated using FEM. The results obtained are plotted on a graph (figure 2). It is observed that for an element size of 1.75 mm and below the output is constant. Thus the quality of mesh is fixed for this element edge length.

To validate the mesh quality, a uniaxial stress of 10 MPa is applied on a model of length 70mm, width 25mm, and depth 3 mm and axial deflection is noted down and the longitudinal Young's modulus is determined. It is obtained as 200MPa which is the input value. Thus the FE model is validated.



Figure 2 Convergence test results

From the figure 3, which is drawn between the length of the beam and %error in Young's modulus (E) for an isotropic material, the following observations are made.

- With increase in 'length' there is a decrease in the % error of the Young's modulus 'E'
- At a span of 70 mm the error is around 2.5%, and at a span of 350mm it is almost zero.

From the figure 4, which is drawn between the width of the beam and %error in Young's modulus (E) for an isotropic material, the following observations are made.

- With the decrease in width there is a decrease in the % error of the Young's modulus 'E'
- At a width of 25mm the %error is above 2% and at a width of 5 mm there is an error of 0.5%.

From the figure 5 which is drawn between the span of the beam and % error in Young's modulus (E_1) for composite material, the following observations are made.

- With the increase in 'length' there is a decrease in the % error of the Young's modulus 'E₁'
- At a span of 70 mm the error is around 8.26%, and at a span of 400mm it is 0.46%

From the figure 6 which is drawn between the length of the beam and % error in Young's modulus (E_2) for composite material, the following observations are made.

- With the increase in 'length' there is a decrease in the % error of the Young's modulus 'E₂'
- At a span of 70 mm the error is around 0.535%, and at a span of 400mm it is 0.246 %.

From the figure 7, which is drawn between the width of the beam and % error in Young's modulus (E₁) for a composite material, the following observations are made.

• With the decrease in width there is a decrease in the % error of the Young's modulus 'E₁'

• At a width of 25mm the %error is above 8.26% and at a width of 5 mm there is an error of 5.45%.

From the figure 8, which is drawn between the width of the beam and % error in Young's modulus (E₂) for a composite material, the following observations are made.

• With the change in width there is no change in the % error of the Young's modulus 'E₂' i.e., there is no effect of change in width on E₂.

From the figure 9 which is drawn between the E_1/E_2 and % error in Young's modulus (E_1) for composites material, the following observations are made.

- With the increase in ratio 'E₁/E₂' there is an increase in the % error of the Young's modulus 'E₁'
- At a ratio of '30' the error is around 14.2%, and at a ratio of '1' it is 1.33%

From the figure 10 which is drawn between the ${}^{4}E_{1}/E_{2}{}^{2}$ and % error in Young's modulus (E₂) for composite material, the following observations are made.

- With the increase in ratio 'E₁/E₂' there is a decrease in the % error of the Young's modulus 'E₂'
- From ' E_1/E_2 '4 to 15 it is almost constant.
- At a ratio of '30' the error is around 14.2%, and at a ratio of '1' it is 1.33%



Figure 3 Variation of % error in 'E' with respect to 'length' for an isotropic material



Figure 4 variation of % error in E with respect to 'width' for an isotropic material



Figure 5 Variation of % error in (E_1) with respect to (length) for a composite material



Figure 6 Variation of % error in (E_2) with respect to (length) for a composite material



Figure 7 Variation of % error in (E_1) with respect to width' for a composite material



Figure 8 Variation of % error in (E_2) with respect to width' for a composite material



Figure 9 Variation of % error in E_1 with respect to E_1/E_2 for a composite material



Figure 10 Variation of % error in E_2 with respect to E_1/E_2 for a composite material

4. Analysis

To find out the reasons for error in flexural modulus, the variation of axial deformation across the height of the beam is plotted for two different cases, one where there is a lot of error $(E_1/E_2=30)$ for a beam of length 70mm, width 25mm and depth 3mm and other where there is no significant error $(E_1/E_2=2)$ for a beam of length 250mm, width 12.5mm and depth 3mm in figures 11 and 12.

In the figure 11 it is observed that the graph obtained is an equation of cubic order. But by Euler's assumptions the graph obtained should be linear. Thus the Euler's assumption of plane crosssection remains same after deformation has failed which resulted in error.

In the figure 12 it is observed that the graph obtained is a linear equation. By Euler's assumptions the graph obtained should be linear. Thus the Euler's assumption is true for lower values of E_1/E_2 'ratio for a beam of length 250mm, width 12.5mm and depth 3mm.



Figure 11 variation of Ux along the depth of a composite material for $(E_1/E_2) = 30$



Figure 12 variation of Ux with respect to depth of a composite material for $E_1/E_2=2$

5. Conclusions

Three-dimensional finite element simulation is made for a 3-point bend test to find the effect of geometrical dimensions on flexural modulus obtained from this test using Euler's bending formula. The analysis is carried out for isotropic and carbon FRP materials and the percentage deviation in transverse modulus due to change in geometry is evaluated. The following conclusions are drawn.

- Span to depth ratio should be more than 50 for a fixed width of 25mm.
- The width of the specimen should be smaller than 5mm for a fixed span to depth ratio of 70/3.
- In case of composite materials where the percentage deviation is more, though the design of 4-point bend experiment is relatively complex, the flexural modulus from this test may give better results as there exists pure bending in the structure.

6. References

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