Effects of Heat Source/Sink on Stagnation Point Flow over A Stretching Sheet

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***Abstract-*Aim of the paper is to investigate the characteristics of a non linear mathematical model of MHD stagnation point flow over a stretching plate. The boundary layer equations with the convective boundary conditions are transferred by a similarity transformation into a system of non linear ordinary differential equations and solved numerically by using fourth order Runge-Kutta integration scheme with shooting method. Numerical results are obtained for velocity, temperature distributions and skin friction coefficient. Further features of the flow and heat transfer for various values of the Prandtl number, stretching parameter and magnetic parameter are analysed and presented through graphs and tables.**

Keywords- Stagnation point flow, Stretching plate, MHD, Heat Transfer, Similarity transformation.

INTRODUCTION

During the past few decades there has been a growing interest to investigate the convective boundary layer flow of fluids in a continuous moving surface because of its extensive applications to many engineering and industrial problems, particularly, aerodynamic extrusion of plastic sheets, polymer, spinning of fibres, cooling of elastic sheets, plasma studies, petroleum industries, MHD power generator, cooling of nuclear reactors and so on. The quality of final product depends on the rate of heat transfer and therefore cooling procedure has to be controlled effectively. Stagnation point flow is a topic of significance in fluid mechanics, because of stagnation point appears in virtually of all flow fields of science and engineering. In some cases, the flow is stagnated by a solid wall, while in others a free stagnation point or a line exists interior of the fluid domain. Carne (1970) was the first to study the convective boundary layer flow over a stretching sheet. Gupta (1977) studied heat and mass transfer on a stretching sheet with suction or blowing. Chen and Char (1988) reported heat transfer of a continuous stretching surface with suction or blowing. Sharma and Jat (1994) considered flow and heat transfer between two vertical plates under viscous fluid injection through porous plate, the other being stretching sheet. Ishak, Nazar and Pop (2006) obtained mixed convection boundary layers in the stagnation point flow towards a stretching vertical sheet. Hayat , Abbas and Sajid (2007) discussed analytic solution of MHD flow of a second grade fluid over a shrinking sheet. Sharma and Singh (2008) presented effect of viscous dissipation and heat source/sink in steady MHD flow near a stagnation point on linearly stretching sheet subject to different thermal boundary conditions. Wang (2008) obtained stagnation flow towards a shrinking sheet. Sharma and Singh (2008) obtained effects of variable thermal conductivity and heat source/sink on MHD flow near a stagnation point on a linearly stretching sheet. Pal (2009) considered heat and mass transfer in stagnation point flow towards a stretching surface in the presence of buoyancy force and thermal radiation. Sharma and Singh (2010) investigated steady MHD natural convection flow with variable electrical conductivity and heat generation along an isothermal vertical plate. Patil, Chamkha and Roy (2012) discussed effects of chemical reaction on mixed convection flow of a polar fluid through a porous medium in the presence of internal heat generation. Yacob and Ishak (2012) studied stagnation point flow towards a stretching/shrinking sheet in a micropolar fluid with a convective surface condition. Mahapatra and Nandy (2013) obtained stability of dual solutions in stagnation-point flow and heat transfer over a porous shrinking sheet with thermal radiation.

Aim of the paper is to investigate stagnation point flow of an incompressible viscous steady fluid over a stretching plate in the presence of magnetic field of uniform strength and heat source or sink.

Mathematical formulation

Consider the steady two dimensional MHD stagnation point flow of an incompressible viscous fluid over a stretching plate which is subjected to a convective boundary condition. It is assumed that the external velocity $U(x)$ and the stretching velocity $u_w(x)$ are of the forms $U(x) = ax$ and $u_w(x) = bx$, respectively where a and b are constants. A magnetic field of uniform strength B is applied normal to the plate in the y-direction, which produces magnetic effect in the x-direction. A heat source or sink is placed within the flow to allow for possible heat generation or absorption effects. Under the above assumptions, the governing equations are

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = U\frac{dU}{dx} + \upsilon\frac{\partial^2 u}{\partial y^2} - \frac{\sigma B^2}{\rho}u, \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2}+Q\left(T-T_\infty\right), \tag{3}$$

where u and v are the velocity components along the x and y directions, respectively, $\upsilon\left(=\frac{\mu}{\rho}\right)$ is the kinematic viscosity, μ is the coefficient of viscosity, σ is the electrical conductivity, ρ is the density of the fluid, T is fluid temperature inside the thermal boundary layer, T_∞ is the fluid temperature in the free stream, α is the thermal diffusivity and Q is the volumetric rate of heat source or sink.

The boundary conditions are

$$\begin{aligned} &y=0: \quad u=u_w(x), \qquad v=0,\\ &\qquad -K\frac{\partial T}{\partial y}(x,0)=h_f\left(T_f-T(x,0)\right)\\ &y\to\infty: \quad u=U(x), \quad T\to T_\infty \end{aligned} \tag{4}$$

where K is the thermal conductivity, h_f is the heat transfer coefficient and T_f is the temperature of the hot fluid. Introducing the following similarity variables and functions

$$\begin{aligned} &\Psi=(\upsilon xU)^{1/2} f(\eta), \quad \theta(\eta)=\frac{T-T_\infty}{T_f-T_\infty},\\ &\eta=\left(\frac{U}{\upsilon x}\right)^{1/2} y, \quad u=axf'(\eta),\\ &v=-(a\upsilon)^{1/2} f(\eta), \end{aligned} \tag{5}$$

into the equations (2) and (3), we get

$$f'''+ff''-Mf'-(f')^2+1=0, \tag{6}$$

$$\frac{1}{\Pr}\theta''+f\theta'+S\theta=0, \tag{7}$$

where, Ψ is the stream function, $M\left(=\frac{\sigma B^2}{\rho a}\right)$ is the magnetic parameter, $S\left(=\frac{Q}{a}\right)$ is the heat generation parameter, $\Pr\left(=\frac{\upsilon}{\alpha}\right)$ is the Prandtl number and prime denotes differentiation with respect to η.

The boundary conditions are reduced to

$$\begin{aligned} &f(0)=0, \qquad f'(0)=\varepsilon,\\ &\theta'(0)=-\gamma[1-\theta(0)],\\ &f'(\infty)\to 1, \ \theta(\infty)\to 0. \end{aligned} \tag{8}$$

where $\varepsilon=\frac{b}{a}\geq 0$ is the stretching parameter. Further, $\gamma=\frac{h_f}{k}\left(\frac{\upsilon}{a}\right)^{1/2}$ is the conjugate parameter for the convective boundary condition. It is noticed that $\gamma=0$ is the insulated plate and $\gamma\to\infty$ is when the surface temperature is prescribed.

Skin friction Coefficient

The shearing stress at the surface is given by

$$\tau_w=-\mu\left(\frac{\partial u}{\partial y}\right)_{y=0}. \tag{9}$$

The skin friction coefficient at the surface, is defined as

$$C_f=\frac{\tau_w}{\rho U^2} \tag{10}$$

$$\Rightarrow C_f \operatorname{Re}^{1/2}=-f''(0), \tag{11}$$

where $\operatorname{Re}=\frac{xU}{\upsilon}$ is the Reynolds number.

Heat transfer Coefficient

The rate of heat transfer at the surface is given by

$$q_w=-\kappa\left(\frac{\partial T}{\partial y}\right)_{y=0}, \tag{12}$$

where κ is thermal conductivity of the fluid.

The Nusselt number is defined as

$$Nu_x=\frac{x}{\kappa}\frac{q_w}{\left(T_f-T_\infty\right)} \tag{13}$$

$$\Rightarrow \frac{Nu_x}{\operatorname{Re}^{1/2}}=-\theta'(0) \tag{14}$$

The governing non-linear boundary layer equations (6) and (7) are solved numerically using Runge- Kutta fourth order integration scheme with shooting integration technique. The numerical values of skin friction and heat transfer coefficients at the surface are derived for different values of physical parameters and presented through Tables. The effects of physical parameters on the velocity and temperature profiles are shown through graphs.

Results and Discussion

Effects of physical parameters on velocity profiles

Figure 1 indicates the influence of the stretching parameter on velocity profiles. It is observed that velocity profiles increases as stretching parameter increases. It is seen from figure 2 that as magnetic parameter (M) increases, the velocity profiles decrease. As magnetic parameter (M) increases, the Lorentz force which opposes the flow also increases leads to enhance deceleration of the flow.

Effects of physical parameter variations on temperature profiles

It is noted from Figure 3 that as stretching parameter increases, the temperature decreases, and the thermal boundary layer thickness also decreases. It is seen from figure 4 that thermal boundary layer thickness increases with increasing values of the magnetic parameter. The temperature profiles increase as the conjugate parameter γ increases as seen from figure 5. Figure 6 depicts, that as heat generation parameter increases, the temperature profiles increase.

It is observed from figure 7 that thermal boundary layer thickness decreases with the increase of Prandtl number. From a physical point of view, if Prandtl number increases, the thermal diffusivity decreases and this phenomenon lead to the decreasing of energy ability that reduces the thermal boundary layer.

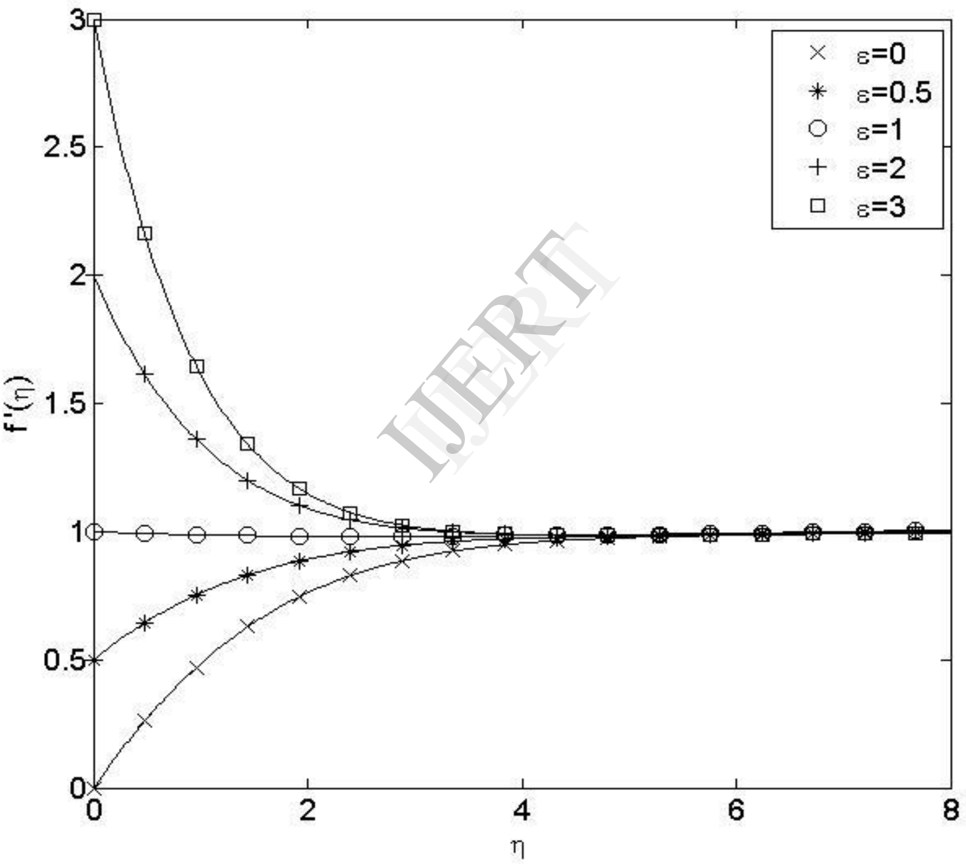

Figure 1. Velocity distribution versus η when $\Pr = 0.72, S = 0.5, \gamma = 1, M = 0.05.$

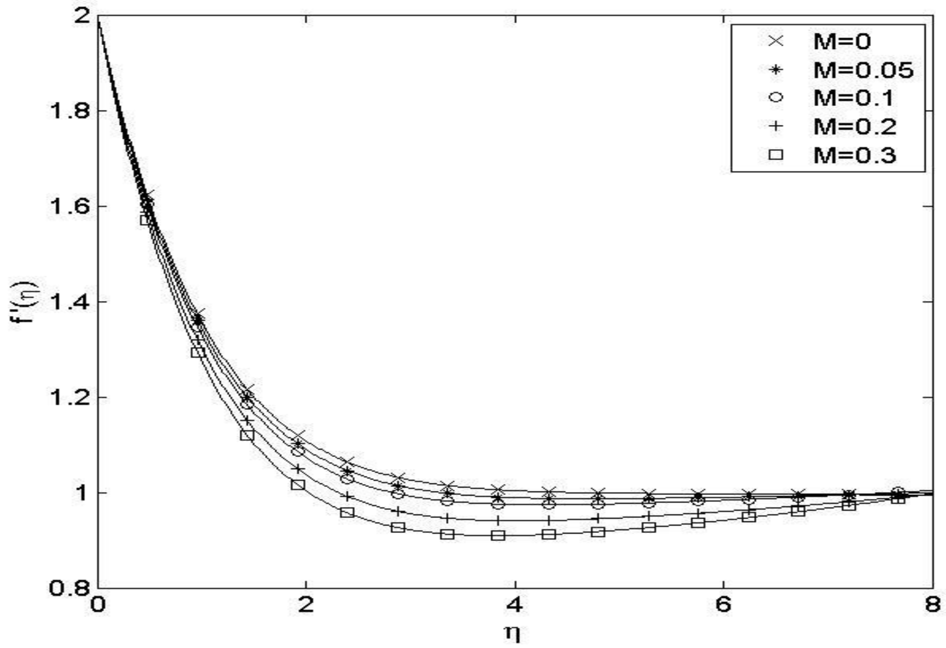

Figure 2. Velocity distribution versus η when $\Pr = 0.72, S = 0.5, \gamma = 1, \varepsilon = 2.$

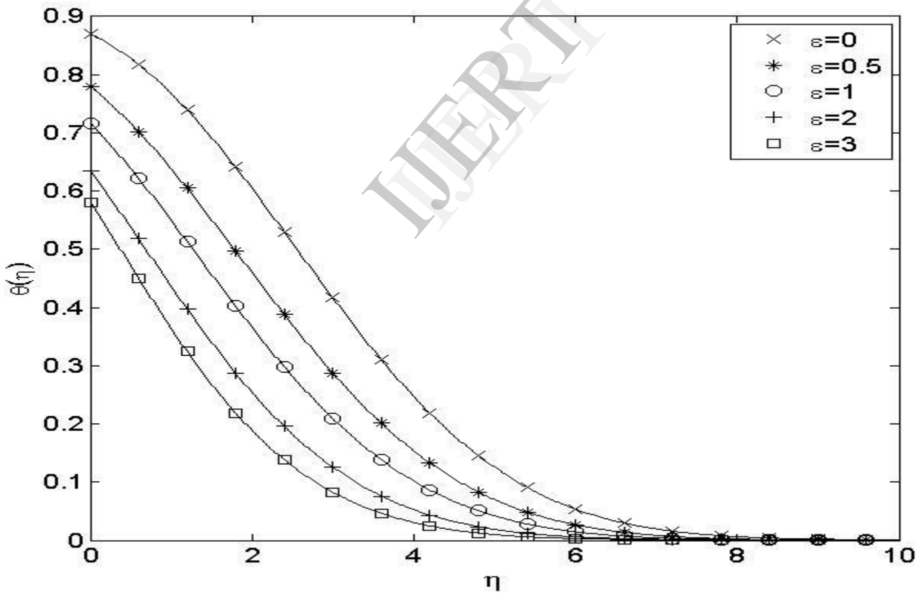

Figure 3. Temperature distribution versus η when $\Pr = 0.72, S = 0.5, \gamma = 1, M = 0.05.$

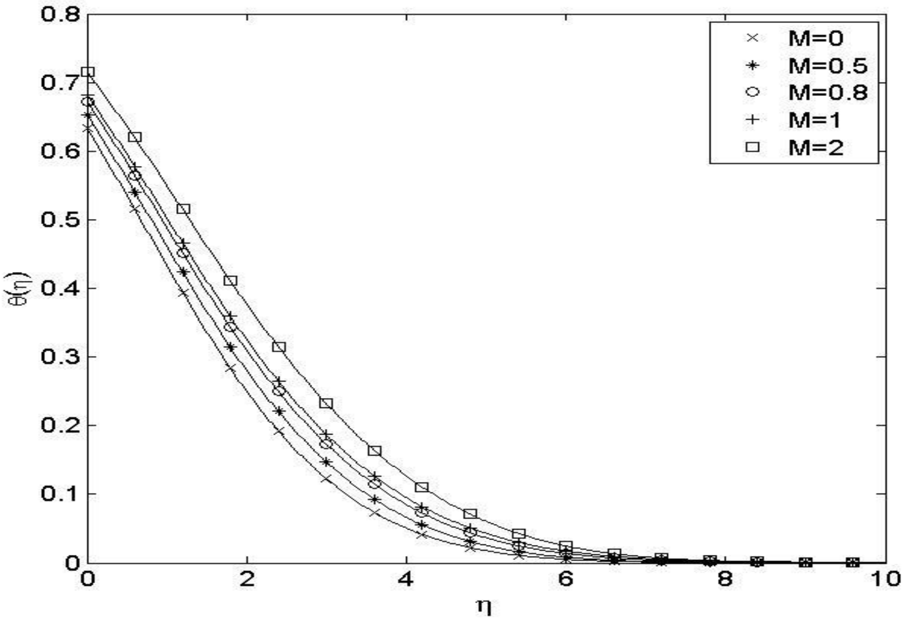

Figure 4. Temperature distribution versus η when $\Pr = 0.72, S = 0.5, \gamma = 1, \varepsilon = 2.$

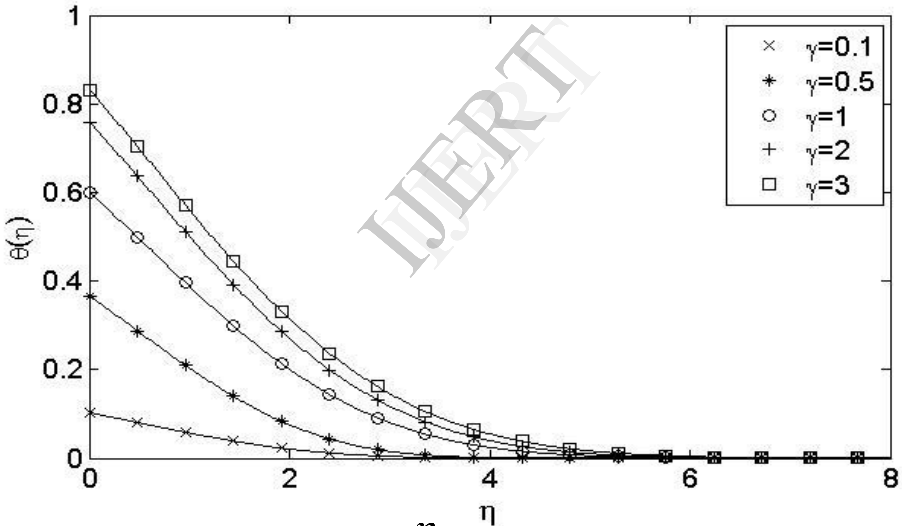

Figure 5. Temperature distribution versus η when $\Pr = 0.72, S = 0.5, \varepsilon = 2, M = 0.05.$

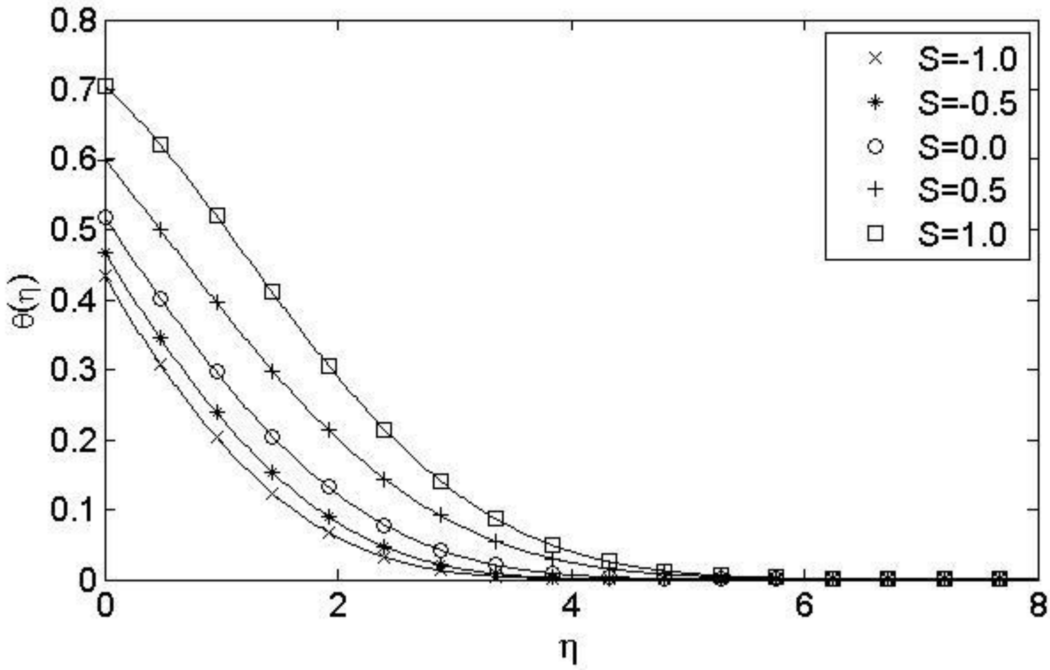

Figure6. Temperature distribution versus η when $\Pr = 0.72, \varepsilon = 2, \gamma = 1, M = 0.05.$

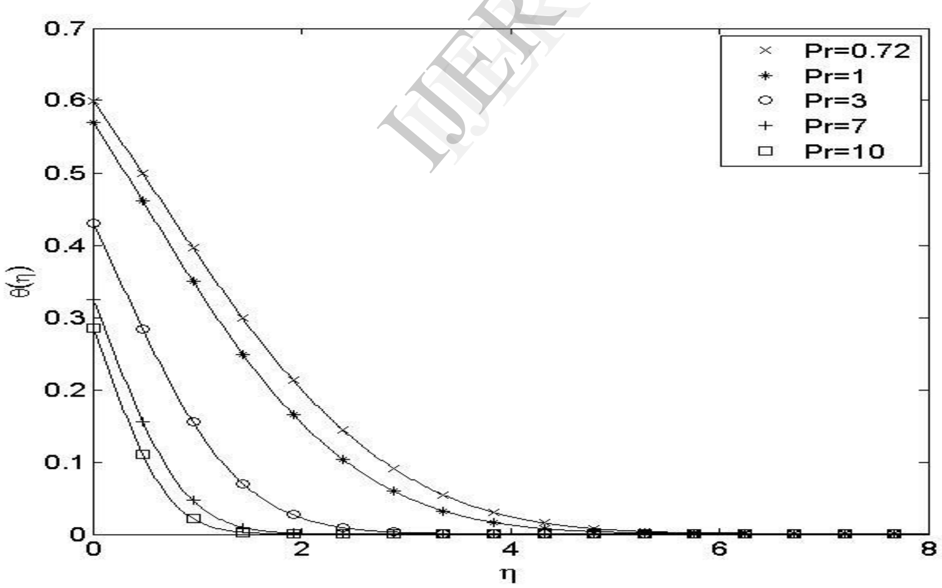

Figure 7.Temperature distribution versus η when $S = 0.5, \gamma = 1, \varepsilon = 2, M = 0.05.$

In order to validate the method used in this study and to judge the accuracy of the present analysis, the numerical values of skin friction coefficient for the stretching sheet are compared with those of Wang [2008], Yacob and Ishak [2012], Mahapatra and Gupta [2013]. These comparisons are shown in Table 1. A good agreement is observed between these results. This lends confidence in the numerical results to be reported subsequently.

CONCLUSION

Steady two dimensional MHD stagnation point flow of an incompressible viscous fluid over a stretching plate is investigated with heat source or sink is placed within the flow to allow for possible heat generation or absorption effects. Numerical calculations are carried out for various values of the physical parameters. The following conclusions are made

1. Fluid velocity profiles decrease due to increase in the magnetic parameter due to generation of Lorentz force.
2. Fluid velocity profiles increase due to increase in stretching parameter.
3. Fluid temperature decreases due to increase in stretching parameter or Prandtl number.
4. Fluid temperature increases due to increase in conjugate parameter, magnetic parameter or heat generation parameter.
5. Skin friction coefficient at the plate surface increases with an increase in intensity of magnetic field, but Nusselt number decreases with an increase in intensity of magnetic field.
6. Skin friction coefficient and Nusselt number both increase with the increase of stretching parameter.
7. Nusselt number increases with the increase of Prandtl number or conjugate parameter, but it decreases due to increase of heat generation parameter.

Table1. Comparison of numerical values of $f''(0)$ when $M=0$ for stretching sheet with different values of ε .

ε	Wang [2008]	Yacob and Ishak [2012]	Mahapatra and Gupta [2013]	Present study
0.0	1.232588	1.232588	1.232588	1.232500
0.1	1.146560	-	1.46560	1.46558
0.2	1.051130	-	1.051131	1.051129
0.5	0.71330	0.713295	0.713296	1.71329
1.0	0	0	0	0
2.0	-1.88731	-1.887307	-1.887308	-1.88730

Table2. Numerical values of skin friction coefficient and Nusselt number at the surface for various values of physical parameters.

ε	M	S	Pr	γ	$-f''(0)$	$-\theta'(0)$
2	0.05	0.5	0.72	1	1.93210	0.36495
3	0.05	0.5	0.72	1	4.32780	0.41889
2	0.5	0.5	0.72	1	2.31780	0.34748
2	1.0	0.5	0.72	1	2.73010	0.31858
2	0.05	1.0	0.72	1	1.61041	0.29475
2	0.05	-0.5	0.72	1	0.50910	0.53242
2	0.05	0.5	1.00	1	1.74012	0.42915
2	0.05	0.5	7.00	1	2.19031	0.67416
2	0.05	0.5	0.72	2	1.73820	0.48026
2	0.05	0.5	0.72	3	1.84512	0.50229

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