

Energy Involvements for Satisfying Conditions of SD Hypothesis

A Theoretical Physics Perspective

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Abstract— The SD Hypothesis addresses a specific scenario in Mechanics, wherein the magnitude of acceleration of a moving body is twice that of the body's velocity at any given instant or interval. However, realizing this magnitude necessitates estimation of the associated energy expenditures. Additionally, an inquiry into the ramifications of a particle attaining this particular energy state is imperative. This exploration will shed light on the calorific outputs we can anticipate, thus facilitating a comprehensive examination of several direct or indirect industries. Through this endeavor, we aim to refine existing models, fostering faster land transportation while concurrently ensuring sustainable fuel management.

Keywords—SD hypothesis, Energy Estimation, Particle Dynamics, Theoretical Physics

I. INTRODUCTION

While examining the stability of the SD phenomenon over a time interval, it was observed that under the influence of a constant or zero force on a body, the SD hypothesis is only feasible in cases of decelerating motion. It is now imperative to investigate the energy consumption when a force acts upon the reference body. Conversely, in scenarios where no external force acts on the particle yet the hypothesis is conditionally satisfied, a scrutiny of the energy transfer during this specific motion specification is warranted. Mathematical equations are established to validate these conditions, further supported by statistical analyses and graphical visualizations of the results. Initially, a broad overview of dynamics and energy conversions is presented, followed by the formulation of equations for the supporting conditions. Finally, a conclusion is drawn regarding the feasibility of the energy requirements at any level. Should the values prove feasible, further research in selected domains can be pursued to advance this inquiry.

II. INVOLVEMENT OF DYNAMICS

It has been noted that the SD hypothesis is only observable in decelerating motion when there is no external force acting on a moving body [2]. However, to extend its applicability to acceleration, an external force must be applied to the body. To investigate the impact of such applied force, a two-tiered study is conducted, focusing on both instantaneous and interval levels. This section provides a broad overview and utilizes Classical Mechanical models to establish the relationship between force, displacement, time, and velocity. The foundational Work-Energy theorem [1] is employed to derive

the equations in this context. Throughout this section, the scenario of a car moving along a road is considered. All the resultant vectors are hence considered unless mentioned.

It is assumed that the mass of the car is m kilograms and is moving with a velocity of magnitude v_{SD} . The kinetic energy (abbreviated as KE_{SD}) of the car can be expressed in terms of mass and velocity as:

$$KE_{SD} = (1/2)(m)(v_{SD}^2) \quad (1)$$

The expression for Kinetic Energy can be derived from the instantaneous SD velocity equation, the condition of the SD hypothesis, and the equation for SD displacement, all of which involve time, acceleration, and position at that particular instant [3]. Equation (2) and (3) are the equations of SD hypothesis (i.e. SD velocity in terms of acceleration) and SD velocity in terms of displacement and time respectively:

$$a_{SD} = 2v_{SD} \quad (2)$$

$$v_{SD} = (x_{SD})/(t_{SD} + t_{SD}^2) \quad (3)$$

The variable a_{SD} represents SD acceleration in (2), x_{SD} , and t_{SD} represent SD displacement and SD time respectively in (3). The values of SD velocity from (2) and (3) are utilized in (1) to derive Kinetic Energy as a function of acceleration in (4) and Kinetic Energy as a function of displacement and time in (5). Besides, since this Kinetic Energy is the particular energy associated with SD hypothesis, the specific term for this parameter is SD Kinetic Energy.

$$KE_{SD} = 2(m)(a_{SD}^2) \quad (4)$$

$$KE_{SD} = \frac{(m)x_{SD}^2}{2(t_{SD} + t_{SD}^2)^2} \quad (5)$$

The newly derived parameter, SD Kinetic Energy, can be expressed mathematically as a function of two variables in (4) and as a function of three variables in (5). Graphically, their representations have been plotted in Fig. (1) and Fig. (2) respectively. It is to be noted that, mass is taken as a constant in all the graphs. The value for mass in this case is chosen to be 1.2 kilograms.

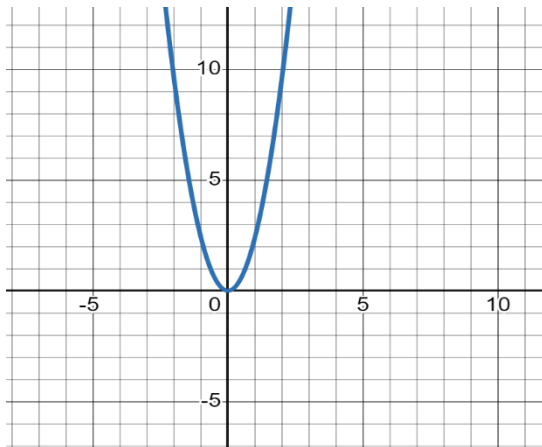


Figure 1: Relation between SD Kinetic Energy and SD acceleration

The following are the axis specifications for Fig.1:

- X-axis: SD acceleration
- Y-axis: SD Kinetic Energy

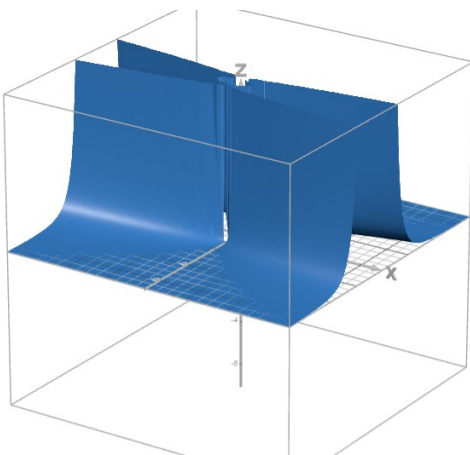


Figure 2: SD Kinetic Energy as a function of displacement and time

The following are the specifications for Fig. 2:

- X-axis: SD displacement
- Y-axis: SD time
- Z-axis: SD Kinetic Energy

Referring to Fig. 2, it's evident that a higher velocity is preferable to achieve a greater SD Kinetic Energy. However, it's important to note that the output energy here, the Kinetic Energy, is a result of all the effective forces and work applied to the car to attain this energy level.

Now that the expressions for Kinetic Energy have been established for a specific instant, it's crucial to examine the force at play during this event. It's important to note that, since this is an instantaneous motion [4] for this subpart, the term "SD time"

used here refers to instantaneous SD time [3]. Additionally, the Force and Energy under consideration are both instantaneous Force and Energy respectively. For estimating the required force, we must consider Newton's second law of motion [5]. Furthermore, since all forces acting on the car are conservative in nature [6], the Work-Energy theorem [1] is applied to establish force as a function of velocity. Subsequently, variables are substituted to establish Force as a function of displacement and time.

According to Newton's second law of motion [5], the equation for force is given as:

$$F = ma \tag{6}$$

Where 'F' represents force, 'm' represents mass of the object and 'a' represents acceleration.

Customizing equation (6) according to the SD hypothesis yields a specific force that satisfies the criteria of the hypothesis. Therefore, the force obtained in equation (7) and subsequent equations will be referred to as SD force, unless otherwise specified.

$$F_{SD} = ma_{SD} \tag{7}$$

Here, F_{SD} is SD force. In general, from (2) and (3), values can be substituted in (7) to generate SD Force in terms of SD velocity and SD Force in terms of SD displacement and SD time. This leads to the following set of equations:

$$F_{SD} = 2mv_{SD} \tag{8}$$

$$F_{SD} = \frac{2mx_{SD}}{(t_{SD} + t_{SD}^2)} \tag{9}$$

Equations (8) and (9) represent different formulations of SD force. It's important to note that all forms of force observed in (7), (8), and (9) are resultant forces stemming from all forces acting on the car [6]. However, the force that directly impacts the car and performs work as a resultant force may slightly differ from these formulations. Looking into the actual forces at play in the scenario, Fig. (3) illustrates the various forces acting on the car at a specific instance, denoted as t_{SD} in this case.

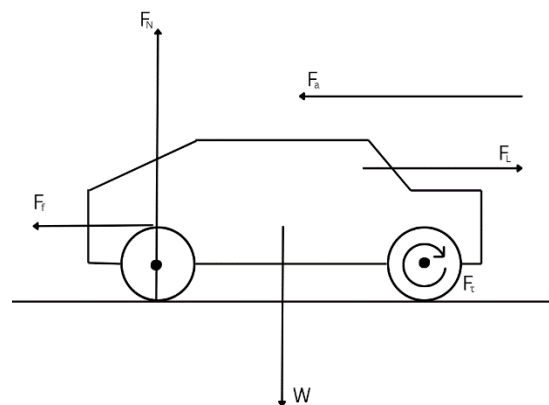


Figure 3: Forces acting on a car moving on a plane

In Fig. (3), 'W' represents the weight of the car, 'F_N' denotes the contact force or Normal force arising from surface contact, 'F_f' signifies frictional force, 'F_a' represents aerodynamic drag, 'F_r' indicates the force generated due to rotational acceleration, and 'F_L' stands for the force attributed to linear acceleration. In circumstances where the car is effectively and linearly accelerating, the resultant force (referred to as 'F_{res}' henceforth) acts effectively in the same direction as the force due to linear acceleration. However, if the road has an incline, forming an angle theta with the horizontal plane, another force known as the hill climbing force [6] comes into play, exerting an additional effect on the car. In this scenario, considering the conditions depicted in Fig. (3), the inclusion of the angle of elevation does not significantly alter the estimations. Therefore, the work done by the resultant force to achieve the SD displacement is expressed as follows:

$$\omega = \lim_{\theta \rightarrow 0} (F_{res})(x_{SD})\cos(\theta)$$

Or, $\omega = (F_{res})(x_{SD})$ (10)

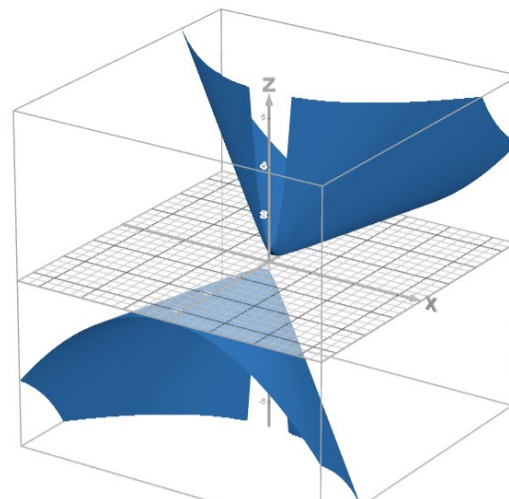


Figure 4: Resultant force as a function of SD velocity and SD displacement

Here, work done is denoted by ω and θ is the angle of elevation. In the absence of any elevation in the plane, the energies associated with the output are primarily dominated by Kinetic Energy, with minor contributions from heat energy and potential energy. However, in scenarios involving elevated terrain (i.e. hilly areas), a notable portion of the total mechanical energy possessed by the car is allocated to potential energy. Since, in this case, Kinetic Energy dominates and others can be neglected for significance, it is safe to assume the other associated energies as U_{others} . Thus, formulating the equation of work using Work-Energy Theorem [1], we get:

$$\omega = (1/2)(m)(v_{SD})^2 + U_{others}$$
 (11)

Equating and simplifying (10) and (11), we get the expression for resultant force in an instant which is given by:

$$F_{res} = \frac{mv_{SD}^2 + U_{others}}{2(x_{SD})}$$
 (12)

Henceforth, for each unit of displacement, the resultant force expected as output can be computed at an instant. Further, (12) can be further transformed such that resultant force is a function of SD displacement and SD time. The following equation is hence formed:

$$F_{res} = \frac{mx_{SD}}{2(t_{SD} + t_{SD}^2)} + \frac{U_{others}}{2(x_{SD})}$$
 (13)

From(2), (3) and (12), (14) can be derived further which establishes resultant force as a function of SD acceleration and time.

$$F_{res} = \frac{ma_{SD}}{4(t_{SD} + t_{SD}^2)} + \frac{U_{others}}{a_{SD}(t_{SD} + t_{SD}^2)}$$
 (14)

Graphically, (12), (13) and (14) are plotted in Fig. (4), (5) and (6). It is noteworthy that the value of other associated energies is taken to be 0.1 joules unless mentioned for graphing the above-mentioned equations.

In Fig. (4), the following specifications have been taken into consideration:

- X-axis: SD velocity
- Y-axis: SD displacement
- Z-axis: resultant force

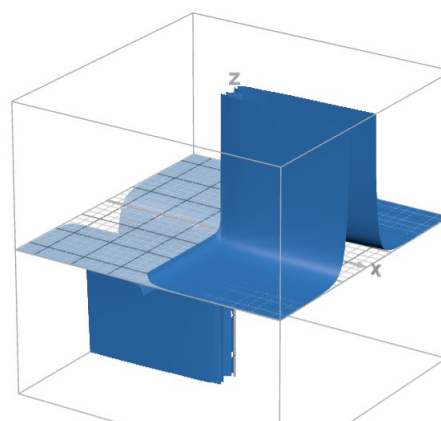


Figure 5: Resultant force as a function of SD displacement and SD time

Specifications for Fig. (5):

- X-axis: SD displacement
- Y-axis: SD time
- Z-axis: resultant force

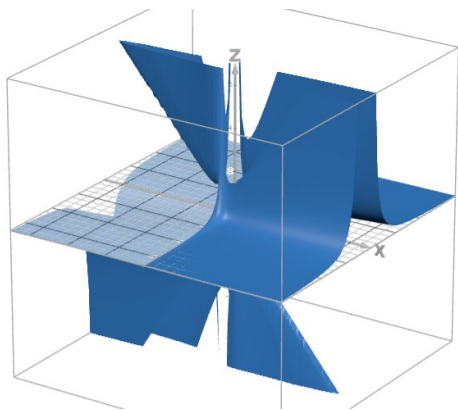


Figure 6: Resultant force as a function of SD acceleration and SD time

Specifications for Fig. (6):

- X-axis: SD acceleration
- Y-axis: SD time
- Z-axis: resultant force

Summarizing the instantaneous dynamics in this scenario, it's evident that the resultant force decreases with time, as evidenced by the clear inverse proportion observed in all forms of resultant force in equations (12), (13), and (14). However, by delving into the applied force responsible for linear acceleration and considering the energy inputs, a more comprehensive analysis can be pursued over a longer interval.

III. POWER INVOLVEMENTS AND FEASIBILITY

The energy input for a moving car is provided in the form of Chemical energy, which is converted into Kinetic energy. Therefore, it's necessary to calculate the Mechanical Power supplied to the car [7] to enable movement. Power can be computed as the rate of work done over infinitesimally small intervals of time. It is mathematically established in the following way:

$$P(t) = \frac{d\omega}{dt} \tag{15}$$

Here P is Power and (15) establishes power as a function of time. For the same, (11) can be restructured to establish work as a function with time as an independent variable. For this, we need to look into the condition of SD velocity for a considerable amount of time [2]

$$\omega = \frac{mx_{SD}^2}{(t_{SD} + t_{SD}^2)^2} \tag{16}$$

Utilizing (16), the expression for Mechanical Power is obtained and is given by

$$P(t) = \left(- \frac{2(mx_{SD}^2)(2t_{SD}+1)}{(t_{SD}+t_{SD}^2)^3} \right) \tag{17}$$

The form of this power is taken out to be negative as it is provided as an input. If we consider the power to be an output, it forms the following equation:

$$P(t) = \left(\frac{2(mx_{SD}^2)(2t_{SD}+1)}{(t_{SD}+t_{SD}^2)^3} \right) \tag{18}$$

An advantage of the derived form of power as of (17) is that, with increasing displacement and timing, the power increases steadily, following a stable curve. This trend is illustrated in Fig. (7).

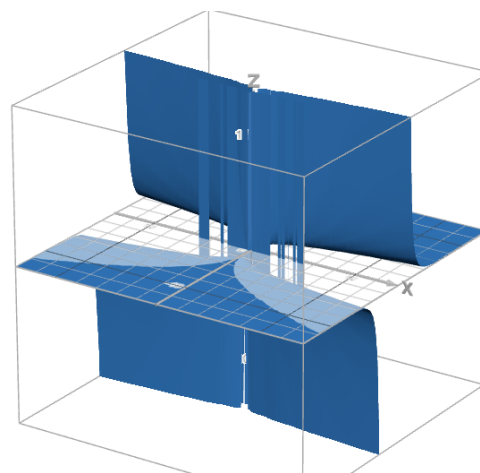


Figure 7: Mechanical Power as a function of SD displacement and SD time

According to this trend, power is needed to be supplied in order to maintain a particular limit in SD hypothesis. It is to be noted that (17) provides the output in joules per second. The power requirements for attaining a particular speed at five different instants in the range of 0 to 100 kilometers per hour is provided in Table 1.

Table 1: Power required for attaining a particular SD velocity

SD Velocity (kilometers per hour)	Power Required (watts)
20	4665.6
40	18662.4
60	41990.4
80	74649.6
100	116640

The speed and power relations outlined in Table 1 indicate that this specific speed is attainable in any standard engine. However, achieving the necessary acceleration poses a concern in this scenario. Further examination of acceleration is crucial to determine whether this model can be applied to existing automobiles. According to the SD hypothesis, a speed of 20 kilometers per hour implies an acceleration of 10 kilometers per square hour or 36 meters per square second. This contrasts sharply with the fixed acceleration limits in automobiles, which typically hover around a much lower value of roughly 2 meters per square second [8].

CONCLUSION

Analyzing the figures and power requirements of the phenomenon, SD hypothesis is feasible in terms of energy and resources and can even be a sustainable way to improve performance of an automobile but it is not a quite stable model for higher speeds and larger units. Although the mathematical stability remains, the regulations and limitations due to safety checks are indeed a barrier to keep this phenomenon feasible for higher values of parameters. Hence, the application of SD Hypothesis or SD Phenomenon won't bring a significant change in the automobile industry as this phenomenon is applied for a very short period of time and ideally for the fixed acceleration, the maximum acceleration will be reached at the very beginning of the motion. However, a significance of the phenomenon lies in Quantum Mechanics with analysis of fields and variations of different particles [9]. Hence, this remains as a theoretical phenomenon which can be applied if the barriers of acceleration can be breached with promising safety in our current situation of roadways. On the other hand, Quantum Mechanically, this can be seen as an independent phenomenon, existing with no such significant applications.

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