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Equation on Circles Using Matrices and Other Methods with Area Analysis

Circle equation methods using exponential, matrices and triangular structures inscribed in circle

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***Abstract*—Here we define a circle using different methods i.e. exponential approximation, generalized coordinate matrix equation and polar coordinate matrix system. We then find radial overlap error and margin. We then use graphical techniques to split the circumference of the circle into 22 parts and diameter into 7 parts. This way we find radius (r) , center point (h,k) , polar coordinate angle (theta) and area (A) of the circle.**

***Keywords*—** circle, exponential, area, pi equation

I. INTRODUCTION

The definition of a circle and its axioms are given below. Definition:

A circle is defined as the locus of a set of points that are equidistant from a given point called the center.

Axiom:

1. The distance from the center to any point on the circle is called the radius.
2. Two circles that have the same radius are congruent.
3. The circumference of a circle is proportional to its radius.
4. The area of a circle is proportional to the square of its radius.
5. The diameter of a circle is twice its radius. The distance around the edge of a circle is called its circumference.

II. UNITS ALL UNITS ARE IN MKS OR SI SYSTEM.

III. EQUATIONS

A. Quarter circle equation:

$$y = \lim_{x \to R} R - [\textstyle\sum_{n=1}^{4} (x \div R)^{(2n+1) \div n} \times n! \div (n-1)!$$

where R = radius of circle > 0

B. Standard form of circle equation:

condition, r = 10 cm; origin O (0,0);
x^2 + y^2 = r^2;
x^2 + y^2 = 100;

C. 3 points diameter form circle equation:

(x-x1)/(x-x2) + (y-y1)/(y-y2) = 0
(x+10)/(x-10) + (y-0)/(y+0) = 0
Now inputting the value of P (x,y) = (2.15,9.8) , A (-10,0) B (10,0). (For 36.87, 53.13, 90 degree angle triangle inscribed in the circle)
Now inputting the value of P(x,y) = (0, 10.1) A (-10,0) B(10,0)
(For 45, 45, 90 degree angle triangle inscribed in the circle)
Now inputting the value of P(x,y) = (4.7, 8.8) A (-10,0) B(10,0)
(For 30, 60, 90 degree angle triangle inscribed in the circle)
Now inputting the value of P(x,y) = (8.2, 5.5) A (-10,0) B(10,0)
(For 16.36, 73.64, 90 degree angle triangle inscribed in the circle)
We get L.H.S = R.H.S (approximately)
Here we have taken the endpoints of the diameter and P(x,y) to be any point on the semi circle.
We know that the angle inscribed in a semi-circle by the diameter is a right angle (90 degrees).
So, for such a scenario the multiplication of the slope of the subtending hands of the angle APB gives us -1 as a result.
Hence the above equation is possible.

Special case:
Now inputting the value of P(x,y) = (5, 8.5) A (-10,0) B(5.1,8.55)
(For 60, 60, 60 degree angle triangle inscribed in the circle)

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D. General Equation of a Circle:

The general equation of a circle is of the form,
x^2 + y^2 +2gx + 2fy +c = 0 where center O (-g,-f)
and r^2 = g^2 + f^2 - c > 0
Let (-g, -f)= (0,0) and r=10;
r^2 = 100;
Therefore, c=-100
Plugging into the general equation,
x^2 + y^2 = 100
The general form of the equation can also be written as:
(x-h)^2 +(y-k)^2 = r^2
where O (h,k) = (0,0) is the origin and r = 10 is the radius of the circle.

E. Cartesian Matrix form of the equation:

(x-h)^2 +(y-k)^2 = r^2
x^2 + -2hx + h^2 + y^2 + -2ky +k^2 = r^2
x=[-10 ,-9.6, -8.35, -5.5, -3.1, -1.3 ,1.55, 4.3, 6.65, 8.2, 9.55, 10, 9.5, 8.15, 6.25, 3.8, 1.9,-1.85, -4.5, -6.85, -8.7, -9.7]
y = [0, 2.8, 5.4, 7.5, 9.05, 9.8, 9.8, 8.95, 7.35, 5.2, 2.5, -0.3, -3.1, -5.65, -7.7, -9.2, -9.9, -9.85, -8.95, -7.4, -4.9,-2.35]T
h(h-2x) + k(k-2y) - r^2 =-(x^2+y^2)
h [h+20, h+19.2, h+16.7, h+11, h+6.2, h + 2.6, h -3.1, h-8.6, h-13.3, h - 16.4, h-19.1, h-20, h-19, h-16.3, h-12.5, h-7.6, h-3.8, h+3.7, h+9.6, h-13.7,h-17.4, h-19.4]
+
k[k-0, k-5.6, k-10.8, k-15, k-18.1, k -19.6, k-19.6, k-17.9, k-14.7, k-10.4, k-5, k+0.6, k+6.2, k+11.3, k+15.4, k+18.4, k+19.8, k+19.7, k+17.9, k+14.8, k+9.8, k+4.7]T
-
r^2
=
[100, 92.16, 69.7225, 30.25, 9.61, 1.69, 2.4, 18.49, 44.22, 67.24, 91.20, 100, 90.25, 66.42, 39.06, 14.44, 3.61, 3.42, 20.25, 46.92, 75.69, 94.09]
+
[0, 7.84, 29.16, 56.25, 81.90, 96.04, 96.04, 80.10, 54.02, 27.04, 6.25, 0.09, 9.61, 31.92, 59.25, 84.64, 98.01, 97.02, 80.10, 54.76, 24.01, 5.52]T

From the above equation, we get,

\[h = [0.99853542, -1.49368842, 1.51244639, -1.30011671, -1.66701501, -1.68557906, 0.87837672, -0.13984872, 0.45892314, -0.13118738, -0.99776207, 0.12907905, 0.31430334, 0.70244807, 0.79429873, 1.62841411, -0.62605197, 0.12717817, -0.28144241, 1.08455679, -0.08719522, 0.03622985] \]
\[k = [-1.5861088, 1.2420173, -1.03770764, 0.84483801, -1.4780324, -1.32193447, 0.7605486, 0.03549951, 1.09032469, 1.14938127, -1.47260717, -0.25446924, 0.76826763, -0.67428599, -0.15609884, -1.5701157, -1.40956782, 1.27420058, 1.46523482, -1.37601269, 0.93328874, -1.34397454] \] \[r = 11.4863092 \]

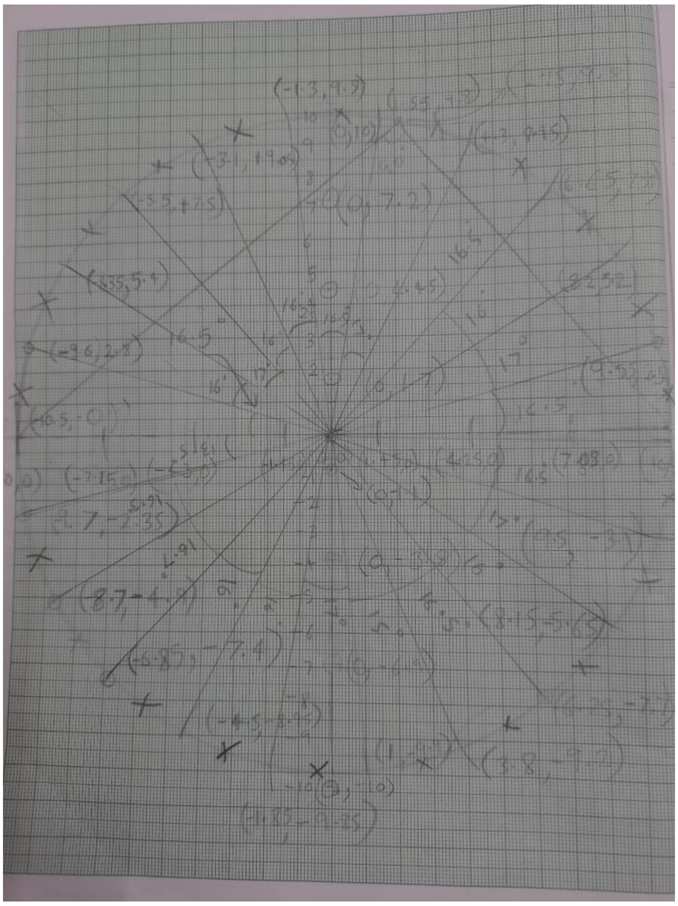

Fig.1: The above image shows the plot of a circle as graphed by hand using error margin.

[redacted]

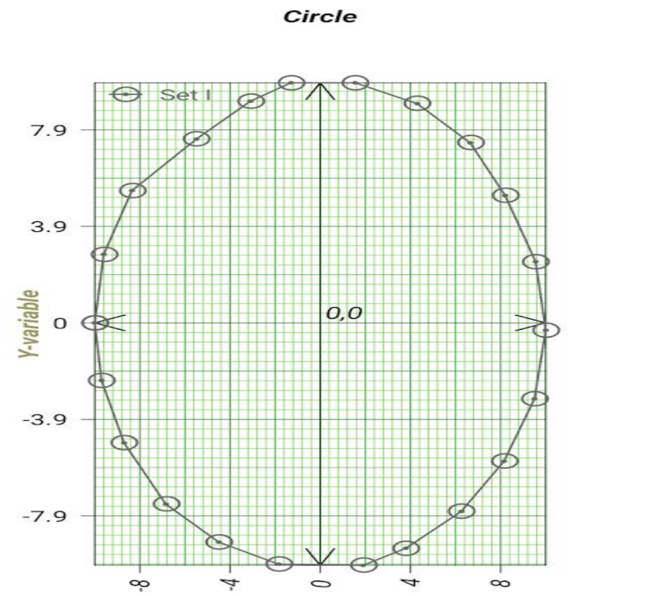

Fig.2 : This is the circle made by graphing 22 equidistant points on the circumference of a r=10, centre O = (0,0) point.

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Radial Overlap Error Margin:

1. By chord length:

Circumference = 2*pi*r =2*3.14*10 = 62.83 cm(ideal)
=2*pi*11.49 (by matrix)
=72.194 cm

Percentage error = (61.555-62.83)/62.83 (chord length)
= -2.03%

2. By chord angle:

Chord Angle = 16.36 degrees
Measured Average = 16.26 degrees
Percentage Error = (16.26-16.36)/16.36
= -0.611%

F. Polar form equation of circle:

We know that,
Ideally,
r = 10 cm and (h,k) = (0,0) and θ = 16.36 degrees
(rCosθ - h)^2 + (rSinθ - k)^2 = r^2
(10.cos(16.36))^2 +(10.sin(16.36))^2 = 100
LHS = RHS
Now, through empirical formula,
r^2.(cos(θ))^2 - 2.r.cos(θ).h + h^2 +
r^2.(sin(θ))^2 - 2.r.sin(θ).k + k^2 = r^2
Now,
θ = [16.5, 16, 17, 16, 16.25, 16.5, 17, 16.5, 16, 17, 16.5, 16.5, 17, 16, 16.5, 15, 17, 16, 16, 16.7, 16.3, 13.5]
Therefore,
r^2.[cos(16.5*pi/180)^2, cos(16*pi/180)^2, cos(17*pi/180)^2, cos(16*pi/180)^2, cos(16.25*pi/180)^2, cos(16.5*pi/180)^2, cos(17*pi/180), cos(16.5*pi/180)^2, cos(16*pi/180)^2, cos(17*pi/180)^2, cos(16.5*pi/180)^2, cos(16.5*pi/180)^2, cos(17*pi/180)^2, cos(16*pi/180)^2, cos(16.5*pi/180)^2, cos(15*pi/180)^2, cos(17*pi/180)^2, cos(16*pi/180)^2, cos(16*pi/180)^2, cos(16.7*pi/180)^2, cos(16.3*pi/180)^2, cos(13.5*pi/180)^2]
-
2.r.h.[cos(16.5*pi/180), cos(16*pi/180), cos(17*pi/180), cos(16*pi/180), cos(16.25*pi/180), cos(16.5*pi/180), cos(17*pi/180), cos(16.5*pi/180), cos(16*pi/180), cos(17*pi/180), cos(16.5*pi/180), cos(16.5*pi/180), cos(17*pi/180), cos(16*pi/180), cos(16.5*pi/180), cos(15*pi/180), cos(17*pi/180), cos(16*pi/180), cos(16*pi/180), cos(16.7*pi/180), cos(16.3*pi/180), cos(13.5*pi/180)]
+
h^2
+
r^2.[sin(16.5*pi/180)^2, sin(16*pi/180)^2, sin(17*pi/180)^2, sin(16*pi/180)^2, sin(16.25*pi/180)^2, sin(16.5*pi/180)^2, sin(17*pi/180), sin(16.5*pi/180)^2, sin(16*pi/180)^2, sin(17*pi/180)^2, sin(16.5*pi/180)^2, sin(16.5*pi/180)^2, sin(17*pi/180)^2, sin(16*pi/180)^2, sin(16.5*pi/180)^2, sin(15*pi/180)^2, sin(17*pi/180)^2, sin(16*pi/180)^2, sin(16*pi/180)^2, sin(16.7*pi/180)^2, sin(16.3*pi/180)^2, sin(13.5*pi/180)^2]
-
2.r.k.[sin(16.5*pi/180), sin(16*pi/180), sin(17*pi/180), sin(16*pi/180), sin(16.25*pi/180), sin(16.5*pi/180), sin(17*pi/180), sin(16.5*pi/180), sin(16*pi/180),sin(17*pi/180), sin(16.5*pi/180), sin(16.5*pi/180), sin(17*pi/180), sin(16*pi/180), sin(16.5*pi/180), sin(15*pi/180), sin(17*pi/180), sin(16*pi/180), sin(16*pi/180), sin(16.7*pi/180), sin(16.3*pi/180), sin(13.5*pi/180)]
+
k^2
=
r^2
So,
With \(r^2 = 0.01\), the solutions for \(h\) and \(k\) are:
h = 0.990
k = 0.099
r = 0.10

G. Area Graphical and Analytical Method

Area of Circle:
Area = pi.r^2 = 0.03142 m^2(Ideal)
Area = pi. r^2 =0.041475 m^2 (by matrix)
Area = 22* θ/360*pi*r^2 (by sector)
= 22 *16.26/360*pi*10^2
= 0.031217m^2

H. Finding the value of pi through graphical method

Chord Length is given by:

$$C = \sqrt{(x1 - x2)^2 + (y1 - y2)^2}$$

c1 = 2.828	c12 = 2.844
c2 = 2.885	c13 = 2.885
c3 = 3.54	c14 = 2.795
c4 = 2.857	c15 = 2.873
c5 = 1.95	c16 = 2.886
c6 = 2.85	c17 = 2.850
c7 = 2.878	c18 = 2.799
c8 = 2.843	c19 = 2.815
c9 = 2.65	c20 = 3.110
c10 = 3.019	c21 = 2.739
c11 = 2.836	c22 = 2.369

Diameter Parts (vertical and horizontal):

d1 = 2.92	d1 = 3.1
d2 = 2.83	d2 = 3.1
d3 = 2.80	d3 = 2.7
d4 = 2.90	d4 = 2.8
d5 = 2.85	d5 = 2.75
d6 = 2.85	d6 = 2.75
d7 = 3.35	d7 = 2.8

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Pi is given by:
Circumference / Diameter = 22/7

$$= \frac{[\, c1 + c2 + \ldots\ldots .c22]}{[\, d1 + d2 + \ldots\ldots\ldots d7]}$$

= 61.555/20.5
= 3.003 (horizontal)
Circumference / Diameter = 61.555/20
= 3.078 (vertical)
= 72.194/2*11.49 (by matrix)
=3.1416
= 2.8*22/2*10
= 3.08
(by sector)

I. Table of Co-ordinates:

No.	x	y	c	d (H)	d (V)	θ
1.	-10	0	2.828	2.92	3.1	16.5
2.	-9.6	2.8	2.885	2.83	3.1	16
3.	-8.35	5.4	3.54	2.80	2.7	17
4.	-5.5	7.5	2.857	2.90	2.8	16
5.	-3.1	9.05	1.95	2.85	2.75	16.25
6.	-0.3	9.8	2.85	2.85	2.75	16.5
7.	1.55	9.8	2.828	3.35	2.8	17
8.	4.3	8.95	2.843			16.5
9.	6.65	7.35	2.65			16
10.	8.2	5.2	3.019			17
11.	9.55	2.5	2.836			16.5
12.	10	-0.3	2.844			16.5
13.	9.5	-3.1	2.885			17
14.	8.15	-5.65	2.795			16
15.	6.25	-7.7	2.873			16.5
16.	3.8	-9.2	2.886			15
17.	1.9	-9.9	2.850			17
18.	-1.85	-9.85	2.799			16
19.	-4.5	-8.95	2.815			16
20.	-6.85	-7.4	3.110			16.7
21.	-8.7	-4.9	2.739			16.3
22.	-9.7	-2.35	2.369			13.5
Total, Avg			61.555, 2.8			357.72, 16.26

IV. ACKNOWLEDGMENT

Lastly, I would like to thank and acknowledge Ishwar Engineering Works company for showing me the correct way to sketch a circle on a graph to find the pi value. Also thanks to all my math teachers and my sisters for providing me with ample research papers on the subject matter of circles.

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