Evaluation of Circularity from Coordinate data using Maximum Distance Point Strategy (MDPS)

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Abstract

Measurement of cylindrical features using Coordinate Measuring Machine (CMM) is one of the important operations in precision engineering industries. The operation necessitates use of efficient computational algorithms as it has to determine radius/diameter of cylindrical features from measured point coordinates. One of the most widely used algorithm for such application is Least-Square Method (LSM) which fits a circle to the points measured using CMM. This paper proposes a new approach termed as Maximum Distance Point Strategy (MDPS) to determine radius/diameter of cylindrical feature for minimizing circularity from measured data-points. The results of MDPS are compared to that of LSM. Moreover, the results of MDPS are also compared with other methods available in literature and it has been found that the results are comparable with the same. It is also demonstrated that the developed methodology offers simplicity in understanding and ease of implementation in computational algorithms.
Keywords: *Coordinate Measuring Machine (CMM), Least-Square Method (LSM), Circularity, Sum of Squared Deviations*

1. Introduction

Many functional components in engineering industries have external or internal cylindrical features. When these components are manufactured, closeness to required dimension is expressed in terms of roundness or circularity. Roundness or circularity can be determined using various instruments such as roundness testing machine, dial gauges, Coordinate Measuring Machines (CMM), gap detectors etc. Among these methods, CMM is widely used in industries due to versatility and ease of operation.

The ANSI Dimensioning and Tolerance Standard Y14.5 [1] defines that the form tolerances on a component must be evaluated with reference to an ideal geometric feature. CMM software evaluates circularity of cylindrical features by establishing a circle as a reference geometric feature from the measured points using Least Squares Method (LSM). LSM is predominantly used to estimate best fit circle for the measured points. LSM fits geometric feature to minimize the sum of squares of deviations in predefined measures.

2. Literature Review

Gander, Colub and Trebel [2] used Gauss-Newton Algorithm (GNA) to minimize the square of error distances for circle. Gauss-Newton algorithm is a modification of Newton's method, which is line-search strategy for finding the minimum of a function, mainly used to solve non-linear least squares problems. If GNA starts nearby the solution, it converges quickly. In other cases, it requires more iteration to converge and sometimes it may not converge at all. Hence, a good initial guess is required for the solution to converge [3].

Shakarji [4] suggested use of Levenberg-Marquardt algorithm (LMA) to minimize the square of error distances for various features including circle. LMA is trust-region strategy which provides a numerical solution to the problem of minimizing non-linear function. LMA is more robust than GNA. However, even for well-behaved functions and reasonable starting parameters, the LMA tends to be a bit slower than the GNA. LMA can also be viewed as improved GNA with trust region approach [4, 5, 6]. Also, convergence of the solution is highly dependent on choice of Levenberg-Marquardt parameter and its selection is challenging.

Chernov and Ososkov [7] proposed two new set of algorithms for full circle-fitting and circular arc namely Iterational Linear Regression Method (ILRM) and Modified Linear Regression Method (MLRM). Although ILRM is well-suited for fitting any size of circular arc including full circle, it is slower. The second suggested method (MLRM) is faster, but works only for small arcs.

Drezner, Steiner and Wesolowsky [8] suggested use of heuristic algorithms for finding a circle whose circumference is close to given set of points. The heuristic uses two efficient algorithms known as minimax and minisum.

All previous methods are establishing the circle from the measured or simulated points. This circle is base feature for evaluating circularity. Attempts have been also made to develop methods for evaluating error in circularity. Murthy and Abdin [9] applied normal least-square fit to determine circularity error but the values obtained are not the minimum for LSM. Instead, they have suggested that simplex search technique is more suitable. To obtain the minimum zone evaluation for sphericity, numerical methods based on the Monte Carlo, Simplex and Spiral Search techniques have also been suggested by Kanada [10]. Murthy [11] compared different algorithms for circularity evaluation and concluded that simplex search is essential and superior to the other methods for evaluating circularity. Shunmugam [12] suggested an alternative approach based on minimum average deviation (MAD) in which different geometric features are established using a search technique. The values obtained by this approach are compared with the ones obtained using least squares and minimum deviation methods. Dhanish and Shunmugam [13] determined minimum zone values using discrete and linear Chebyshev approximations which is applied directly to form data as well as coordinate data provided by CMM. An algorithm suggested by Dhanish [14] guarantees the minimum value of circularity error. Kim and Kim [15] proposed an algorithm for least squares evaluation of circularity which takes geometrical approximation of the orthogonal Euclidean distance in measuring deviational errors of sample data over very small arc so that the assessment criterion of normal least squares is faithfully implemented. Wang, Hossein Cheraghi and Masud [16] formulated a nonlinear optimization problem to find circularity error based on the minimum radial separation criterion. Samuel and Shumugam [17, 18] suggested methods based on computational geometric techniques to deal with CMM measured data and form data.

The present work aims to define a strategy that finds best fit circle for given set of data points to minimize circularity and it is named as "Maximum Distance Point Strategy (MDPS)". For the purpose of comparison, results of MDPS are compared with LSM and CMM results. The results of MDPS are also compared with results of methods published in references [9] and [17].

This is a customized approach to find the best fit circle for evaluating the circularity rather than addressing a general unconstrained nonlinear problem. It is based on the postulate that "*A unique circle passes through any three non-collinear points in a plane*". Hence, selection of three points (triplet) plays important role to fit the best circle.

3. Point Selection

The selection procedure for triplet (A, B, C) is as follow.

1. Let $P_i(x_i,\ y_i)$, $i = 1,2,\dots,n$ and $n > 2$, be the CMM measured set of n points.
2. Select a point from P_i and name it as A, which is first point in triplet.
3. Calculate distance from point A to each point P_i using equation 2.1.
   $AP_i = \sqrt{(x_a - x_i)^2 - (y_a - y_i)^2}$ (3.1)
   where, AP_i is distance from point A to point P_i, $i = 1,2,\dots,n$,
   $x_{a,}\ y_a$ are coordinates of point A,
   $x_{i,}\ y_i$ are coordinates of point P_i.
4. Select second point from P_i, $i = 1,2,\dots,n$ (second point in triplet) for which AP_i is maximum. Name it as B.
5. Point C (third point in triplet) is selected from P_i, $i = 1,2,\dots,n$ such that its normal distance from line AB is maximum.
   To determine maximum normal distance, the following expression is used.
   $\mathrm{d(P_i)} = |(\mathrm{y_B} - \mathrm{y_A}) * (\mathrm{x_i} - \mathrm{x_A}) + (\mathrm{x_B} - \mathrm{x_A}) * (\mathrm{y_i} - \mathrm{y_A})|$ (3.2)
   where, $\mathrm{i} = 1,2,\dots,\mathrm{n}$
   $\mathrm{x_{a,}\ y_a}$ are coordinates of point A,
   $\mathrm{x_{b,}\ y_b}$ are coordinates of point B,
   $\mathrm{x_{i,}\ y_i}$ are coordinates of point $\mathrm{P_i}$.

The selection procedure is repeated for each point $\mathrm{P_i}$, $\mathrm{i} = 1,2,\dots,\mathrm{n}$. Hence, there are n triplets and n candidate circles passing through the triplets.

In figure 3.1, selection procedure for point 1 as first point in triplet is shown. Since, distance between point 1 and point 3 is the maximum amongst all points, point 3 is selected as second point in triplet. Point 5 is selected as third point in triplet as its distance from line AB is the maximum. The circle shown in figure 3.1 is candidate circle for point 1.

Amongst all candidate circles, circles which are far from the solution are eliminated heuristically as discussed in section 4.3. The average of center coordinates of the selected circles and the average radii of these circles represent the center and radius of the best fit circle for a given set of points.

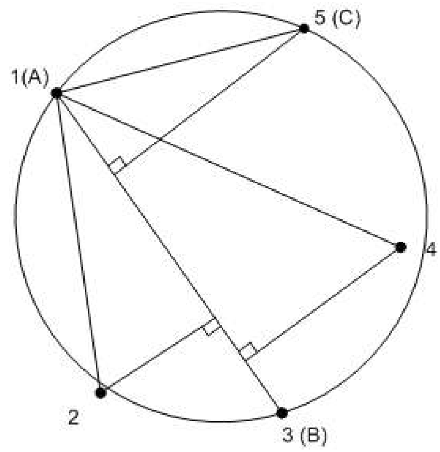

Figure 3.1 Selection Procedures for Triplet.

4 Formulations

For $P_i(x_i, y_i)$, $i = 1,2, \ldots, n$ and $n > 2$.

4.1 Least Square Method

A circle with the center (x_0, y_0) and radius r_0 is found such that it minimizes the sum of squared deviations. The circle equation in an implicit form can be written as

$$f(x,y) = (x - x_0)^2 + (y - y_0)^2 - r_0^2 = 0$$

The deviation of distance for a point P_i, $i = 1,2, \ldots, n$ may be explicitly written as

$$e_i = \sqrt{(x_i - x_0)^2 + (y_i - y_0)^2} - r_0; \; i = 1,2, \ldots, n \quad (4.1)$$

The sum of squared deviations is then described as

$$e_s = \sum_{i=1}^{n} e_i^2 = \sum_{i=1}^{n} \left[\sqrt{(x_i - x_0)^2 + (y_i - y_0)^2} - r_0\right]^2 \quad (4.2)$$

4.2 Circularity error

Denote the maximum value among the deviations e_i, $i = 1,2, \ldots, n$ as e_{max} and the minimum value as e_{min}. Then, the circularity error h can be computed as (refer Figure 4.1)

$$h = e_{max} - e_{min} \quad (4.3)$$

According to the minimum zone criterion given by ANSI Standard Y14.5 [1], the center (x_0, y_0) and radius r_0 of an ideal circle should be determined such that the circularity h is the minimum.

4.3 Maximum Distance Point Strategy

Fix a point, say, P_k and select two other points as explained in section 3. Let the coordinates of the points be (x_a, y_a), (x_b, y_b) and (x_c, y_c). Solve the following system of linear algebraic equations

$$2(x_b - x_a)x_0 + 2(y_b - y_a)y_0 = (x_b^2 - x_a^2) + (y_b^2 - y_a^2)$$

$$2(x_c - x_a)x_0 + 2(y_c - y_a)y_0 = (x_c^2 - x_a^2) + (y_c^2 - y_a^2)$$

for center of the circle (x_0, y_0) and calculate r_0 using $r_0 = \sqrt{(x_a - x_0)^2 + (y_a - y_0)^2}$. This is the circle passing through (x_a, y_a), (x_b, y_b) and (x_c, y_c). Repeating the procedure by fixing each point P_i, $i = 1, 2, \ldots n$, n–centers and n–radii are found. To select the best fit circle following heuristic method is used.

1. Let
$$e_k = \sum_{i=1}^{n} \left[\sqrt{(x_i - a_k)^2 + (y_i - b_k)^2} - r_k\right]^2$$
(a_k, b_k) is center and r_k is radius of kth circle where, $k = 1,2, \ldots n$.
2. The mean and standard deviation of e_k, $k = 1,2, \ldots n$ are found.
3. The circles with e_k less than or equal to mean of e_k, $k = 1,2, \ldots n$ are selected.
4. Calculate the mean of the coordinates of centers and radii of these selected circles. This gives center and radius of the best fit circle.
5. Calculate circularity using equation (3.3) for the circle found in step 4.
6. Steps 1 to 5 are followed for (n – m) number of circles; where m is number of circles which are not selected in step 3.
7. If circularity calculated in step 5 is less than circularity calculated in previous iteration, go to step 7. Otherwise go to step 8.
8. The circle found in second last iteration is the claimed best fit circle.

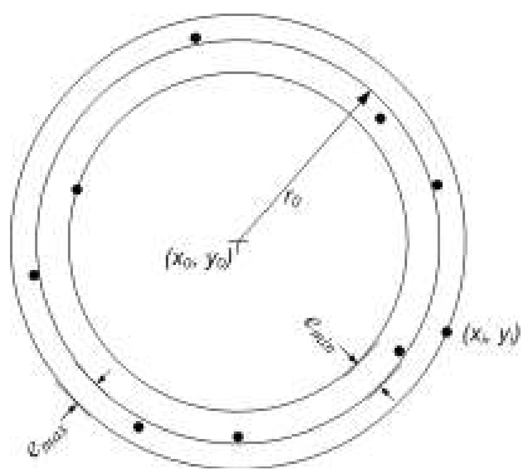

Figure 4.1 Definition of circularity error.

5 Results and Discussion

MATLAB programs for evaluating circularity by MDPS and LSM were executed on computer with Intel atom processor, 800 MHz clock speed and 1 GB RAM. The programs were run for CMM measured data set. A hole (circular feature) is measured using SCAN facility available on CMM. The SCAN facility ensures that points in the dataset are uniformly spaced. These measured points are tabulated in Table 1.

Table 2 shows results of circularity (h) evaluation for the dataset presented in Table 1. The results of MDPS and LSM are expressed up to six decimal places. It can be observed that circularity error obtained by MDPS is less than that of LSM. It can also be observed that the same is more than that obtained by CMM result. The CMM results were available up to three decimal places. If circle coordinates and radius values obtained by MDPS are rounded to third decimal place, the circularity error is the same as that obtained by CMM. Table 2 also shows the comparison of sum of squared deviation (e_s). It can be observed that sum of squared deviation of MDPS is minimum amongst all.

Simplex search is superior to the other methods in many cases [11]. Murthy and Abdin [9] have applied simplex search to find a circle to minimize the circularity error on simulated dataset. The same dataset is used to evaluate MDPS. The circularity error obtained by MDPS appears to be more than simplex search, but it is less than that obtained through LSM (refer Table 3). The MDPS can be used as starting solution for simplex search which can reduce number of iterations in finding the circle by simplex search method.

The programs are also executed for the data presented in table 1(a) of Samuel and Shunmugam [17]. Table 4 summarizes the results for circularity evaluation. It can be seen that the MDPS gives less circularity error than that of LSM, MCC (Maximum Circumscribe Circle) and MIC (Minimum Inscribe Circle). The circularity error obtained by MZ (Minimum Zone) is less than that of MDPS, but sum of squared deviation is higher.

6. Conclusions

The present paper proposes an approach termed as MDPS to determine dimensions of a cylindrical feature from CMM measured point datasets. MDPS is a simple method to understand and to implement amongst similar methods. It gives better results compared to Least Square Method (LSM) for CMM measured points (Table 2). It is also good on evaluating simulated dataset (Table 3). Results of MDPS are comparable with that of Simplex Search method. This method can be used as starting solution for Simplex Search as its solution is closer to Simplex Search compared to LSM. The MDPS results are also showing its potential compared to Maximum Circumscribe Circle (MCC), Minimum Inscribe Circle (MIC) and Minimum Zone (MZ) (Table 4). The developed methodology has great potential for implementation in CMM software for evaluation of circular features.

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Table 1: CMM measured dataset.

I	x_i	y_i	i	x_i	y_i	i	x_i	y_i
1	182.096	115.902	26	227.545	89.313	51	228.262	141.939
2	182.197	113.365	27	229.708	90.658	52	226.034	143.155
3	182.508	110.845	28	231.741	92.17	53	223.724	144.178
4	183.027	108.362	29	233.655	93.859	54	221.329	145.008
5	183.753	105.926	30	235.412	95.693	55	218.869	145.637
6	184.682	103.554	31	237.017	97.675	56	216.369	146.059
7	185.799	101.28	32	238.45	99.784	57	213.842	146.271
8	187.102	99.105	33	239.699	102	58	211.318	146.272
9	188.589	97.035	34	240.757	104.309	59	208.793	146.064
10	190.236	95.106	35	241.621	106.703	60	206.293	145.646
11	192.045	93.315	36	242.281	109.159	61	203.833	145.021
12	193.99	91.686	37	242.735	111.665	62	201.433	144.193
13	196.067	90.223	38	242.977	114.193	63	199.118	143.172
14	198.262	88.935	39	243.009	116.714	64	196.892	141.96
15	200.563	87.834	40	242.83	119.249	65	194.777	140.569
16	202.941	86.93	41	242.44	121.762	66	192.784	139.006
17	205.384	86.229	42	241.842	124.229	67	190.925	137.281
18	207.878	85.734	43	241.043	126.635	68	189.214	135.406
19	210.409	85.448	44	240.045	128.968	69	187.67	133.4
20	212.951	85.375	45	238.854	131.216	70	186.292	131.263
21	215.492	85.514	46	237.485	133.347	71	185.1	129.026
22	218.012	85.864	47	235.933	135.37	72	184.097	126.693
23	220.492	86.423	48	234.232	137.242	73	183.292	124.287
24	222.925	87.19	49	232.38	138.967	74	182.691	121.823
25	225.272	88.152	50	230.383	140.539	75	182.295	119.313

Table 2: Results of circularity evaluation for measured points tabulated in Appendix A.

	x_0 *(mm)*	y_0 *(mm)*	r_0 *(mm)*	*h* (µm)	*Sum of squared deviation,* e_s
MDPS	212.559041	115.834956	30.462729	1.151146	6.2964×10^{-006}
LSM	212.559001	115.834943	30.462731	1.170239	1.6659×10^{-005}
CMM result	212.559	115.835	30.463	1.125702	1.1785×10^{-005}

MDPS – Maximum Distance Point Strategy, LSM – Least Square Method

Table 3: Results of circularity evaluation for data presented in Muthy and Abdin [9].

	x_0 *(mm)*	y_0 *(mm)*	r_0 *(mm)*	*h* (µm)	*Sum of squared deviation,* e_s
MDPS	-2.363043	-0.526847	16.165762	0.972108	1.830282
LSM	-2.088	-1.206	16.027	1.829877	5.689571
Simplex	-2.1268	-1.1095	- *	0.955	- *

*R(Radius) is not presented in the reference, hence Sum of squared deviation cannot be calculated

Table 4: Results of circularity evaluation for data presented in Samuel and Shunmugham [17].

	x_0 *(mm)*	y_0 *(mm)*	r_0 *(mm)*	*h* (µm)	*Sum of squared deviation,* e_s
MDPS	39.999741	30.002065	25.003333	2.3737	7.5750×10^{-006}
LSM	40.0002	30.0012	25.0030	4.0930	1.1302×10^{-005}
MCC	40.0000	30.0014	25.0040	3.6102	1.7539×10^{-005}
MIC	40.0000	30.0010	25.0020	4.2930	1.7539×10^{-005}
MZ	39.9998	30.0022	25.0030	2.2621	6.3538×10^{-005}

MCC – Maximum Circumscribe Circle, MIC – Minimum Inscribe Circle, MZ – Minimum Zone