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Existence of New Sets in an Ideal Topological Spaces

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***Abstract*:- An ideal topological space is a triplet (X, τ, I), where X is a nonempty set, τ is a topology on X, and I is an ideal of subsets of X. In this article, we introduce L*-perfect, R*-perfect, and C*-perfect sets in ideal spaces and study their properties. We obtained a characterization for compatible ideals via L*-perfect sets. Also, we obtain a generalized topology via ideals which is finer than τ using L*-perfect sets on a finite set.**

Key words- L*-perfect sets, R*-perfect sets, and C*-perfect sets.

1. **Introduction and Preliminaries** The contributions of Hamlett and Jankovic (D. Jankovic and T. R. Hamlett, 1990) in ideal topological spaces initiated the generalization of some important properties in general topology via topological ideals. The properties like decomposition of continuity, separation axioms, connectedness, compactness, and resolvability (G. Aslim, A. Caksu Guler, and T. Noiri, 2005)(E. Ekici and T. Noiri, 2008) have been generalized using the concept of ideals in topological spaces. By a space (X, τ), we mean a topological space X with a topology τ defined on X on which no separation axioms are assumed unless otherwise explicitly stated. For a given point x in a space (X, τ), the system of open neighborhoods of x is denoted by $(x) = \{U \in \ : x \in U\}$. For a given subset A of a space (X, τ), cl(A) and int(A) are used to denote the closure of A and interior of A, respectively, with respect to the topology.

A nonempty collection of subsets of a set X is said to be an ideal on X, if it satisfies the following two conditions: (i) If $A \in \mathrm{I}$ and $B \subseteq A$, then $B \in \mathrm{I}$; (ii) If $A \in \mathrm{I}$ and $B \in \mathrm{I}$, then $A \cup B \in \mathrm{I}$. An ideal topological space (or ideal space) (X, τ, I) means a topological space (X, τ) with an ideal I defined on X. Let (X, τ) be a topological space with an ideal I defined on X. Then for any subset A of X, $A*(\mathrm{I}, \tau) = \{x \in X/A \cap U \notin \mathrm{I}$ for every $U \in N(x)\}$ is called the local function of A with respect to I and τ. If there is no ambiguity, we will write $A*(\mathrm{I})$ or simply $A*$ for $A*(\mathrm{I}, \tau)$. Also, cl$*(A) = A \cup A*$ defines a Kuratowski closure operator for the topology τ*(I) (or simply $\tau*$) which is finer than τ. An ideal I on a space (X, τ) is said to be condense ideal if and only if $\tau \cap \mathrm{I} = \{\emptyset\}$. X* is always a proper subset of X. Also, X=X* if and only if the ideal is condense.

Lemma 1 (D. Jankovic and T. R. Hamlett, 1990). Let (X, τ) be a space with I_1 and I_2 being ideals on X, and let A and B be two subsets on X. Then (i) $A \subseteq B \Rightarrow A^* \subseteq B^*$; (ii) $\mathrm{I}_1 \subseteq \mathrm{I}_2 \Rightarrow A^*(\mathrm{I}_2) \subseteq A^*(\mathrm{I}_1)$; (iii) $A^* = cl(A^*) \subseteq cl(A)$ (A^* is a closed subset of $cl(A)$); (iv) $(A^*)^* \subseteq A^*$; (v) $(A \cup B)^* = A^* \cup B^*$; (vi) $A^* - B^* = (A - B)^* - B^* \subseteq (A - B)^*$; (vii) for every $I \in \mathrm{I}$, $(A \cup I)^* = A^* = (A - I)^*$.

2. *-Perfect, R*-Perfect, and C*-Perfect Sets

In this section, we define three collections of subsets L, R and C in an ideal space and study some of their properties.
Definition 5. Let (X, τ, I) be an ideal topological space. A subset A of X is said to be (i) $L*$-perfect if $A - A* \in \mathrm{I}$, (ii) $R*$-perfect if $A* - A \in \mathrm{I}$, (iii) $C*$-perfect if A is both $L*$-perfect and $R*$-perfect. The collection of $L*$-perfect sets, $R*$-perfect sets, and $C*$-perfect sets in (X, τ, I) is denoted by L, R, and C, respectively. Remark 6. (i) If $\mathrm{I} = \{0\}$, then L $= \{A \subseteq X : A \subseteq A* = \mathrm{cl}(A)\}$ = the collection of all $*$ -dense-in-itself sets = $\wp(X)$, R $= \{A \subseteq X : A* \subseteq A\}$ = the collection of all $\tau*$ -closed sets = $\{A \subseteq X : A = A* = \mathrm{cl}(A)\}$ = the collection of all closed sets in (X, τ) = the collection of all $*$ -perfect sets, C $= \{A \subseteq X : A* \subseteq A, A \subseteq A* \} = AX : A = A* = \mathrm{cl}(A)$ = the collection of all closed sets in (X, τ) = the collection of all $*$ -perfect sets. (1) (ii) If $\mathrm{I} = \mathrm{P}(X)$, then L = T = $\wp(X)$. (iii) If $\tau \sim \mathrm{I}$, then L = $\wp(X)$ (by Theorem 4(iv)).

In this thesis we are going to present some existing theorems, propositions and lemma that shall be applied to establish the existence of new sets in an ideal topological space. These theorems state some properties of L*-perfect set which used to obtain a characterization for compatible and a generalize topology via ideals which is finer than τ.

3. Main result

Theorem 1. A subset A of a topological space X is closed if and only if A contains each of its accumulation points, i.e. A ∪ A* (Seymour, 1965).

Proof; Let $p \in (A \cup A^*)^c$. Since $p \notin A^*$, ∃ an open set G such that $p \in G$ and $G \cap A = \emptyset$ or $\{p\}$ however, $p \notin A$; hence in particular, $G \cap A = \emptyset$. We can also claim that $G \cap A^* = \emptyset$. If $g \in G$, then $g \in G$ and $G \cap A$ where G is an open set. So $g \notin A^*$ and thus $G \cap A^* = \emptyset$. Accordingly, $G \cap (A \cup A^*) = (G \cap A) \cup (G \cap A^*) = \emptyset \cup \emptyset = \emptyset$. $G \subset (A \cup A^*)^c$. Thus p is an interior point of $(A \cup A^*)^c$ which is therefore an open set. Hence A ∪ A* is closed.

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Proposition. 2 Manoharan and Thangavelu (2013) Let (X, τ, I) be an ideal space and $A \subseteq X$. The set A is L^*-perfect if and only if $F \subseteq A{-}A^*$ in X implies that $F \in \mathrm{I}$.

Proof. Assume that A is an L^*-perfect set. Then $A{-}A^* \in \mathrm{I}$. By heredity property of ideals, every set $F \subseteq A{-}A^*$ in X is also in I. Conversely, assume that $F \subseteq A{-}A^*$ in X implies that $F \in \mathrm{I}$. Since $A{-}A^*$ is a subset of itself, by assumption $A{-}A^* \in \mathrm{I}$. Hence A is L^*-perfect.

Theorem 3. (Manoharan and Thangavelu, 2013; Srivastarva and Gaur, 2020).
Let (X,τ) be a space with ideal I on X.
Then the following are equivalent $\tau \sim \mathrm{I}$.

(i) If A has a cover of open sets each of whose intersection with A is I, then A is in I.
(ii) For every $A \subseteq X$, $A \cap A^* = \emptyset \Rightarrow A \in \mathrm{I}$.
(iii) For every $A \subseteq X$, $A - A^* \in \mathrm{I}$.
(iv) For every τ^*-closed subset A, $A - A^* \in \mathrm{I}$.
(v) For every $A \subseteq X$, if A contains no nonempty subset B with $\mathrm{A} \subseteq B$, then $A \in \mathrm{I}$.

Definition 4. Srivastarva and Gaur, (2020) Let (X, τ, I) be an ideal topological space. A subset A of X is said to be L^*-perfect if $A{-}A^* \in \mathrm{I}$

Remark 5. If $\mathrm{I} = \{\emptyset\}$, then $\mathrm{L} = \{A \subseteq\ : A \subseteq A^* = \mathrm{cl}(A)\}$ = the collection of all * -dense-in-itself sets = $\wp(X)$.

Proposition 6.In an ideal space (X, τ, I), every τ^*-closed set is L*-perfect.

Proof. Let A be a τ^*-closed set. Therefore, $\ \subseteq A^*$. Hence $A - A^* = \phi = \mathrm{I}$. Therefore, A is an L*-perfect set.

Example 7. Let (X, τ, I) be an ideal space with $X = \{a, b, c\}$, $\tau = \{\phi, X, \{a\}\}$, and $\mathrm{I} = \{\phi, \{b\}\}$.The set $\{a, c\}$ is L*-perfect set which is not a τ^*-closed set and hence not a regular-I-closed set.

Proposition 8. Manoharan and Thangavelu (2013) In an ideal space (X, τ, I), every *-dense-in itself set is an L^*-perfect set.

Proof. Let A be a *-dense-in-itself set of X. Then $\ \subseteq A^*$. Therefore, $A{-}A^* = \phi \in \mathrm{I}$. Hence A is an L^*-perfect set.

Corollary 9. Manoharan and Thangavelu (2013) In an ideal space (X, τ, I)

(i) every I-dense set is L^*-perfect,
(ii) every I-open set is L^*-perfect,
(iii) every almost strong β-I-open set is L^*-perfect,
(iv) every almost I-open set is L^*-perfect,
(v) every regular-I-closed set is L^*-perfect,
(vi) every fI-set is L^*-perfect.

Proof. Since all the above sets are *-dense-in-itself, by Proposition 8 which said In an ideal space (X, τ, I), every *-dense-in itself set is an L^*-perfect set. Therefore, these sets are L^*-perfect.

Proposition10. Manoharan and Thangavelu (2013) In an ideal space (X, τ, I),

(i) empty set is an L^*-perfect set,
(ii) X is an L^*-perfect set if the ideal is codense.

Proof.

(i) Since $\phi{-}\phi^* = \phi \in \mathrm{I}$, the empty set is an L^*-perfect set.
(ii) We know that $X{=}X^*$ if and only if the ideal I is codense. Then $X{-}X^* = \phi \in \mathrm{I}$. Hence the result follows.

Remark 10.1. from the above theorems, corollaries and proposition we prove that finite union and intersection of L*-perfect sets are again L*-perfect set. Using these results, we obtain a new topology for the finite topological spaces which is finer than τ^*-topology. In Ideal spaces, usually $A{\subset}B$ implies $A^* \subset B^*$. We observe that there are some sets A and B such that $A{\subset}B$ but $A^* = B^*$.

Example 11.Let (X, τ, I) be an ideal space with $X = \{a, b, c, d\}$, $\tau = \{\phi, X, \{a, c\}, \{d\}, \{a, c, d\}\}$, $\mathrm{I} = \{\phi, \{c\}, \{d\}, \{c, d\}\}$. Here the sets $A = \{a\}$ and $B = \{a, b\}$ are such that $A \subseteq B$, but $A^* = B^* = \{a, b, c\}$.

Proposition12.(Manoharan and Thangavelu, 2013; Sebastian and Manoharan, 2018)
Let (X, τ, I) be an ideal space. Let A and B be two subsets of X such that $A{\subseteq}B$ and $A^* = B^*$; then A is L^*-perfect if B is L^*-perfect.

Proof. Let B be an L^*-perfect set. Then $B - B^* \in \mathrm{I}$. Now, $A{-}A^* = A{-}B^* \subseteq B{-}B^*$. By heredity property of ideals, $A{-}A^* \in \mathrm{I}$. Hence A is L^*-perfect.

Corollary 13. (Manoharan and Thangavelu, (2013) Let (X, τ, I) be an ideal space. Let A and B be two subsets of X such that $A \subseteq B \subseteq cl^*A$; then A is L^*-perfect if B is L^*-perfect

Proof. Since $A \subseteq B \subseteq \mathrm{cl}^*A$, $A^* \subseteq B^* \subseteq (\mathrm{cl}^*A)^* = A^*$. Hence $A^* = B^*$. Therefore, the result follows from Proposition 12.

Proposition 14. Manoharan and Thangavelu (2013). If a subset A of an ideal topological space (X, τ, I) is R^*-perfect set and A^* is L^*-perfect, then $A{\cap}A^*$ is L^*-perfect

Proof. Since A is R^*-perfect, $A^*{-}A \in \mathrm{I}$. By a Lemma $A^*{-}\mathrm{B}^* = (A - \mathrm{B})^* - \mathrm{B}^* \subseteq (A - \mathrm{B})^*$. for every $I \in \mathrm{I}$, $(A \cup I)^* = A^* = (A - I)^*$. Therefore, $(A^* \cup (A^* - A))^* = A^{**} = (A^* - (A^*{-}A))^*$.This implies $A^{**} = (A{\cap}A^*)^*$. Therefore, we have $A{\cap}A^* \subseteq A^*$ with $(A \cap A^*)^* = A^{**}$. By Proposition 14, $A{\cap}A^*$ is L^*-perfect if A^* is L^*-perfect set. Hence $A{\cap}A^*$ is L^*-perfect.

Proposition 15. . (Manoharan and Thangavelu 2013; Srivastarva and Gaur, 2020)..
If A and B are L^*-perfect sets, then $A{\cup}B$ is an L^*-perfect set

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Proof. Since A and B are L^*-perfect sets, $A-A^* \in$ I and $B - B^* \in$ I. Hence by finite additive property of ideals, $(A-A^*) \cup (B-B^*) \in$ I. Since $(A\cup B) - (A\cup B)^* = (A\cup B) - (A^* \cup B^*) \subseteq (A-A^*) \cup (B-B^*)$, by heredity property $(A\cup B) - (A\cup B)^* \in$ I. This proves that $A\cup B$ is an L^*-perfect set.

Corollary16. . (Manoharan and Thangavelu, 2013; Sebastian,and Manoharan, 2018; Srivastarva and Gaur, 2020) Finite union of L^*-perfect sets is an L^*-perfect sets.

Proof. Since $A_1, A_2, A_3, \dots, A_n$ and $B_1, B_2, B_3, \dots, B_n$ are L^*-perfect sets, $A_n - A_n^* \in$ I and $B_n - B_n^* \in$ I. Hence by finite additive property of ideals, $(A_n - A_n^*) \cup (B_n - B_n^*) \in$ I. Since $(A_n \cup B_n) - (A_n \cup B_n)^* = (A_n \cup B_n) - (A_n^* \cup B_n^*) \subseteq (A_n - A_n^*) \cup (B_n - B_n^*)$, by heredity property $(A_n \cup B_n) - (A_n \cup B_n)^* \in$ I. This proves that $A_n \cup B_n$ is an L^*-perfect set.

Proposition 17. If A and B are L*-perfect sets, then $A\cap B$ is an L*-perfect set.

Proof. Suppose that A and B are L*-perfect sets. Then $A - A^* \in$ I and $B - B^* \in$ I. By finite additive property of ideals, $(A - A^*) \cup (B - B^*) \in$ I. Since $(A \cap B) - (A^* \cap B^*) \subseteq (A - A^*) \cup (B - B^*)$, by heredity property $(A \cap B) - (A^* \cap B^*) \in$ I. Also $(A \cap B) - (A \cap B)^* \subseteq (A \cap B) - (A^* \cap B^*) \in$ I. This proves the result.

Corollary 18. Finite intersection of L*-perfect sets is an L*- perfect set.

Proof. Since $A_1, A_2, A_3, \dots, A_n$ and $B_1, B_2, B_3, \dots, B_n$ are L^*-perfect sets, then $A_n - A_n^* \in$ I and $B_n - B_n^* \in$ I. Hence by finite additive property of ideals, $(A_n - A_n^*) \cup (B_n - B_n^*) \in$ I. Since $(A_n \cap B_n) - (A_n^* \cap B_n^*) \subseteq (A_n - A_n^*) \cup (B_n - B_n^*)$, by heredity property $(A_n \cap B_n) - (A_n^* \cap B_n^*) \in$ I. Also $(A_n \cap B_n) - (A_n \cap B_n)^* \subseteq (A_n \cap B_n) - (A_n^* \cap B_n^*)$, This proves that $A_n \cap B_n$ is an L^*-perfect set.

Proposition 19. If $(X, \tau,$ I) is an ideal topological space with X being finite, then the collection L is a topology which is finer than the topology of τ^*-closed sets.

Proof. By proposition 10, X and ϕ are L^*-perfect sets. By Corollary 16, finite union of L^*-perfect sets is an L^*-perfect set, and by Corollary 18, finite intersection of L*-perfect sets is L*-perfect. Hence the collection L is a topology if X is finite. Also, by Proposition 6 every τ^*-closed set is an L*- perfect set. Hence the topology L is finer than the topology of $\tau*$-closed sets if X is finite.

Proposition 20.In an ideal space $(X, \tau,$ I), $\{\tau^*\text{-}closed\ sets\} \cup$ I $\subseteq$ L.

Proof. The proof follows from Propositions 8 and 10. The following example shows that $\{\tau^*$-closed sets$\} \cup$ I $\neq$ L.

Example 21.Let $(X, \tau,$ I) be an ideal space with $X = \{a, b, c\}$, $\tau = \{\emptyset, X, \{a\}\}$, and I $= \{\emptyset, \{b\}\}$. The collection of τ^*-closed sets is $\{\emptyset, X, \{b\}, \{b, c\}\}$ and L $= \{\emptyset, X, \{b\}, \{b, c\}, \{a, c\}\}$. Now, $\{\tau^*$-closed sets$\} \cup$ I $= \{\emptyset, X, \{b\}, \{b, c\}\} \neq$L.

Proposition22 (Manoharan and Thangavelu, 2013, Sebastian,and Manoharan, 2018).
.Let $(X, \tau,$ I) be an ideal space and $A\subseteq X$. The set A is L*-perfect if and only if $F\subseteq A - A^*$ in X implies that $F \in$ I

Proof. Assume that A is an L*-perfect set. Then $A - A^* \in$ I. By heredity property of ideals, every set $F \subseteq A - A^*$ in X is also in I. Conversely assume that $F \subseteq A - A^*$ in X implies that $F \in$ I. Since $A - A^*$ is a subset of itself, by assumption $A - A^* \in$ I. Hence A is an L*-perfect.

Theorem 23.Let (X, τ) be a space with an ideal I on X.
Then the following are equivalent. (i) $\tau \sim$ I. (ii) If A has a cover of open sets each of whose intersection with A is I, then A is in I. (iii) If $A\subseteq X$, then $A\cap A^* = \phi \Rightarrow A\in$ I. (iv) If $A\subseteq X$, then $A - A^* \in$ I. (v) If $A\subseteq X$ and A is L*-perfect set, then $A\Delta A^* \in$ I. (vi) For every $\tau*$-closed subset A, $A - A^* \in$ I. (vii) For every $A\subseteq X$, if A contains no nonempty subset B with $B \subseteq B^*$, then $A \in$ I (Manoharan and Thangavelu, 2013).

Proof. To prove this theorem, it is enough to prove (iv) $\Rightarrow$ (v) $\Rightarrow$ (vi). Others follow from Theorem 3. (iv) $\Rightarrow$ (v) follows from a Proposition if a subset A of an ideal space $(X, \tau,$ I) is C*-perfect, then $A\Delta A^* \in$ I. Suppose that $A\Delta A^* \in$ I. Since $A - A^* \subseteq A\Delta A^*$, by heredity property $A - A^* \in$ I. Hence (v) $\Rightarrow$ (vi).

4. L*-Topology

By corollary 9 and Proposition 17, we observe that the collection L satisfies the conditions of being a basis for some topology and it will be called as L* c $(\tau,$ I). We define L*$(\tau,$ I) $= \{A \subseteq X / X - A \in$ L* c $(\tau,$ I)$\}$ on a nonempty set X. Clearly, L*$(\tau,$ I) is a topology if the set X is finite. The members of the collection L*$(\tau,$ I) will be called L*- open sets. If there is no confusion about the topology τ and the ideal I, then we call L*$(\tau,$ I) as L*-topology when X is finite.

Definition 24. A subset A of an ideal topological space $(X, \tau,$ I) is said to be L*-closed if it is a complement of an L*-open set.

Definition 25.Let A be a subset of an ideal topological space $(X, \tau,$ I). One defines L*-interior of the set A as the largest L*-open set contained in A. One will denote L*-interior of a set A by L* $-$ int(A).

Definition 26.Let A be a subset of an ideal topological space $(X, \tau,$ I). A point $x\in A$ is said to be an L*-interior point of the set A if there exists an L*-open set U of x such that $x \in U \subseteq A$.

Definition27. Let $(X, \tau,$ I) be an ideal space and $x\in X$. One defines L*-neighborhood of x as an L*-open set containing x. One denotes the set of all L*-neighborhoods of x by L*-(x). (Jankovic and Hamlett, 1990)

Proposition 28.In an ideal space $(X, \tau,$ I), every τ^*-open set is an L*-open set.

Proof. Let A be a τ^*-open set. Therefore, $X - A$ is a τ^*-closed set. That implies that $X - A$ is an L*-closed set. Hence A is an L*-open set.

Corollary 29. The topology L*$(\tau,$ I) on a finite set X is finer than the topology $\tau^*(\tau,$ I).

Proof. The proof follows from Proposition 28.

Corollary 30. For any subset A of an ideal topological space $(X, \tau$, I), (A) is an L*-open set.

Proof. The proof follows from Proposition 28.

Remark 31.(i) Since every open set is an L*-open set, every neighborhood U of a point $x \in X$ is an L*-neighborhood of x. (ii) If $x \in X$ is an interior point of a subset A of X, then x is an L*-interior point of A. (iii) From (ii), we have int$(A) \subseteq$ int*$(A) \subseteq$ L*-int(A), where int*(A) denotes interior of A with respect to the topology τ^*.

Theorem 32. Let A and B be subsets of an ideal space $(X, \tau$, I) with X being finite. Then the following properties hold. (i) L*-int$(A) = \cup\{U : U \subseteq A$ and U is an L*-open set$\}$. (ii) L*-int(A) is the largest L*-open set of X contained in A. (iii) A is L*-open if and only if A=L*-int(A). (iv) L*-int(L*-int(A)) = L*-int(A). (v) If $A \subseteq B$, then L*-int$(A) \subseteq$ L*-int(B).

Proof. Let A be a subset of an ideal topological space $(X, \tau$, I). One defines L*-interior of the set A as the largest L*-open set contained in A. One will denote L*-interior of a set A by $L^* - \text{int}(A)$. a point $x \in A$ is said to be an L*-interior point of the set A if there exists an L*-open set U of x such that $x \in U \subseteq A$. And let $(X, \tau$, I) be an ideal space and $x \in X$. We defines L*-neighborhood of x as an L*-open set containing x. We denote the set of all L*-neighborhoods of x by L*-(x).

Definition 33. Let A be a subset of an ideal topological space $(X, \tau$, I). One defines L*-closure of the set A as the smallest L*-closed set containing A. One will denote L*-closure of a set A by L*-cl(A).

Remark 34. For any subset A of an ideal topological space $(X, \tau$, I), L*-cl$(A) \subseteq$ cl*$(A) \subseteq$ cl(A).

Theorem 35. Let A and B be subsets of an ideal space $(X, \tau$, I) where X is finite. Then the following properties hold: (i) L*-(A) = $\cap\{F : A \subseteq F$ and F is L*-closed set$\}$. (ii) A is L*-closed if and only if A=L*-(A) (iii) L*-(L*-(A)) = L*-(A) (iv) If $A \subseteq B$, then L*-$(A) \subseteq$ L*-(B).

Proof. Let A be a subset of an ideal topological space $(X, \tau$, I). We define L*-closure of the set A as the smallest L*-closed set containing A. We will denote L*-closure of a set A by L*-cl(A).

Theorem 36.Let A be a subset of an ideal space $(X, \tau$, I). Then the following properties hold: (i) L*-int$(X - A) = X$ – L*-$cl(A)$; (ii) L*-$cl(X - A) = X$ – L*-int(A).

Proof. Let A be a subset of an ideal topological space $(X, \tau$, I). We define L*-interior of the set A as the largest L*-open set contained in A. We will denote L*-interior of a set A by $L^* - \text{int}(A)$, a point $x \in A$ is said to be an L*-interior point of the set A if there exists an L*-open set U of x such that $x \in U \subseteq A$, and we define L*-closure of the set A as the smallest L*-closed set containing A. One will denote L*-closure of a set A by L*-cl(A).

5. CONCLUSION

Based on the research conducted we can easily observe that some new set in an ideal topological space exist. We also observed that L*-perfect form topology. Therefore in this journal we studied the existence of new set in an ideal topological space by presenting theorems and proposition some examples are also given to illustrate theorems that can be easily employed on the topological space.

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