

# Experimental Identification of Thrust Dynamics for a Multi-Rotor Helicopter

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**Abstract**—In this paper we propose a simple experimental procedure for an identification of the thrust dynamic for a multi-rotor helicopter. As a case study, we experimentally confirm that the transfer function from a PPM speed command for an ESC (electrical speed controller) driving a BLDC (brushless DC) motor to the thrust force generated by a propeller can be modeled as a simple first-order system

**Keywords**— Multi-rotor Helicopter; Drone; Thrust Dynamics

## I. INTRODUCTION

Multi-rotor helicopters informally called as *drones*, attracted much attention and interest from both academic community and general public. Various applications of such helicopter for search and rescue operation, mapping, aerial photograph, surveillance and so on, have been proposed in diverse field. Recently toy-level commercial drones are also widely used in academic community, e.g. see [1,2,3].

A small battery-powered drone is composed of mechanical frames, a set of motors with propellers, battery and electrical boards for stabilization, sensing, communication and navigation guidance. A typical drone, which is considered in our case study, is shown in Fig. 1. For a controller design and implementation for drones one needs to have a dynamic modeling of a whole drone system.



Fig. 1 A Typical Drone (Case Model)

Drones typically have simple mechanical structures and hence a dynamic modeling of its mechanical part, i.e., the rigid body dynamics, is well known [3].

A real difficulty however is how to characterize the dynamic behavior of a BLDC motor, ESC and aerodynamic forces and moments generated by propellers installed on motors. This difficulty also comes from the fact that a commercial motor control unit, commonly called as an ESC (electrical speed controller) for a BLDC motor, has its own dynamical property, a set of selectable operational modes, communication protocol and even its own user-configurable speed controller. In fact a precise dynamic modeling of an ESC itself is a hard task [4].

In this paper we propose an experimental approach for characterizing the dynamics of propeller thrust. The dynamic input-output relation from the ESC command to the thrust force is regarded as an unknown *black-box* subsystem whose transfer function needs to be identified.

## II. EXPERIMENT

### A. Sensor Design

We designed and implemented two sensors; a force sensor to measure the thrust of a propeller and an optical sensor for the angular velocity of motor.

A thrust force is measured with a low cost load-cell type scale. The scale is composed of a strain gauge and an IC amplifier HX711 from AVIA Semiconductor © which includes an instrument amplifier and a 24-bits AD converter. The digital signal from HX711 is captured by a microprocessor module (Arduino Due ©) and then converted to an analog signal again for an easy monitoring with an oscilloscope.

A simple optical sensor system is made in order to measure the rotational angular velocity of a propeller. This sensor is composed of an infrared LED, an optical transistor and an OP amplifier in order to ensure a high voltage signal when the blade reflects an infrared ray from LED.

The force sensor and the velocity sensor are installed as shown in Fig. 2.

As a first step, the thrust sensor is calibrated making use of objects whose weights are measured with another precise scale. For this, we made the configuration of propeller and load cell in Fig. 2 upside down and hang objects from the motor center with a wire.

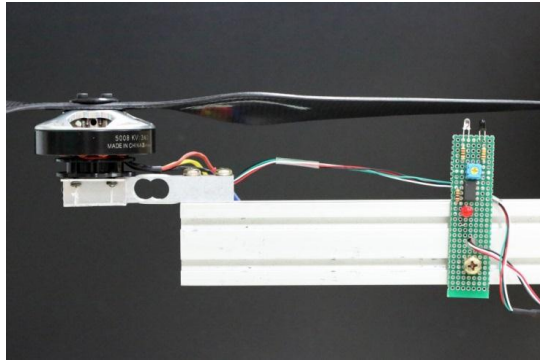


Fig. 2 Sensor Configuration

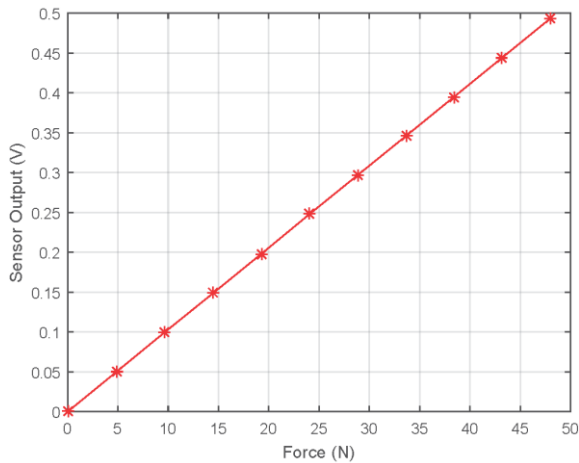


Fig. 3 Force Sensor Output

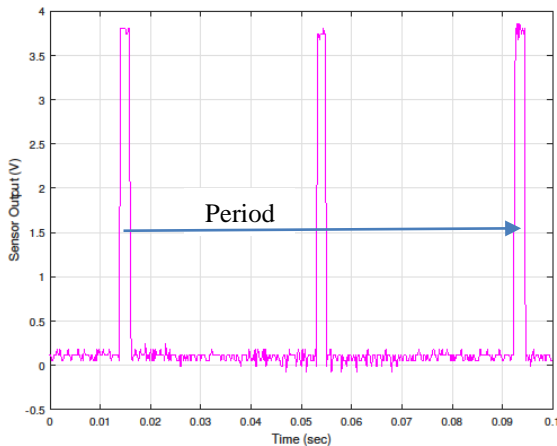


Fig. 4 Velocity Sensor Output

The relation between known weights of objects and the output of thrust sensor is shown in Fig. 3. From a linear curve fitting of the data in Fig.3, we were able to obtain a linear relation between the thrust force and the load cell output.

The output of rotational velocity sensor was shown in Fig.4. From the rising edge of every second pulse we could measure the period, which gives the angular frequency of the motor rotor.

TABLE I. COMPONENTS SPECIFICATION

BLDC Motor	Motor Outer Diameter	58.5 mm
	Stator Diameter	50.0 mm
	Speed per Volt	340 RPM /V
	Stator Number	12
	Motor Poles	14
	Weight	168 g
Propeller	Length	18 inches
	Pitch	5.5 inches
	Material	carbon fiber
	Blade Root Thickness	3.3 mm
Load Cell	Capacity	5 kg
	Resistance	1000 $\Omega$
	Material	Aluminum
	Nonlinearity	0.05 %
ESC	Output (continuous)	40 A
Battery	Type	LiPo
	Capacity	10000 mAh
	Nominal Voltage	22.2 V
	Discharging Rate	25C

### B. Motor Electronic Speed Controller (ESC)

From a historical reason within the author's knowledge, most commercial ESC's are following a special driving protocol. Typically, the driving signal is PPM (pulse position modulation) signal with 50 Hz frequency (fixed mostly) and the duty ratio is between 5% and 10%.

In our experiment a PPM signal is made with a function generator (Agilent 33220A). With this signal, we were able to find that our BLDC motor starts to move with 6% duty ratio and reaches its maximum speed when the duty ratio reaches around 8.8 %.

### C. Components Technical Specification

Several technical specifications of components which were used in our experiment are given in Table 1.

## III. RESULTS

### A. Static Thrust

By changing the duty ratio of ESC over the range (6.0, 8.8) % with a step size 0.2%, we have measured the outputs of force and velocity sensors.

A measurement of angular velocity sensor with different duty ratios gave the results in Fig. 5. It is remarkable that with a small duty ratio the relation between the duty ratio and angular velocity is almost linear.

In addition, from the data of force sensor, we have obtained the relation between duty ratio and thrust force as shown in Fig. 6. By combining Fig. 5 and Fig. 6, we were able to find the relation between propeller angular velocity and *static* thrust force as shown in Fig. 7. It is clear from this result that the thrust force is a quadratic function of angular velocity. From a curve fitting of data in Fig. 7 we have found that the quadratic relation can be explicitly given

$$T_{exp} = 1.24 \times 10^{-6} \omega_{RPM}^2 \quad (1)$$

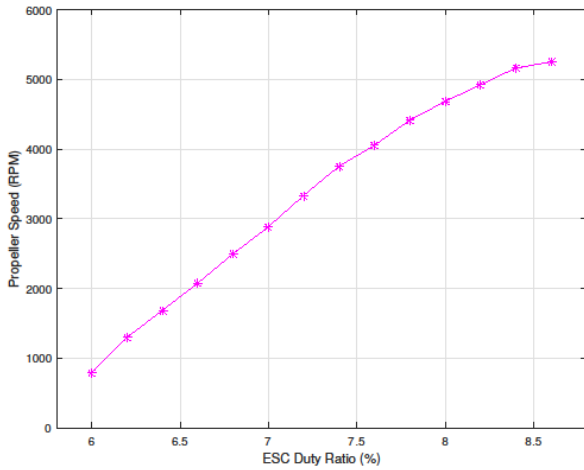


Fig. 5 Angular Velocity versus Duty Ratio

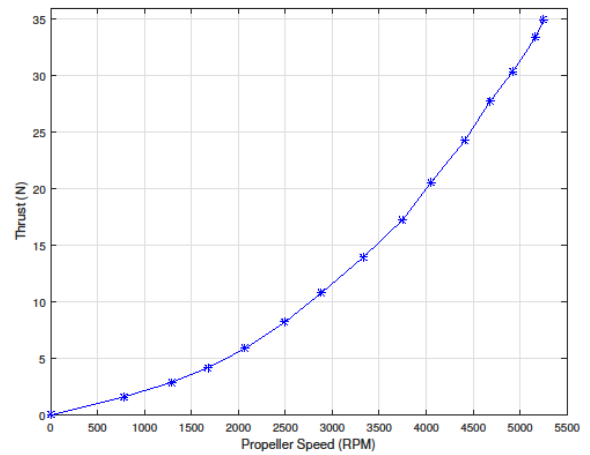


Fig. 7 Static Thrust versus Velocity

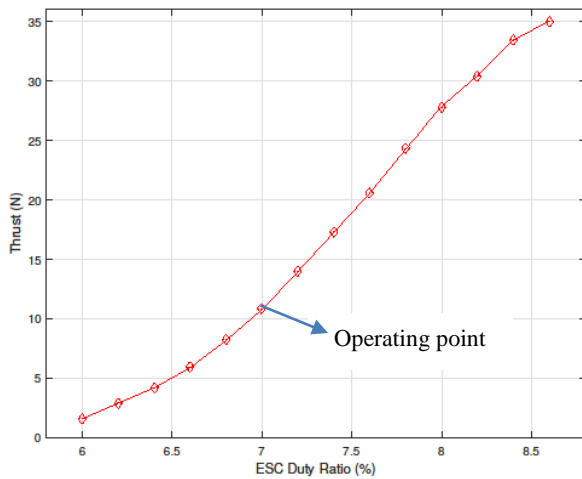


Fig. 6 Thrust versus Duty Ratio

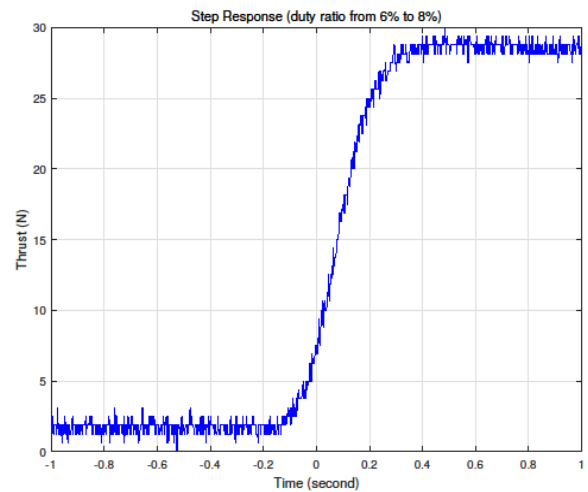


Fig. 8 Thrust Step Responses

where  $T$  denotes the thrust force in Newton and  $\omega_{RPM}$  denotes the angular velocity in RPM.

A well-known theoretical value of the thrust is given as

$$T_{theory} = \rho \frac{\pi d^2}{4} \times \frac{p^2}{3600} \omega_{RPM}^2 \quad (2)$$

where  $\rho = 1.225 \text{ kg/m}^3$  is the air density,  $d$  and  $p$  denote the diameter and pitch of a propeller in meter unit [5]. By plugging our data  $d = 18$  (inches) and  $p = 5.5$  (inches) in Table 1 into the above thrust equation (2), we obtained the following theoretical relation between angular velocity and static thrust force which is quite close to our experimental result;

$$T_{theory} = 1.09 \times 10^{-6} \omega_{RPM}^2 \quad (3)$$

### B. Dynamic Thrust

In order to characterize the dynamic behavior of thrust force, we captured the response of thrust force sensor after applying a step (PPM) command to our ESC unit as shown in Fig. 8.

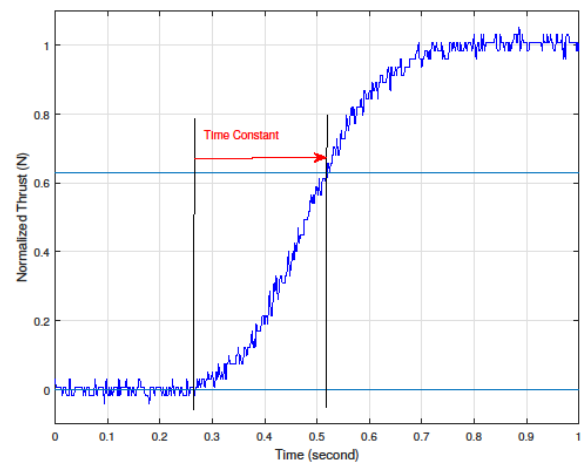


Fig. 9 Normalized Step Response

The data in Fig. 8 strongly suggests that the dynamic response of thrust force with respect to a step PPM command for ESC unit can be modeled as a first-order system, e.g. see [6].

For a more precise estimation of thrust dynamics, we normalized the responses in Fig.8 to obtain the normalized step response in Fig. 9. From Fig. 9, we could experimentally

obtain the rising time  $\tau \cong 0.25$  (second) of thrust force output with respect to the PPM command input for ESC unit. Consequently we could identify the transfer function between PPM command  $\Delta U(s)$  and thrust force  $\Delta T(s)$  as follows;

$$\Delta T(s) = \frac{k}{s + 1/\tau} \Delta U(s) \quad (4)$$

where  $k$  is an unknown constant which will be determined later and  $\Delta T(s)$ ,  $\Delta U(s)$  are the Laplace transforms of the thrust force and PPM command for ESC, respectively.

For an estimation of the constant  $k$  in (4), that is, the DC gain of the transfer function (4), we could use the experimental results in Fig. 6. We choose an operating (nominal) point around

$$\begin{aligned} U_{nom} &= 7.0 \% \\ T_{nom} &= 10.8 \text{ N} \end{aligned} \quad (5)$$

for a PPM command  $U = U_{nom} + \Delta U$ . In addition, from a linear approximation of the relation between  $\Delta T(s)$ ,  $\Delta U(s)$  around the operating point  $(U_{nom}, T_{nom})$  in Fig.6, we could find a proportional coefficient 14.50 which corresponds to the DC gain of (4). This allowed us to find an equality relation

$$k = \frac{14.50}{\tau} = 58.0$$

and thus finally we could find the following transfer function

$$\frac{\Delta T(s)}{\Delta U(s)} = \frac{58}{s + 4} \quad (6)$$

which is valid around our choice of an operating point at  $(U_{nom}, T_{nom}) = (7\%, 10.8 \text{ N})$ .

Note that the operating point should be chosen from a required thrust of a single propeller in an actual design of a drone controller,

#### IV. CONCLUSION

From a series of elementary experiments along with some simple analysis, we could identify both the static and dynamic property of the propeller thrust for a multi-rotor helicopter. Even though our experiment was quite elementary without any special instruments, our experimental results showed good agreement with theoretical predictions. We believe our results in this paper should be generalized and confirmed with more general communication protocols and user-programmable parameters of ESC in future.

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