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Factorial Experimental Modelling of Ball Milling Response for Baban Tsauni (Nigeria) Lead-Gold Ore.

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Abstract

An empirical model of the ball milling response of Baban Tsauni (Nigeria) lead-gold ore was developed in this work by applying factorial experimental design. The four factors considered were, mill speed, steel ball diameter, grinding time and the ratio of mass of ball to mass of ore samples. The factors were treated at two levels and the measurable ball milling response was the size reduction ratio. The student t-test was used for determining the significance of the factors. All the factors were found to have both main and interactive effect on ball milling response. Predicted values calculated using model equation were in good

agreement with experimental values of the response $(R^2 = 0.997)$. The mathematical model indicated that ball milling output increased with increase in grinding time, ball mass to ore sample mass ratio and very slowly with increase in mill speed. The response decreased with increase in ball size. This work established a model for selecting the parameters required for a desired size reduction ratio. The work also indicated that ball milling response is a function of the feed particle size.

Key words: four factors at two levels, ball milling response, model equation, experimental design.

1.0 Introduction

Energy consumption in cumminution is the highest in mineral dressing processes [13], [19] and conservation of energy can be achieved by controlling the conditions that affect grinding response.

A well structured factorial experimental design makes it possible for effects of several variables on the performance of a system to be determined with a few

experimental runs and a line of action for product or process improvement is identified. The effects of interactions between factors can also be examined. The data generated from the designed experiment are used to develop a modelling equation which is subjected to significance test and analysis of variance to determine quality of the model before validation and optimization [1], [14], [11] [3].

2.0 Literature review

The resistance of materials to breakage is often expressed as its grindability and the Bond's Work index has generally been accepted as the measure of grindability in the mineral industry [2], [20], [4], [8]. The effects of operating conditions on the performance of mills have been observed over the years [15], [10], [19]. It has also been indicated that the product size, the power consumption and production rate are influenced by the size of the ball utilised in a ball milling operation [6], [9], [18]. The number of balls and particle density have also been indicated as factors that affect the performance of a ball mill [15].

In a factorial design at two levels each of the factors are taken at two treatment levels: low (-1) and high (+1). The numbers of factors which are perceived to influence the process determine the number of

experimental runs for a full factorial design at two

levels. When three factors are considered to be of importance the factorial design denoted by 2³ will require eight experimental runs [11], [15]. As a rule each of the runs must be unique and the order of run is usually random in order to avoid structural error. The modelling equation for the 2³ factorial experiments above is given by:

$$Y=b_0+b_1X_1+b_2X_2+b_3X_3+b_{12}X_1X_2+b_{13}X_1X_3+b_{23}X_2X_3+b_{123}X_1X_2X_3$$

where b_0 , b_1 , b_2 to b_{123} are coefficients which are determined from measurable responses of eight runs; X_1 , X_2 , X_3 are the coded values of the factors of interest and Y is a measurable performance indicator [15]. A well designed experiment provides the eight responses required to solve eight simultaneous equations that are obtainable from Equation 1.

It is indicated that a good modelling information can be obtained from a half factorial design of experiments[11]. Particularly when a large number of factors appear to be of interest, a half factorial design provides a means of screening out the insignificant factors by carrying out smaller number of test runs. It is a subset of full experimental design. Further improvement design at more levels with only significant factors from the screening stage experiment leads to optimum conditions.

3.0 Experimental Procedures

The cylindrical mill that was used has internal diameter of 125mm and length of 175mm. The mill did not have lifters. An initial sample preparation aimed at ensuring a homogeneous feed for subsequent test runs was carried out. Crushed ore samples from jaw crusher and roll crusher (in that order) were subjected to the same grinding conditions of mill speed, ore mass to ball mass ratio, ball size and grinding time of 10minutes [5]. The products of this batch milling operations were mixed thoroughly and passed through a Jones riffling sampler, until sets of 0.51kg samples were obtained.

Four factors were considered in this work, namely; mill speed (x_1) , ball diameter (x_2) , holding time (x_3) and Mass of ball to mass of ore ratio (x_4) . The factors were considered at two levels. The high level was coded +1 and the low level -1. The form of the modelling equation adopted for this work is given by,

$$Y = b_0 + b_1 X_1 + b_2 X_2 + b_3 X_3 + b_4 X_4 + b_{12} X_1 X_2 + b_{13} X_1 X_3 + b_{14} X_1 X_4 + b_{23} X_2 X_3 + b_{24} X_2 X_4 + b_{34} X_3 X_4$$

$$2$$

where all the parameters are same as defined for Equation 1. Table 1 shows the coded and actual values of the factors for the test runs. To avoid structured error, the experimental runs were performed in a random order. There are eleven unknown parameters in Equation 1 and twelve unique responses were obtained for the simulation of a modelling equation while the replicated test runs were used for analysis of variance. The response that was measured was breakage reduction ratio given by, 80% passing size of feed divided by 80% passing size of the product $(F_{80}/P_{80} [5])$. The set of twelve unique responses were used with Equation 1 on a MATLAB programme to generate the coefficients for the modelling equation. The significance of main effects was tested with student t-test. The validation and optimum test runs were carried out and the responses were subjected to residual analysis.

Table 1: The coded and actual values of the factors for the test runs

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Coded	$\mathbf{x}_1 =$	$x_2 = Ball$	$x_3=$	x ₄ =
values	Speed	diameter	Grinding	M_B/M_O
	(rpm)	(mm)	time	
			(minutes)	
-1	90.6	20	5	2
+1	131.1	40	15	6

4.0 Results and Discussion

The results of particle size analyses for the feeds used for factorial experimental design of the effects of grinding conditions are presented in Figure 1. The 80% passing size was determined from the equation of the line. The equation of the line in Figure 1 is,

$$y = -0.00002x^2 + 0.0958x - 26.849$$

The eighty per cent passing size of the feed (F_{80}) was calculated by substituting y = 80 into Equation 3.

Therefore,

$$x^2 - 4790x + 5342450 = 0.$$

Solving the quadratic equation yielded, $x = F_{80} =$ 1767.644µm.

The products of each unique grinding condition were subjected to particle size analyses and the 80% passing sizes (P80) were calculated in the same way as for the

The summary of the responses in terms of ratio of F_{80}/P_{80} for twelve unique grinding conditions, which were required for the calculation of the eleven coefficients of Equation 2, are presented in Table 2.

There are eleven unknown parameters in Equation 2 and matrix methods yielded easy solution.

$$\underline{y} = \underline{X} \cdot \underline{\beta}$$
 5 z = column matrix of the responses (column

where y = column matrix of the responses (column 13)on Table 2)

 $X = an m by n calculation matrix of coded values of <math>x_k$ deduced from Equation 1 (Table 3, column 2 to 12 by row 2 to 13), with b_0 represented by one and $\underline{\beta}$ = column matrix of the coefficients. Multiplying Equation 2 by \underline{X}^{T} and rearranging yielded, $\underline{\beta} = (\underline{X}^{T} \cdot \underline{X})^{-1} \cdot \underline{X}^{T} \cdot \underline{y}$

$$\underline{\mathbf{S}} = (\underline{\mathbf{X}}^{\mathrm{T}} \cdot \underline{\mathbf{X}})^{-1} \cdot \underline{\mathbf{X}}^{\mathrm{T}} \cdot \underline{\mathbf{y}}$$

The calculation matrix (Table 3) was used with Equation 6 in a MATLAB programme to generate the coefficients of the modelling equation (Equation 2) and these coefficients are presented in Table 4.

In order to test for the significance of the coefficients by using student's t-test, the variance of the effect estimates and the average effect were calculated from the variance of the replicated observations [17], [12], [5]. The calculated variance for the replicated test runs are presented in Table 5.

Adopting the expression given by [3], the variance in the test run condition G₁ became,

$$S_1^2 = (u_{11} - \overline{u}_1)^2 / (n-1) + (u_{12} - \overline{u}_1)^2 / (n-1) + (u_{13} - \overline{u}_1)^2 / (n-1)$$

$$7$$

where u_{1i} = individual responses in G_{1i} for i = a, b and c, \bar{u}_1 = mean of the responses in G_1 and n = number of replicates. Substituting the responses yielded,

$$S_1^2 = (0.000441 + 0.000169 + 0.001089)/(3-1) = (0.001699)/2 = 0.0008495$$

By a similar calculation, the variance in the replicated test run condition G_8 was found to be 0.0009. The pooled variance (S_p^2) is the mean of all variance from replicated test runs. Thus,

$$S_p^2 = (0.0008495 + 0.0009)/2$$

$$= 0.000875$$

According to Devor et al. (2007) the sample variance of an effect is calculated by,

$$S_E^2 = \frac{4(S_p^2)}{N}$$
 8

where N = the total number of test runs used for the model. In this work N = 16 and hence,

$$S_E^2 = \frac{4(0.000875)}{16} = 0.00021875$$

The standard error (s.e) of the effects is the square root of the sample variance and hence,

s.e =
$$\sqrt{(0.00021875)} = 0.0147902$$

The sample variance of the average (S_{av}^2) is given by, $S_{av}^{2} = S_{p}^{2}/N$

$$= 0.000875/16 = 0.0000547.$$

Therefore,

$$s_{av} = \sqrt{(0.0000547)} = 0.0073959$$

4.1 Hypotheses test

The t-statistics was calculated for each effect estimate (under the hypothesis; H_0 : $\mu_{effect} = 0$) by,

$$t=(E_i-\mu_{effect})/s.e$$
 10

where E_i = effect estimate = $2xb_i$ for i = 1, 2, 3, ... 34and the b_i are the coefficients of the anticipated model

The alternative hypothesis is, $H_0: \mu_{effect} \neq 0$.

Substituting for $b_1 = -0.1107$ yielded $E_1 = -0.221$ and by Equation 4.20,

t=
$$(E_1-\mu_{effect})/(0.0147902)$$
 = $(-0.221-0)/(0.0147902)$ = -14.942 .

For the average effect, $b_0=2.2908=E_0$ and the t-value

$$t = \frac{E_0 - \mu_{effect}}{S_{av}} = \frac{2.2902}{0.0073959} = 309.735$$

The null hypothesis requires that μ_{effect} should be 0. The null hypothesis was rejected when the calculated t-value of an effect or average effect was greater than corresponding t-distribution at a desired confidence level [17], [12], [3]. A 95% confidence level was considered adequate in this work. The corresponding t-distribution value for four degrees of freedom (t_{4,0.975} or t_{4,0.025}) was read from standard t-test

table [11] as 2.776. Table 6 presents the calculated associated t-values for all the estimated effects.

The results obtained when the estimated effects were subjected to T-test (Table 6) showed that the calculated absolute t values for all the effects were higher than $t_{4,0.975}$ (or $t_{4,0.025}$). This implied that the main effects and interaction effects affected the grinding outputs which were measured in terms of size reduction ratio- F₈₀/P₈₀ at a confidence level of 95% [3], [11]. This meant that interactions between the four factors under consideration were important in addition to the individual effects of the factors.

4.2 The equation for the model

The implication of rejecting the null hypothesis for all the effects was that the model equation should contain all the coefficients of Table 5. The resulting modelling equation for predicting future grinding outputs then became Equation 12.

$$Y' = 2.2908 - 0.1107X_1 - 0.0572X_2 + 0.6941X_3 + 0.8131X_4 + 0.0258X_1X_2 + 0.0663X_1X_3 + 0.1536X_1X_4 - 0.1424X_2X_3 - 0.2072X_2X_4$$
 12 + 0.4491X_3X_4

An empirical model is normally checked for possible presence of structured error before validation and adoption for future prediction. This was done by subjecting the model to residual analysis. The calculated residuals of all the responses used for generating the model and their percentages are presented on Table 7.

The actual residuals and their percentages (as functions of the actual responses) are shown on the tenth and eleventh columns of Table 7 respectively. It was observed that the per cent residuals varied from -0.737% to a maximum of 4.884%. The levels of these residuals were quite low and more so when the complex environment in a grinding mill is considered. Figure 2 shows a plot of predicted responses against the actual responses. A correlation of 0.997 was found column of Table 8). This observation supported the submission above on the effect of feed size. Moreover a considerable change in the grinding time from the initial mean value caused a shift in the mean particle size in the bulk sample. This explained the slight

between the predicted values and the actual values.

The modelling equation indicated that positive coded values of X_1 (grinding speed), X_3 (grinding time) and X₄ (M_B/M_O) enhanced grinding output. This implied that grinding output increased as grinding speed, holding time and mass of ball to mass of ore ratio increased above 110.5rpm (or 0.849V_c), 10 minutes and ratio of 4 respectively. On the contrary negative values of X_2 improved the grinding response. Positive (or negative) coded value is relative to the mean values used in the experiments.

Figure 3 show the predicted responses as each factor was varied within a feasible range while the other three factors were kept at the highest level of their experimental values. The responses converged to 3.975 which marks the best response within the range of values used in the experiment.

4.3 Model validation

The results of breakage responses of nine model validation test runs are shown in Table 8.

The results of the first three validation test runs showed very small deviation from the predicted values with residuals ranging from 0.97% to 2.225%. The results showed reasonable deviation from the predicted values when the feed sizes (V4, V5,....V9) were altered from the value used in generating the model.

Two reasons were most likely responsible for this increase in deviation: (1) the difference in the feed size and (2) the effect of grinding time on the mean particle size in the bulk samples in the mill. It is fairly well known that the grindability of an ore increases with decrease in particle size [16], [7], [20]. A change in feed size directly implied a change in the bulk particle size if all other conditions (ball size, mill speed, grinding time and ball mass to sample mass ratio) were kept constant. The 80% feed size for the factorial grinding experiment was 1767.644µm. The divergence between predicted and actual responses increased with decrease in the 80% passing size of the feed (last

difference in residuals recorded for samples ground for 25minutes and 20minutes.

When the complex environment in grinding mills was considered, the model gave a good prediction of ball milling response of Baban Tsauni ore.

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5.0 Conclusion

This research work has established a mathematical relationship between the grinding response of Baban Tsauni lead –gold ore and ball milling conditions. All the factors investigated in this work have direct and interactive effect on the grindability of the ore. The model equation will serve as a valuable tool for comminution flow sheet design and production planning. The effect of feed size on breakage response was observed in this work. An adequate correction factor that will account for the effect of feed size on grindability is desired to make the model more versatile.

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Appendix: Figures and Tables.

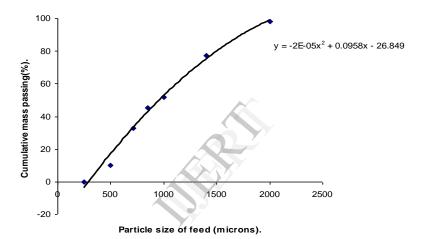


Figure 1: Particle size analysis for factorial experimental design feeds

Table 2: The grinding conditions and summary of responses for 2^{4**} factorial grinding experiments.

 $Y = F_{80}/P_{80}$ Coded values

	X ₁	X ₂ (Ball	X ₃	X ₄	P ₈₀ (microns)	
	(speed)	size)	(grinding	(M_B/M_O)		
			Time)			
G1a	-1	-1	-1	-1	1350.01	1.309356
G1b	-1	-1	-1	-1	1342.51	1.316671
G1c	-1	-1	-1	-1	1296.61	1.363281
G2	1	-1	-1	1	883.97	1.999665
G3	-1	1	-1	1	980.25	1.803258
G4	1	1	-1	-1	1489.18	1.186991
G5	-1	-1	1	1	389.6	4.537074
G6	1	-1	1	-1	1212.02	1.458428
G7	-1	1	1	-1	945.66	1.869217
G8a	1	1	1	1	459.76	3.84471
G8b	1	1	1	1	452.68	3.904842
G8c	1	1	1	1	456.21	3.874628
G9	1	1	1	-1	1088.06	1.624583
G10	1	1	-1	1	926.24	1.908408
G11	1	-1	1	1	367.93	4.804294
G12	-1	1	1	1	468.63	3.771939

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Table 3.	Calculation	matrix for	tactorial	experimental	design of	t orindino	conditions

Test	b_0	X_1	X_2	X_3	X_4	X_1X_2	X_1X_3	X_1X_4	X_2X_3	X_2X_4	X_3X_4	Y=
												F_{80}/P_{80}
G_{lav}	1	-1	-1	-1	-1	1	1	1	1	1	1	1.33
G_2	1	1	-1	-1	1	-1	-1	1	1	-1	-1	2.000
G_3	1	-1	1	-1	1	-1	1	-1	-1	1	-1	1.803
G_4	1	1	1	-1	-1	1	-1	-1	-1	-1	1	1.187
G_5	1	-1	-1	1	1	1	-1	-1	-1	-1	1	4.537
G_6	1	1	-1	1	-1	-1	1	-1	-1	1	-1	1.458
G_7	1	-1	1	1	-1	-1	-1	1	1	-1	-1	1.869
$G_{8av} \\$	1	1	1	1	1	1	1	1	1	1	1	3.875
G_9	1	1	1	1	-1	1	1	-1	1	-1	-1	1.625
G_{10}	1	1	1	-1	1	1	-1	1	-1	1	-1	1.908
G_{11}	1	1	-1	1	1	-1	1	1	-1	-1	1	4.804
G_{12}	1	-1	1	1	1	-1	-1	-1	1	1	1	3.772

Table 4: The calculated coefficients of Equation 1

			24020 11			orrection .	or Equate				
Coefficients	b_0	b_1	b_2	b_3	b_4	b_{12}	b_{13}	b_{14}	b_{23}	b_{24}	b ₃₄
Values	2.29	-0.111	-0.057	0.694	0.8131	0.0258	0.0663	0.1536	-0.142	- 0.207	0.4491

Table 6: Coefficients, effect estimates and associated t values for grinding experiment

Coefficients b ₀ b ₁ b ₂ b ₃	Values 2.2908	Effects E ₀	Effect estimate	Associated t value	t ₄ , _{0.025} t ₄ , _{0.975}
b_1 b_2		E			
b_2		\mathbf{E}_0	2.291	309.7392	-2.776
	-0.1107	E_1	-0.221	-14.942	or+2.776
b_3	-0.0572	E_2	-0.114	-7.7078	
- 3	0.6941	E_3	1.388	93.8459	
b_4	0.8131	E_4	1.626	109.9377	
b_{12}	0.0258	E_{12}	0.052	3.5158	
b_{13}	0.0663	E_{13}	0.133	8.9924	
b ₁₄	0.1536	E_{14}	0.307	20.7570	
b_{23}	-0.1424	E_{23}	-0.285	-19.2695	
b ₂₄	-0.2072	E_{24}	-0.414	-27.9915	
b ₃₄		E_{34}	0.898		

Table 7 Residuals of grinding experiment responses.

Test	Run		Coded	values		y=	$Y = F_{80}/y$	Υ'	Residuals	Per cent
	order	\mathbf{X}_1	X_2	X_3	X_4	(P ₈₀)	(Actual	(Predicted	Y-Y'	
							response)	response)		residuals
G_{1a}	5	-1	-1	-1	-1	1350.01	1.309	1.3	0.0127	0.967
G_{1b}	4	-1	-1	-1	-1	1342.51	1.317	1.3	0.02	1.517
G_{1c}	1	-1	-1	-1	-1	1296.61	1.363	1.3	0.0666	4.884
G_2	9	1	-1	-1	1	883.97	2.000	2.03	-0.034	-1.692
G_3	10	-1	1	-1	1	980.25	1.803	1.84	-0.033	-1.832
G_4	3	1	1	-1	-1	1489.18	1.187	1.22	-0.033	-2.806
G_5	8	-1	-1	1	1	389.6	4.537	4.57	-0.033	-0.737
G_6	2	1	-1	1	-1	1212.02	1.458	1.49	-0.033	-2.254
G_7	6	-1	1	1	-1	945.66	1.869	1.9	-0.033	-1.781
G_{8a}	11	1	1	1	1	459.76	3.845	3.98	-0.131	-3.397
G_{8b}	13	1	1	1	1	452.68	3.905	3.98	-0.07	-1.804
G_{8c}	16	1	1	1	1	456.21	3.875	3.98	-0.101	-2.598
G_9	7	1	1	1	-1	1088.06	1.625	1.56	0.0665	4.092
G_{10}	12	1	1	-1	1	926.24	1.908	1.84	0.0673	3.527
G_{11}	15	1	-1	1	1	367.93	4.804	4.74	0.067	1.394
G_{12}	14	-1	1	1	1	468.63	3.772	3.71	0.0666	1.767

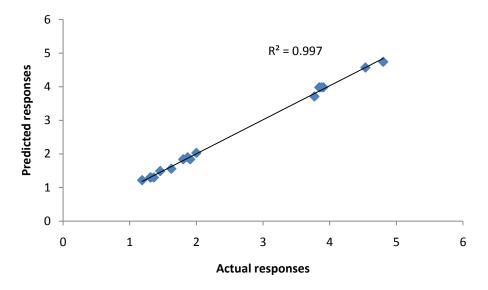


Figure 2: Correlation between the predicted and the actual grinding responses

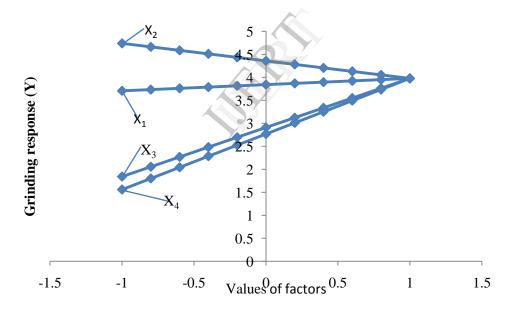


Figure 3: Response plot of the effect of each factor when others remain constant.

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Table 8: Summary of grinding model validation and optimum test runs results.

Test	Cod	ed val	lues		(F_{80})	y=	Y= F ₈₀ /y	Υ'	Residuals
	X ₁	X ₂	X ₃	X ₄]	(P ₈₀)			Y-Y'
V1	1	-1	1	1	1767.644	369.5	4.783881	4.7373	0.046581
V2	-1	1	1	1	1767.644	466.447	3.789595	3.7053	0.084295
V3	1	1	1	1	1767.644	452.55	3.905964	3.9753	-0.06934
V4	1	-1	3	1.515	1350	218	6.19271	8.74	-2.547
V5	1	-1	3	1.504	1342.5	220.35	6.09258	8.711	-2.618
V6	1	-1	3	1.524	1489.2	211.98	7.0251	8.761	-1.736
V7	1	-1	2	1.506	1296.6	228.463	5.67536	7.138	-1.462
V8	1	-1	2	2.005	883.97	221.72	3.98688	8.171	-4.184
V9	1	-1	2	1.89	926.24	213.87	4.33086	7.933	-3.603

