

# Fe Analysis On The Effect Of Flexural Steel On Punching Shear Of Slabs

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## Abstract

In this paper, an advanced finite element analysis of reinforced concrete slabs have been carried out in an effort to ascertain the effect of flexural reinforcement on punching shear capacity of RC slabs. The FE analysis consists of 3D modeling of concrete elements, separate and discrete modeling of reinforcing bar. Nonlinear material model for concrete is capable of simulating cracking in tension as well as crushing in compression. Non-linear arc-length solution algorithm has been applied to simulate post peak response. The validity of FE analysis was verified through comparison with available experimental data obtained by the authors. Cracking pattern and load-deflection behavior of the slabs have also been investigated. Code-specified strength of the specimen was calculated in accordance with the American, British, Canadian, German and Australian codes. It has been observed that punching shear is not effectively estimated in some of the codes. Findings of the study in the design codes will result in a more rational design of structural systems where punching phenomenon plays a vital role.

**Keywords:** FE Analysis, Punching shear, flexural reinforcement.

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## Introduction

For the design of flat plates, flat slabs, bridge decks and column footings, the punching shear strength of concrete in the vicinity of columns, concentrated loads or reactions is one of the major critical criteria to govern the design. Punching shear failure is normally a brittle type of failure and occurs when shear stress around a slab column connection exceeds the shear capacity of the slab, resulting in the column and the part of the slab punching through the slab (Tan and Teng 2005).

Some of the present codes do not acknowledge the possible effect of flexural reinforcement on the punching shear behavior of reinforced concrete slabs. Design codes such as the American code (ACI 318-2011), Canadian Standard (CSA-A23.3-04 (R2010)) and Australian code (AS 3600-2009) do not reflect the influence of the flexural reinforcement ratio on the punching capacity of slab-column connections. Others codes like the British (BS 8110-1997) and German codes (DIN 1045-1: 2008) consider the effect of flexural reinforcement on the punching shear capacity of slabs. Some codes do not take adequate account of the possible role of specimen size and slab thickness (Lovrovich and McLean 1990; Mitchell et al. 2005).

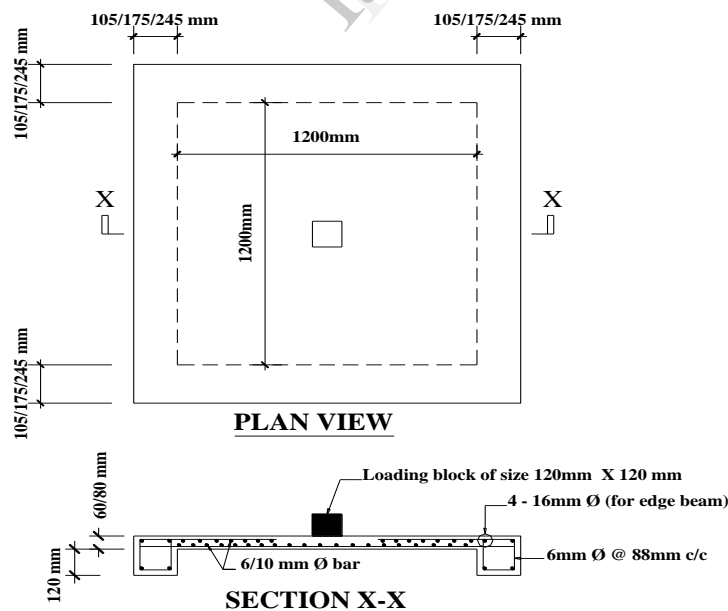
An advanced FE investigation has been carried out on the behavior of punching shear characteristics of concrete slab in presence of flexural reinforcement. At first stage, FE model has been developed to simulate relevant experiments carried out earlier (Alam et al. (2009)). Good agreement has been observed between numerical FE simulation and experiment, which establish the validity of FE model. Later on the same FE model has

been used to carry out a systematic parametric study to investigate the effect of various design parameters.

## Experimental Works

### *Specimen details*

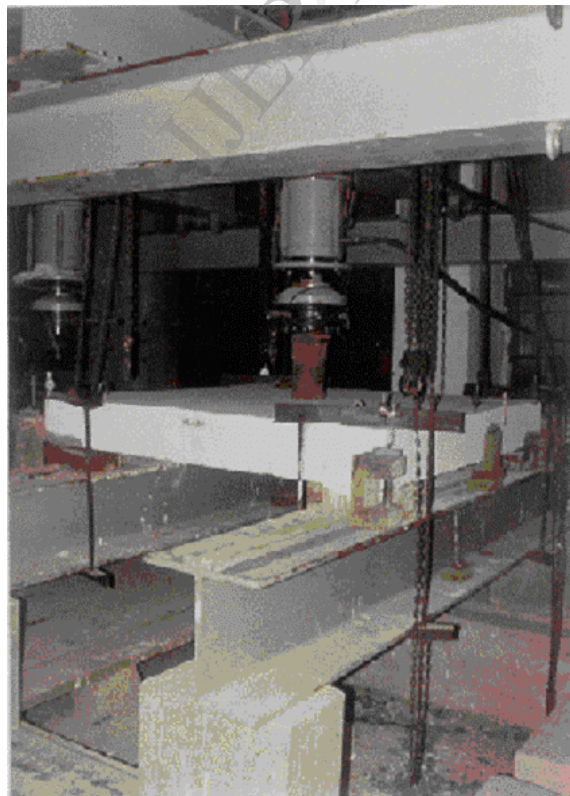
Total 15 square slab specimens with and without edge beam have been constructed and tested in this study. Typical plan and sectional details of a slab is shown in Figure 1. The concrete used in the specimens consisted of ordinary Portland cement, natural sand and crushed stone aggregate with maximum size of 10 mm. The water-cement ratio for concrete was 0.45. Both 6 mm and 10 mm diameter bars having average yield strength of 421 MPa were used in the slab panels and stirrup of edge beams. 16 mm diameter bars with average yield strength 414 MPa were provided as flexural reinforcements in the edge beams. Details data for experimental working would be obtained in researcher's other papers (Alam and Amanat (2012), Alam et al. (2009)).



**Figure 1.** Details of a typical model slab and reinforcement with edge beam.

### *Testing*

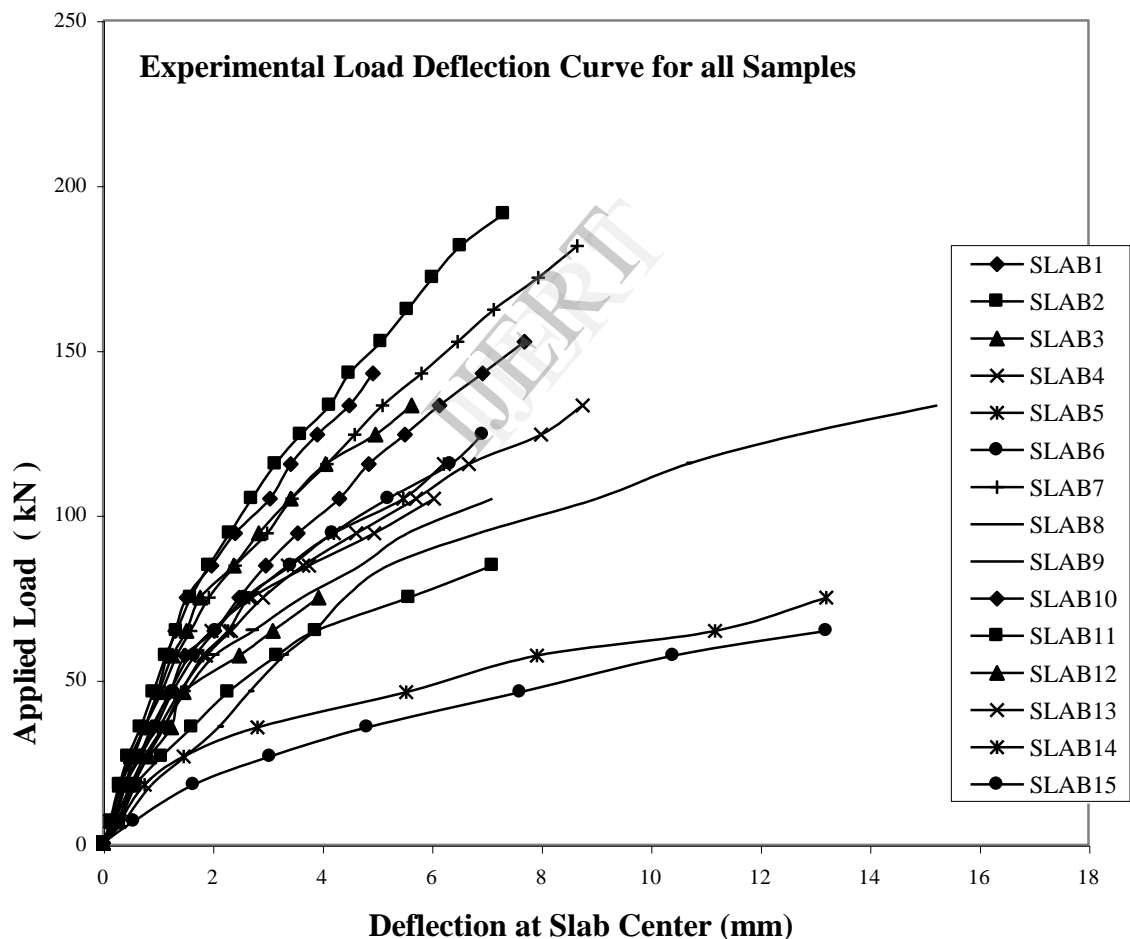
A test rig, consisting mainly of steel girder and 300 kN capacity hydraulic jack was used for the purpose of loading the slabs of various sizes. Each slab was subjected to concentrated loading at the geometric center through a steel plate of 120mm x 120mm size, simulating a concentrated load. Four steel blocks were used at each corner of slab as support. During testing, corners of each sample were properly anchored by means of heavy joists, which were connected to structural floor. Loading was applied at an approximately constant rate up to the peak load and deflections were measured simultaneously. Failure occurred suddenly in all specimens and loading was stopped after failure. The testing set up is shown in Figure 2. There was one LVDT at the mid-span to measure the central slab deflection; one LVDT was placed at the middle span of one of the edge beams to measure the central vertical deflection of the edge beam and four LVDTs at the corner of edge beams to assess the performance of the supports.



**Figure 2.** Test rig and testing arrangement.

### *Test Results*

All the models underwent punching type of failure with inherent brittle characteristics. Most of the slab samples failed at a load much higher than that load predicted by most of the codes. Load-deflection curve for all tested slab is shown in Figure 3. The deflection of slabs having higher percentage of reinforcement is smaller than those of lower reinforcement for same applied load. Cracking pattern of the top surface of all the slabs were very much localized and approximately had a size of average 120mm x 120mm. The cracking patterns at the bottom surface of slabs having low percentage of reinforcement were more severe than those having higher percentages of steel.

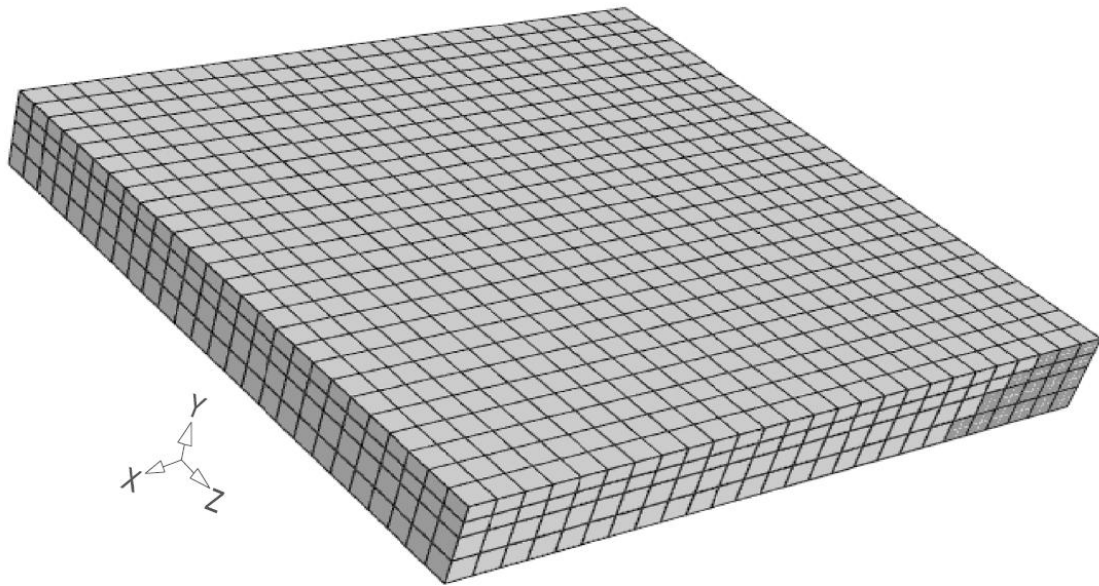
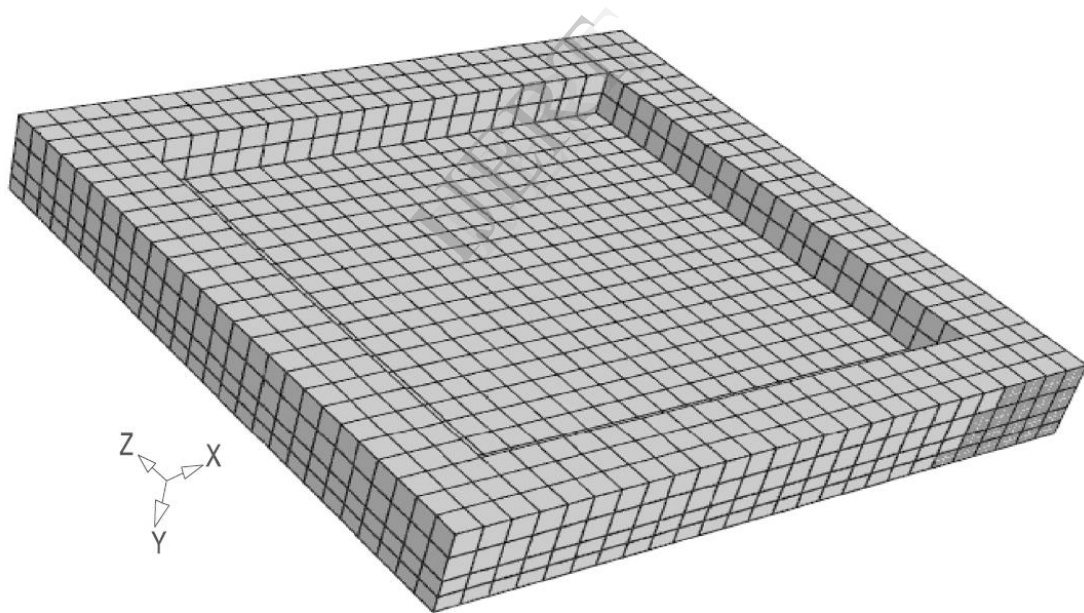


**Figure 3.** Experimental load deflection curve of slabs during testing

## FINITE ELEMENT ANALYSIS

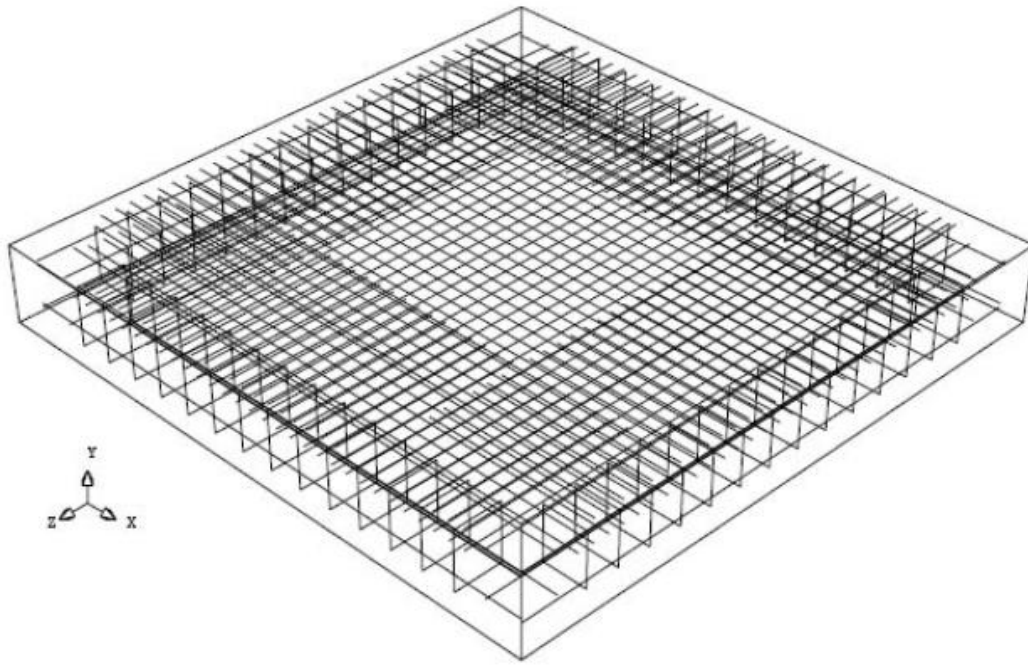
The finite element program DIANA was used in this study. The element adopted was twenty-node isoparametric solid brick element (elements CHX60). The program is capable to represent both linear and non-linear behavior of the concrete. For the linear stage, the concrete is assumed to be an isotropic material up to cracking. For the non-linear part, the concrete may undergo plasticity and/or creep. Gaussian 2x2x2 integration scheme was used which yields optimal stress points. The total strain approach is used with fixed smeared cracking; i.e., the crack direction is fixed after crack initiation. For present study, ideal behavior of concrete for tension and Thorenfeldt compression curve are used. A constant shear retention factor = 0.01 was considered for the reduction of shear stiffness of concrete due to cracking.

The full-scale geometry of all slabs was modeled and meshed model of a typical slab is shown in Figure 4. The reinforcement mesh in a concrete slab was modeled with the bar reinforcement embedded in the solid element. In finite element mesh, bar reinforcements have the shape of a line, which represents actual size and location of reinforcement in the concrete slab and beam. Thus in the present study, reinforcements are used in a discrete manner exactly as they appeared in the actual test specimens. The constitutive behavior of the reinforcement modeled by an elastoplastic material model with hardening. Tension softening of the concrete and perfect bond between the bar reinforcement and the surrounding concrete material was assumed. This was considered reasonable since welded mesh reinforcement was used in the tests. Typical reinforcement in finite element model is shown in Figure 5. The steel reinforcement behaves elastically up to the Von Mises yield stress of 421 MPa for slab and 414 MPa for edge beam.

**( a )****( b )**

**Figure 4.** Meshed model of a FE typical slab showing ( a ) top surface, ( b ) bottom surface.





**Figure 5.** Embedded reinforcement in a FE typical slab model.

The four edge beams of the slab were vertically restrained at their corners, as in the experimental set-up. One corner had all transnational degrees of freedom fixed, while diagonally opposite of that corner was fixed with two degrees of freedom so as to prevent the slab from moving and rotating in its own plane. Loading was applied within at 120 mm x 120mm area of central portion of slab model at the top surface to simulate actual experimental loading.

A commonly used modified Newton–Raphson solving strategy was adopted, incorporating the iteration based on conjugate gradient method with arc-length control. The line search algorithm for automatically scaling the incremental displacements in the iteration process was also included to improve the convergence rate and the efficiency of analyses. Second order plasticity equation solver solved physical non-linearity with total



strain cracking. Reinforcement was evaluated in the interface elements. Accuracy checked by the norms of residual vector.

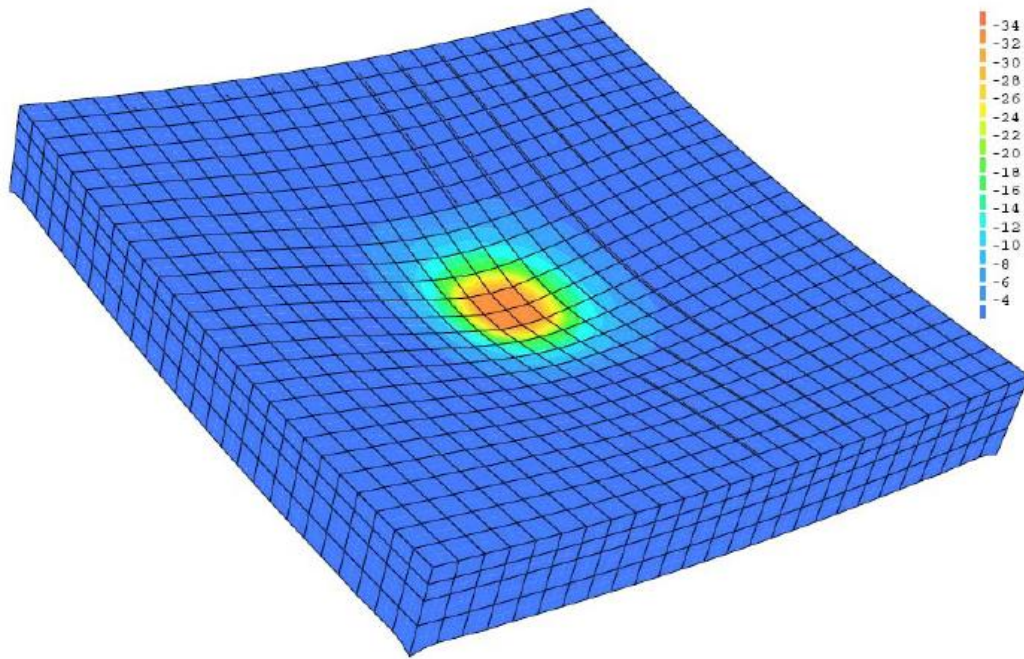
## DISCUSSIONS

Test results obtained from this study have been analyzed and it has been found that ultimate punching shear capacity and behavior of slab samples are dependent on flexural reinforcement ratio of slabs. In this paper, apart from studying the effect of flexural reinforcement, slab deflection and cracking patterns are also included to evaluate the actual punching shear behavior of slabs.

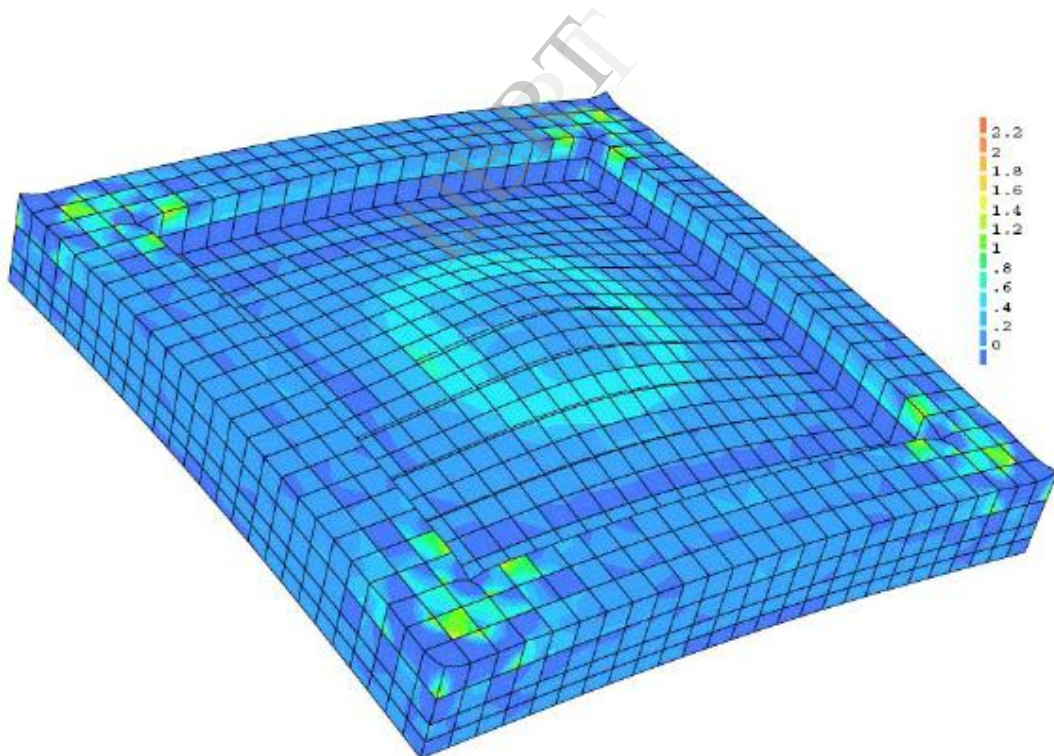
### *Load-Deflection Behavior*

It appeared that, deflection for both 80mm and 60mm thick slabs are very close for all types of reinforcement ratio. Deflection of 80mm thick slabs is smaller than that of 60mm thick slab for same loading. Deflection is also very close for slabs of same thickness but with different reinforcement ratio. A little higher deflection was observed for slabs with less reinforcement at same level of loading.

Typical deflected shape and stress contour of FE slab model is shown in Figure 6. Experimental failure on top surface of slab model as shown in Figure 12(b) was very localized which is represented analytically by stress contour on top surface as shown in Figure 6(a). Compressive stress developed on top surface and tensile stress developed on bottom surface at the central region of slab is shown in Figure 6. Maximum compressive stresses produced on top surface, which are concentrated around and within the loading block. Higher value of tensile stress developed outside of the loading block as shown in Figure 6(b) and indicative to failure surface at that portion.



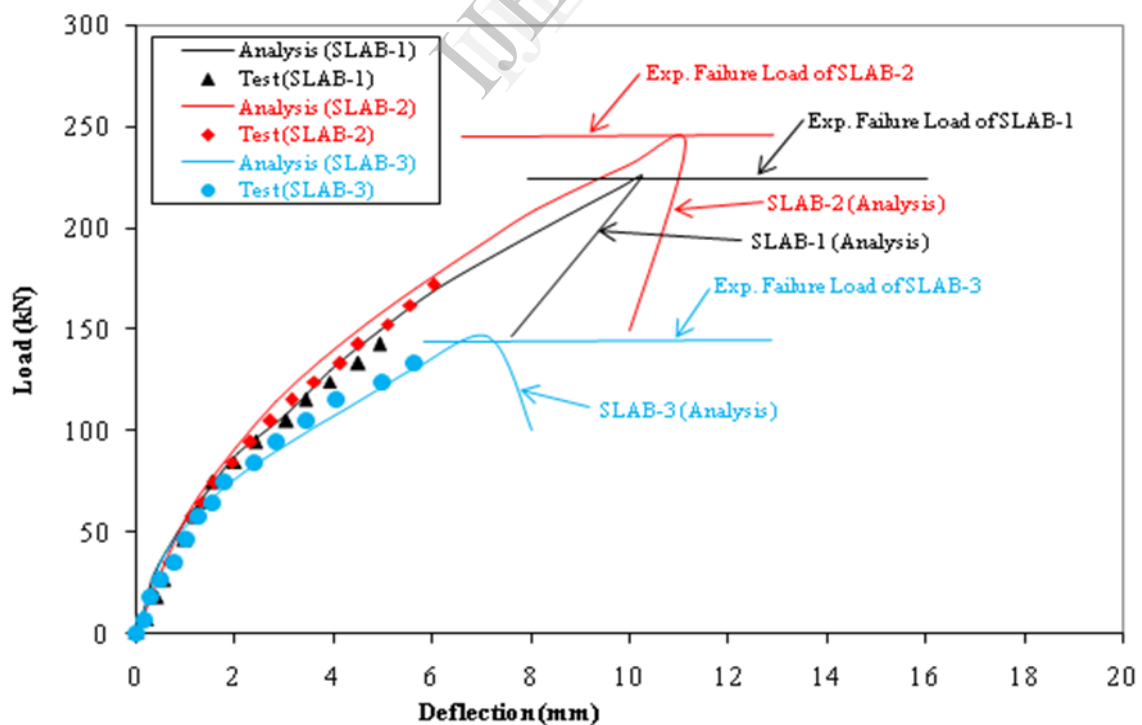
( a )



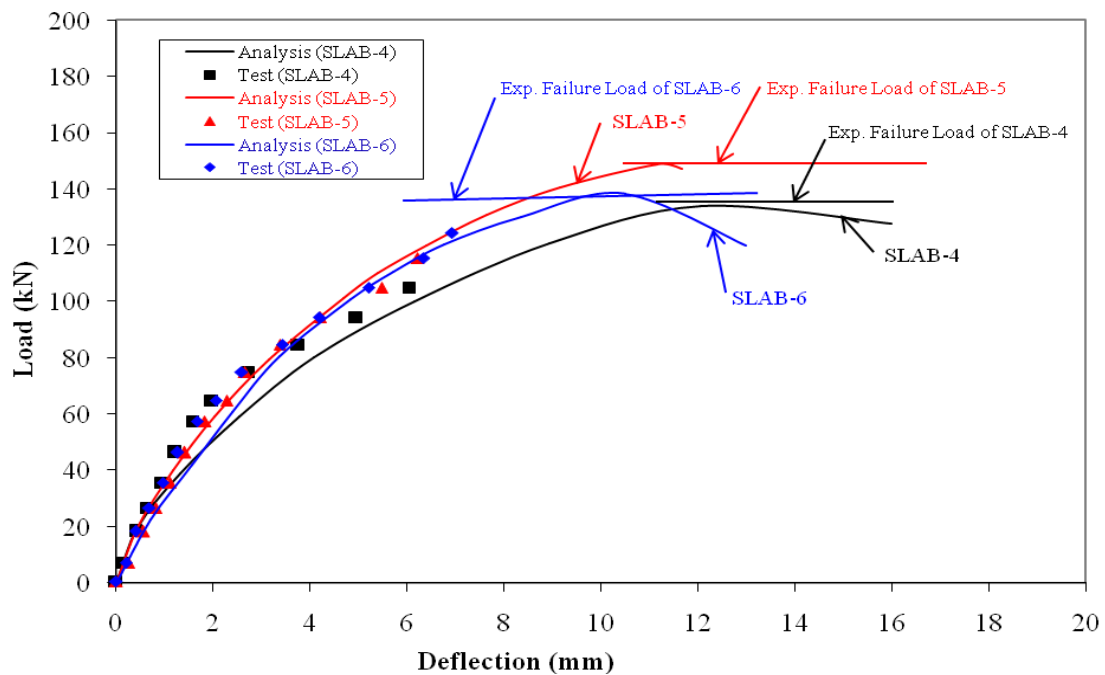
( b )

**Figure 6.** Deflected shape and stress contour shown on ( a ) top surface ( b ) bottom surface of typical slab model.

Load deflection curve for both experiment and FE analysis of all slabs are shown in Figures 7 to 11. Due to instrumental limitation, complete experimental load-deflection curves upto failure load could not be traced. A horizontal line in all load-deflection curves drawn in Figures 7 to 11 showing experimental failure load. It is clear from Figures 7 to 11 that analytical load deflection behavior of all model slabs are reasonably matched with experimental result. In case of same width of edge beam, variation of deflection occurred due to the variation of slab thickness and reinforcement ratio. Deflections of slabs without edge beam are higher than all other slabs with edge beam as shown in Figure 11. It is obvious that flexural reinforcement play a significant role in the behavior of RC slab subjected to punching force. Similar trend of load deflection behavior of numerical analysis and experimental data indicate to have similar nature of other parameters for structural designing of slab.

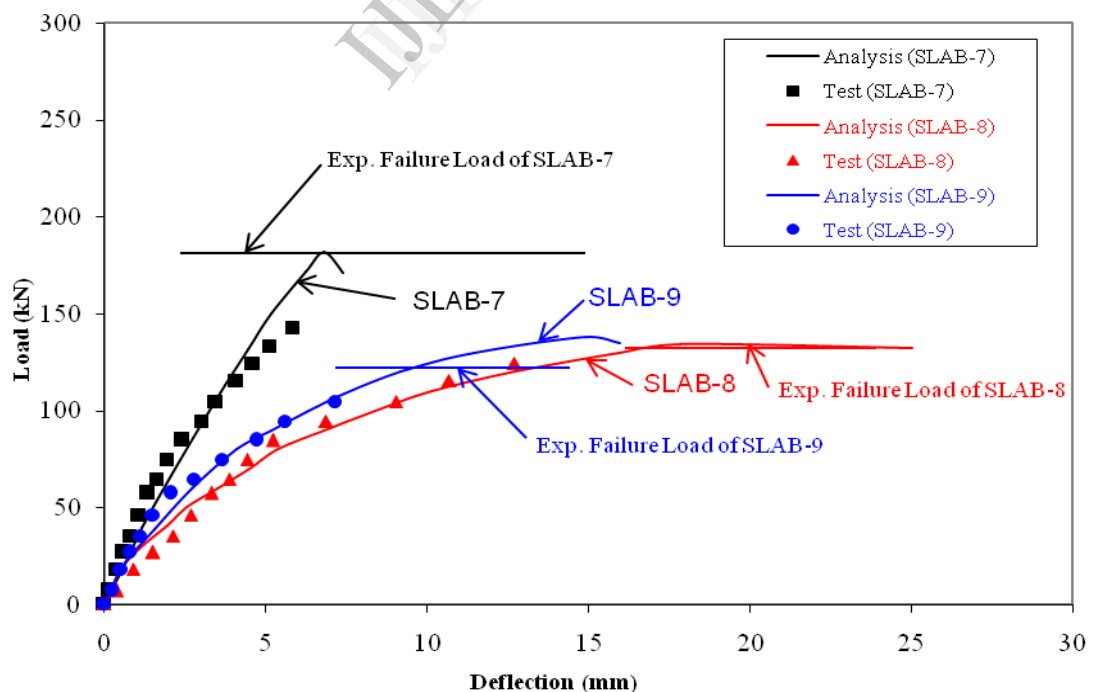


**Figure 7.** Load-deflection curves of analyzed and tested model having  
slab thickness = 80mm and width of edge beam = 245mm.



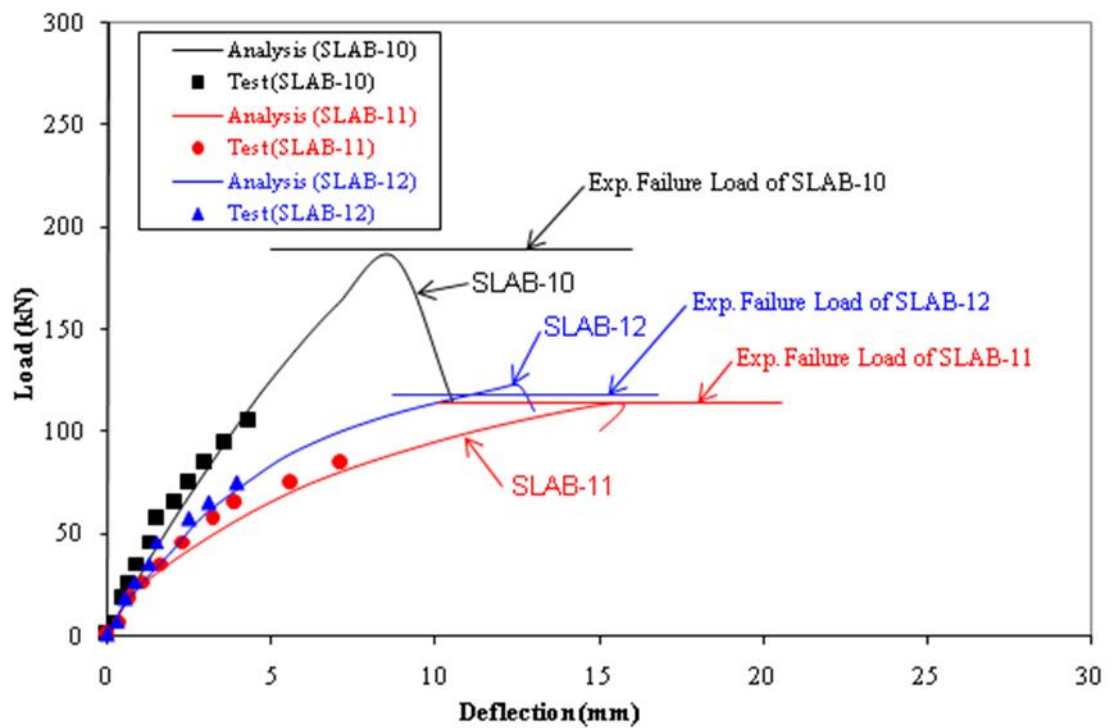
**Figure 8.** Load-deflection curves of analyzed and tested model having

slab thickness = 60mm and width of edge beam = 245mm.

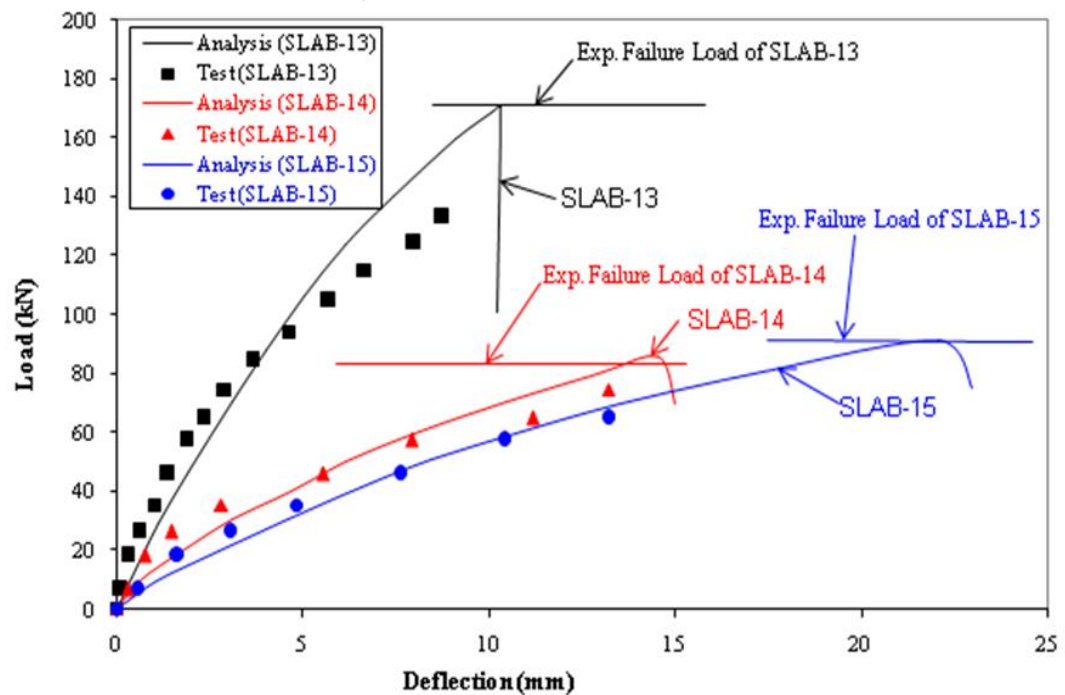


**Figure 9.** Load-deflection curves of analyzed and tested model having

width of edge beam = 175mm.



**Figure 10.** Load-deflection curves of analyzed and tested model having width of edge beam = 105mm.



**Figure 11.** Load-deflection curves of analyzed and tested model having no edge beam.

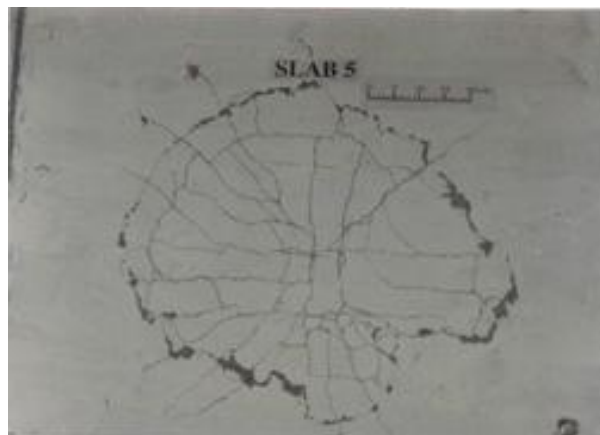
The value of deflection decreased, in general, as the slab thickness increases. Again, the heavily reinforced slabs, on the whole, underwent lesser deflections and showed slightly higher stiffness. Higher reinforcement increases tensile strength capacity at extreme fibre of slab, which causes lesser deflection.

The tensile strength of concrete is an important property because the slab will crack when the tensile stress in the extreme fibre is exceeded. Due to increase of load, crack width and depth will also increase which results increase of slab deflection. Higher the slab thickness, due to increase of effective depth of concrete, tensile strength at extreme fibre will be lower for slab loading. Due to higher section modulus of 80mm thick slab, deflection of 80mm thick slab is smaller than that of 60mm slab.

### *Cracking*

Typical cracking pattern of tested slab is shown in Figure 12. During the tests, the development of cracking and the width of cracks were carefully observed and monitored at various load levels. Cracking on the underside of the slabs developed as a series of cracks radiating from the centrally loaded area. As the load increased, the widths of the cracks increased as expected. For the same width of edge beam, the crack widths of the normally reinforced ( $\rho=1.0$  percent) and heavily reinforced ( $\rho=1.5$  percent) slabs were found to be smaller than those of lightly reinforced slabs ( $\rho=0.5$  percent). For lower amount of reinforcement ( $\rho=0.5$  percent), number of cracks was small and more spalling occurred than others. For higher level of flexural reinforcement ( $\rho=1.5$  percent), cracks were concentrated in the middle portion of the slab.





(a) bottom surface



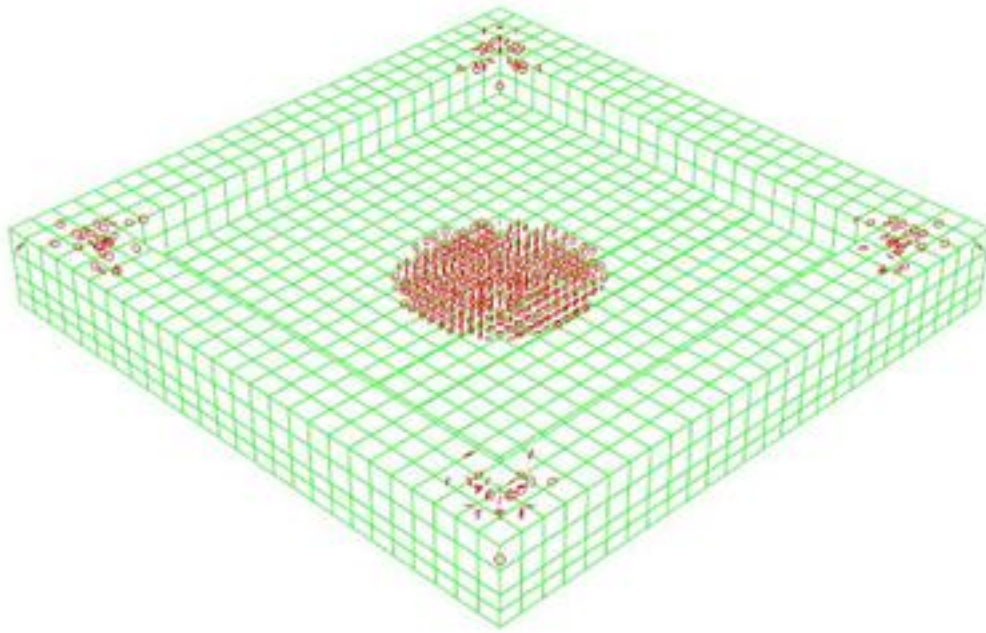
(b) top surface

**Figure 12.** Typical cracking pattern a model slab after testing

For lower level of reinforcement ratios ( $\rho = 0.50\%$  and  $1.0\%$ ), some cracking of the slab is present in the immediate vicinity of the column, but punching occurs before yielding of the entire slab reinforcement. For higher reinforcement ratios (test with  $\rho = 1.5\%$ ), punching occurs before any yielding of the reinforcement takes place, in a very brittle manner. In this case, the strength of the slab is lower.

Figure 13 shows the crack pattern of finite element model of a typical slab for applied load of 180 kN, where uniaxial principal strain characteristics is used. Cracks at the bottom surface are propagated toward edge beam and major cracking area is concentrated to central region of slab. The major cracking produced a circular bounded area in both analysis and experiment.

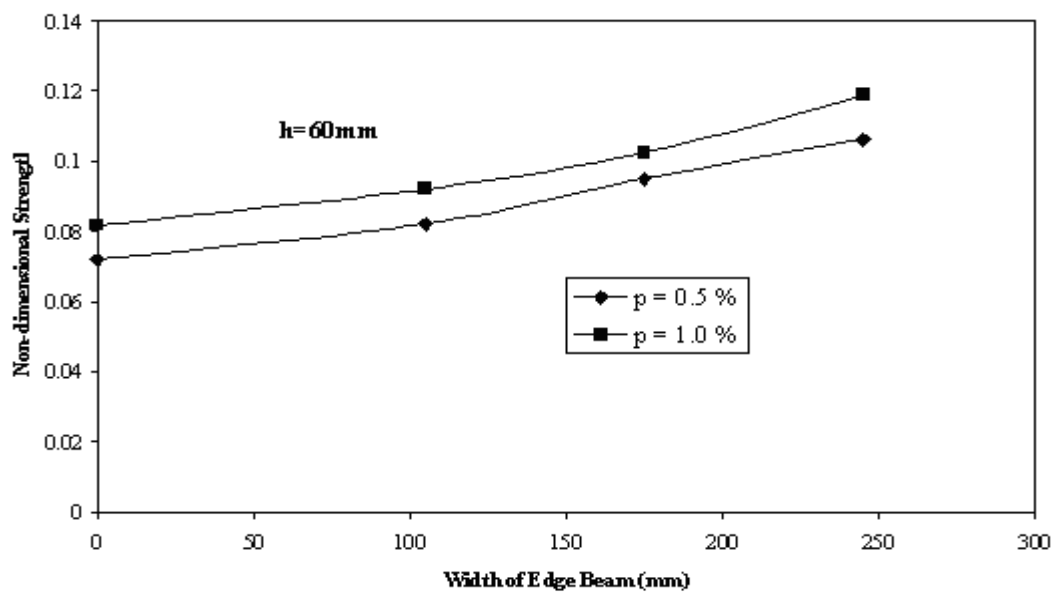
It can be concluded that cracking patterns of slab for testing and analysis are similar as shown in Figures 12 and 13.



**Figure 13.** Cracking pattern of a typical slab at bottom surface.

#### *Effect of Flexural Reinforcement*

The ultimate non-dimensional shear strengths  $P_u / f_c b_0 d$ , [where,  $d$ =effective depth of slab,  $b_0 = 4 * (\text{width of loading block} + d)$ ] of various slabs, have been shown in Figures 14. In this cases non-dimensional punching shear strength has been calculated by dividing the corresponding ultimate load by the product of the compressive strength of concrete and critical surface at half the effective depth away from the perimeter of loaded area. As it would be expected, the load-carrying capacity of the test slab panels increased with the addition of steel reinforcement.



**Figure 14.** Effect of reinforcement for slab thickness of 60mm.

Hallgren (1996) reported from test results of 240 mm thick slab-column assemblies that gain in stress at punching failure when reinforcement ratio is increased from 0.9 to 1.6 percent is very little. Marzouk and Hussein (1991) had shown from test results of 120mm and 150mm thick slab models, the increase in punching failure load with the increase in reinforcement ratio from 0.9% to 2.3%. Vanderbilt (1972) shows the average increase in punching shear stress with the increase of reinforcement ratio from 1.3% to 2.6%.

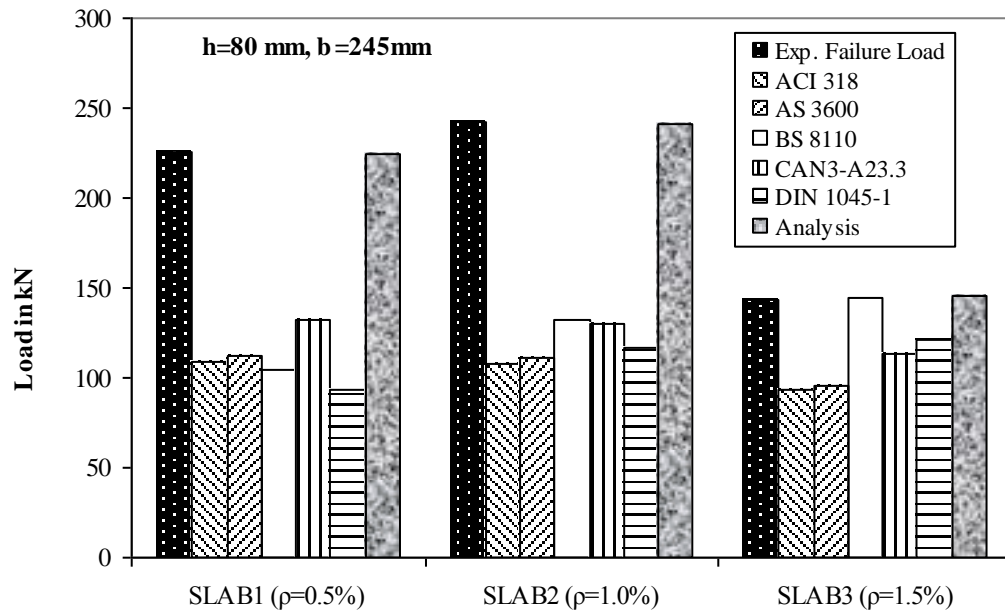
Dilger et al (2005) studied over one thousand test results and concluded that, a distinct decrease in punching shear resistance with decreasing reinforcement ratio. He added, a concentration of flexural reinforcement in the vicinity of the column seems to lead to a small increase in the punching shear strength only if the reduced bar spacing does not lead to a reduction in the bond strength along the bars. Gardner (2005) noted that while increasing the flexural steel increases the punching shear capacity, the behavior of the connection becomes more brittle and practically reinforcement should never be less than

0.5% and will rarely exceed 2% in real slabs. Significant yielding of flexural reinforcement produces large crack, which decrease the effective area resisting the shear. If it is assumed that little or no shear can be transferred through the portion of the depth of slab that is cracked, it is easy to conclude that the width and hence the depth of the crack have a significant influence on the shear capacity of the connection.

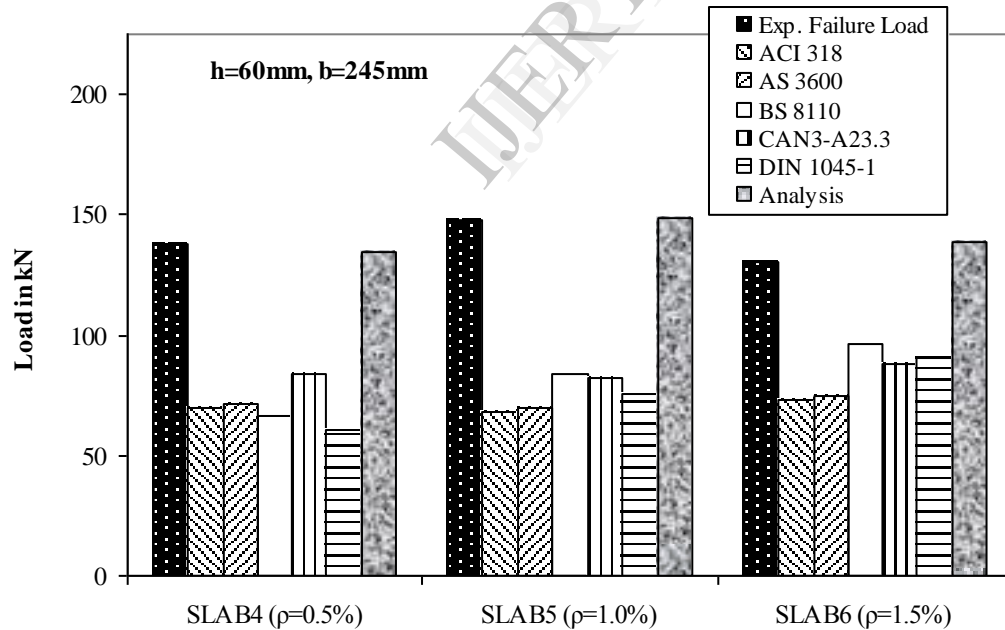
### **COMPARISON OF TEST RESULTS AND ANALYSIS WITH DIFFERENT CODE OF PREDICTIONS**

A comparison of the experimental failure loads, analytical failure and the punching shear capacity predicted by various codes is shown in Figures 15 to 18. It is to be noted that the nominal safety factor, partial safety factors, reduction factors, etc. have been removed in this exercise and the actual strength of concrete of individual slabs has been considered while plotting the graphs.

Figures 15 and 16, represent punching load carrying capacity of slabs having  $h=80\text{mm}$  of SLAB1, SLAB2, SLAB3 and  $h=60\text{mm}$  of SLAB4, SLAB5, SLAB6 with same width of edge beam ( $b=245\text{mm}$ ). It is evident from these figures that the punching load carrying capacity having higher the slab thickness is higher than smaller one. Punching shear strength capacity with 1.5% flexural reinforcement as calculated by British, Canadian and German codes are close to the experimental load carrying capacity.

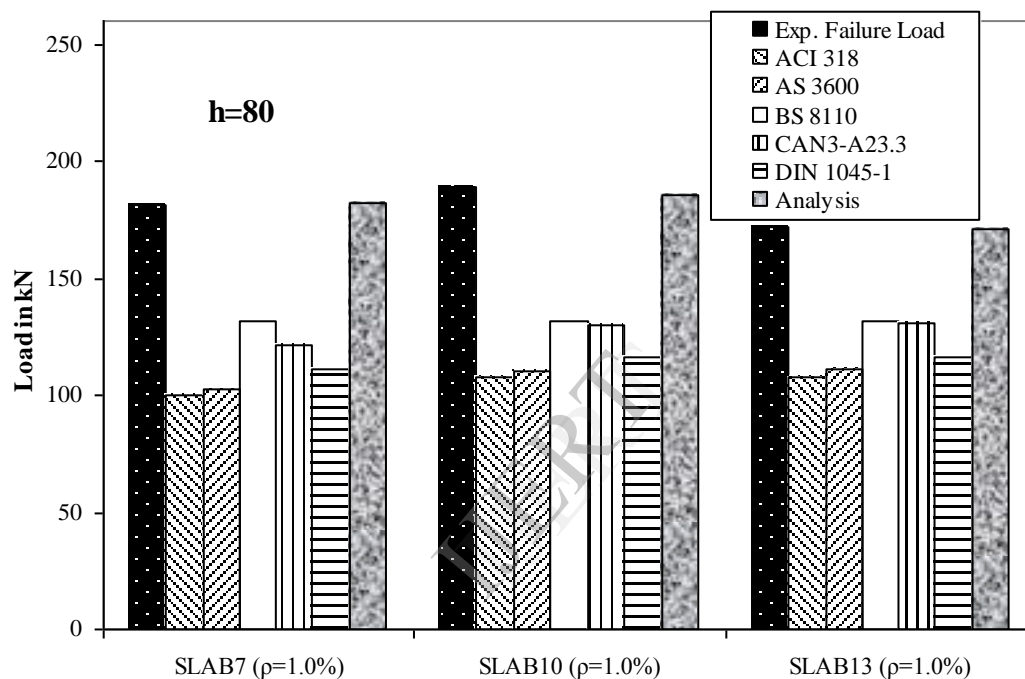


**Figure 15.** Comparison of test results with different code of prediction at  $h=80\text{mm}$  and  $b=245\text{mm}$ .



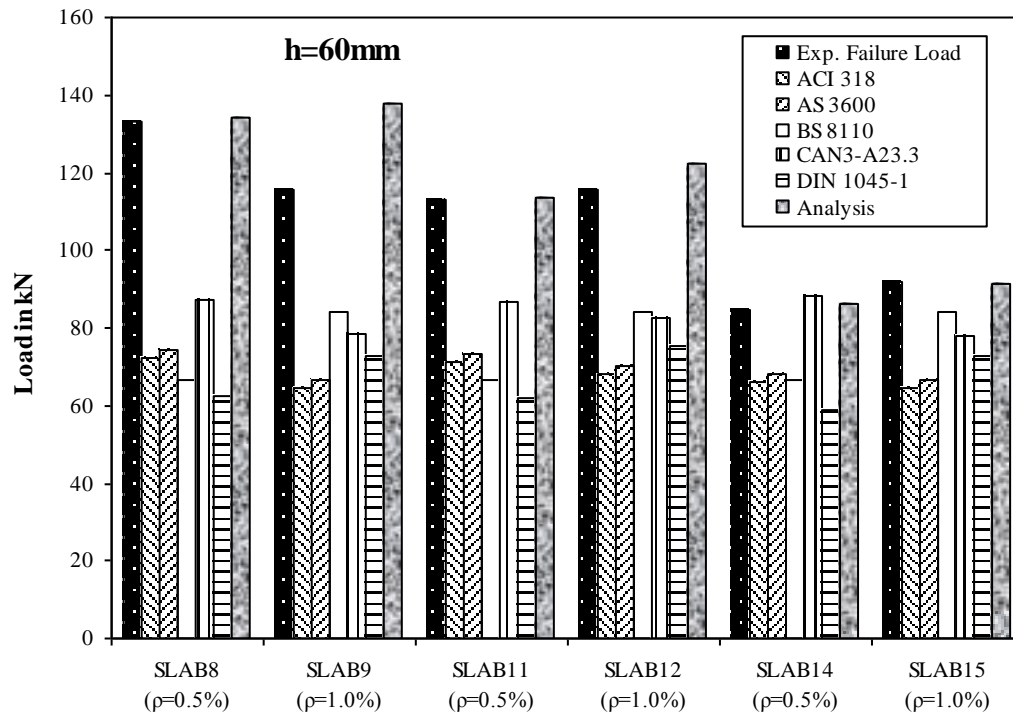
**Figure 16.** Comparison of test results with different code of prediction at  $h=60\text{mm}$  and  $b=245\text{mm}$ .

In Figure 17, it is observed that punching shear capacity of slabs having 1.0 percent reinforcement and 80mm thickness, load carrying capacity in accordance with British and Canadian are very close. This tendency is also evident in Figure 18 for 1% flexural reinforcement. It appears that for slab samples without edge restraint (SLAB14 and SLAB15), load carrying capacity predicted by the Canadian code is very close to experimental punching capacity.



**Figure 17.** Comparison of test results with different codes at same slab thickness of  $h=80\text{mm}$ .





**Figure 18.** Comparison of test results with different codes at same slab thickness of  $h=60\text{mm}$ .

In general, it was observed that experimental and analytical punching failure load of most of the slabs was higher than punching load carrying capacity calculated by the American (ACI 318-2011), Australian (AS 3600-2009), British (BS 8110-97), Canadian (CSA-A23.3-04 (R2010)) and German (DIN 1045-1 : 2008) codes. BS 8110-97 and Canadian (CSA-A23.3-04 (R2010)) codes are comparatively close to the experimental and analytical punching failure load. The American (ACI 318-2011) and Australian (AS 3600-2009) codes are very close to each other in all slabs.

From the above discussion, it can be concluded that some of the present codes are not sufficiently capable for predicting the punching shear strength of reinforced concrete slabs. For all the slabs tested having more than 0.5% flexural reinforcement, the prediction of American and Australian codes are most conservative. On the other hand,

although British code (which considered effect of reinforcement) predictions are on the conservative side, its prediction of punching failure load is better than the others.

## CONCLUSIONS

The experimental work presented includes testing of fifteen model slabs with different reinforcement ratios to examine the effect of reinforcement ratio on punching shear capacity.

In experiment, all slabs failed in a punching mode when subjected to punching load at the slab center, which is also simulated in FE analysis. It has been found that flexural reinforcements embedded in the flat slab play a significant role on punching shear capacity of slabs.

Load-deflection behavior and ultimate failure of both experiment and numerical analysis is matched in this study. Thus FE model for punching shear behavior of reinforced concrete slabs can effectively be used to analyze actual behavior of flexural reinforcement on punching shear behavior of reinforced concrete slabs and provide a virtual testing scheme of structures to explore their behavior under different loadings and other effects under different conditions.

Although British and German codes recognize the influence of percentages of steel, American, Australian and Canadian codes completely ignore the possible influence of the amount of flexural reinforcement in formulating its equations for punching shear capacity of slabs. The provisions of all these codes may, thus, be reviewed to accommodate the influence of flexural steel more rationally.

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