International Journal of Engineering Research & Technology (IJERT)
ISSN: 2278-0181
Vol. 1 Issue 10, December- 2012

Flow Of A Couple Stess Fluid Through A Porous Layer Bounded by Parallel Plates

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Abstract The flow of a couple stess fluid through a porous layer bounded by parallel plates is investigated. The expressions for the velocity and the temperature are obtained in terms of exponential functions. The mass flow rate and its fractional increase are determined. The effect of permeability and couple stress parameters on the velocity and temperature are discussed.

Keywords Couple stress fluid, Parallel plates, Porous layer, Mass flow rate.

1. Introduction

The flow of a non-Newtonian fluid through and past porous media is of wide spread importance in various branches of science and technology. The flow of a non –Newtonian fluid through and past porous media in of wide spread importance in various branches of science and Technology. With the growing importance of non-Newtonian fluids in modern technology, industries, the investigations on such fluids are desirable. During recent years the theory of polar fluids has received much attention and this is because the traditional Newtonian fluids can't precisely describe the characteristics of the fluid flow with suspended particles. The study of such fluids have applications in a number of processes that occur in industry such as the extrusion of polymer fluids, solidification of liquid crystals, cooling of metallic plate in a bath, exotic lubrications and colloidal and suspension solutions. In the category of non-Newtonian fluids couple stress fluid has distinct features , such as polar effects. The theory of polar fluids and related theories are models for fluids whose micro structure is mechanically significant. The constitutive equations for couple stress fluids were given by Stokes (1966). The theory proposed by Stokes is the simplest one for micro fluids, which allows polar effects such as the presence of couple stress, body couples and non-symmetric tensor. Couple stresses are found to appear in noticeable magnitude in fluids with very large molecules. The couple stress effects are considered as a result of the action of one part of a deforming body on its neighbourhood. This theory has wide results and applications in mechanics of bio fluids, colloidal fluids, liquid crystals and for pumping fluids such as synthetic lubricants. This theory has wide results and applications in mechanics of bio-fluids, colloidal –fluids liquid crystals and for pumping fluids such as synthetic lubricant. The theory of Stokes has applied for the study of some simple lubrication problems see Bujurke and Jayaraman, (1982). Since the long chain hyaluronic acid molecules are found as additives in synovial fluids, they are modeled as couple stress fluids in human joints. The presence of small amounts of additives in a lubricant can improve the bearing performance by increasing the lubricant viscosity and thus producing an increase in the load capacity. These additives in a lubricant also-reduce the coefficient of friction and increase the temperature range in which bearing can operate.

Based on the couple – stress theory of Stokes, Valanis and Sun (1969), Chaturani and Kaloni (1976), Chaturani and Upadthya (1976) have proposed various theoretical models obtained from these three models are in good agreement with experimental results.

Further Chaturani and Pralhad (1981) studied a three layered flow model for blood flow and they assumed that the top and bottom layers consist of plasma and the middle layer consist of red-cell suspension (couple-stress fluid). Recently Malashetty and Umavathi (1999) discussed the effects of couple stresses on the free Convection flow in a vertical channel. Free convection flow of an electrically conducting couple stress fluid and a couple stress fluid for the radiating medium in a vertical channel has been studied by Umavathi (1999,2000).

Keeping in mind the importance and applications of non Newtonian (Couple stress) fluids, the flow of a couple stress fluid through a porous layer bounded by parallel plates is investigated. The expressions for the velocity and the temperature are obtained. The mass flow rate and its fractional increase are determined. The effect of permeability and couple stress parameters on the velocity and temperature are discussed.

2. Mathematical Formulation

Consider the flow of a couple stress fluid through a porous medium bounded by parallel plates. The permeability of the

International Journal of Engineering Research & Technology (IJERT)
ISSN: 2278-0181
Vol. 1 Issue 10, December- 2012

porous medium is taken as k . The lower and upper plates are maintained at fixed temperatures T_1 and T_2 respectively. The x-axis is taken along the central line of the channel and y-axis perpendicular to it. The width of porous channel is 2h as shown in figure 4.1

To derive the basic equations of the problem, we make the following assumptions.

I. The flow in the x-direction is driven by a constant pressure gradient.

II. The flow is steady and fully developed with negligible body forces so that all the physical quantities except the pressure are functions of y only.

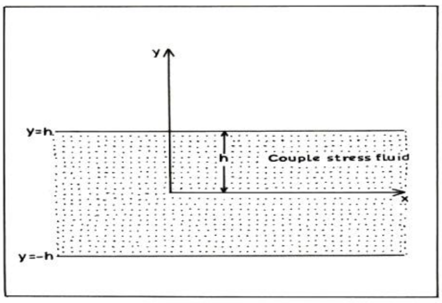

Figure1: Physical model

Under these assumptions the basic equations of the flow are given below.

Basic Equations

$$\frac{\partial u}{\partial x}=0 \qquad (1.1)$$

$$\eta\frac{\partial^4 u}{\partial y^4}-\frac{\mu}{\in}\frac{\partial^2 u}{\partial y^2}+\frac{\mu}{K}u=-\frac{\partial P}{\partial x} \qquad (1.2)$$

$$\frac{\partial^2 T}{\partial y^2}+\frac{\mu}{k}\left(\frac{\partial u}{\partial y}\right)^2=0 \qquad (1.3)$$

Boundary Conditions

$$u = 0 \text{ at } y = h \qquad (1.4)$$

$$u = 0 \text{ at } y = -h \qquad (1.5)$$

$$\frac{d^2 u}{dy^2}=0 \text{ at } y=\pm h \qquad (1.6)$$

$$T=T_2 \quad \text{at} \quad y=+h \qquad (1.7)$$

$$T=T_1 \quad \text{at} \quad y=-h \qquad (1.8)$$

1.4 Non- dimensionalization of the flow quantities

We introduce the following quantities in order to make the basic equations and boundary conditions dimensionless.

$$\bar{x}=\frac{x}{h}; \quad \bar{y}=\frac{y}{h}; \quad \bar{u}=\frac{u}{U}; \quad \bar{p}=\frac{p}{\rho u^2};$$

$$\theta=\frac{T-T_2}{T_1-T_2}$$

In view of the above dimensionless quantities, the equations (1.1)-(1.8) take the following form. The asterisks (*) are neglected here after.

$$\frac{\partial u}{\partial x}=0 \qquad (1.9)$$

$$\frac{\partial^4 u}{\partial y^4}-b^2\frac{\partial^2 u}{\partial y^2}+c^2 u=Pa^2 \qquad (1.10)$$

where

$$b^2=\frac{a^2}{\in}; c^2=\frac{a^2}{Da}; a^2=\frac{\mu h^2}{\eta};$$

$$P=-Re\frac{\partial p}{\partial x}$$

$$\frac{d^2\theta}{dy^2}=-E_e \quad Pr \quad \left(\frac{du}{dy}\right)^2 \qquad (1.11)$$

where

$$u = 0 \qquad \text{at } y = 1 \qquad (1.12)$$

$$u=0 \qquad \text{at } y = -1 \qquad (1.13)$$

$$\frac{\partial^2 u}{\partial y^2}=0 \qquad \text{at } y = \pm 1 \qquad (1.14)$$

$$\theta=0 \qquad \text{at } y = -1 \qquad (1.15)$$

$$\theta=1 \qquad \text{at } y = 1 \qquad (1.16)$$

1.5 Solution of the Problem

Solving equation (1.10) subject to the boundary conditions (1.12) – (1.14) we obtain the velocity field as.

$$u=C_1e^{A_1 y}+C_2e^{-A_1 y}+C_3e^{B_1 y}+C_4e^{-B_1 y}+\frac{Pa^2}{c^2} \qquad (1.17)$$

Where

$$C_1=C_2=\frac{F\left(e^{-A_1}-e^{A_1}\right)}{e^{2A_1}-e^{-2A_1}}$$

ISSN: 2278-0181
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$$C_3 = C_4 = \frac{G\left(e^{-B_1} - e^{B_1}\right)}{e^{2B_1} - e^{-2B_1}}$$

$$F = \frac{Pa^2b^2}{c^2} \quad G = \frac{-a^2}{\left(a^2 - b^2\right)} \frac{Pa^2}{c^2}$$

$$A_1 = \sqrt{\frac{b^2 + \sqrt{b^4 - 4c^2}}{2}} \qquad B_1 = \sqrt{\frac{b^2 - \sqrt{b^4 - 4c^2}}{2}}$$

$$P = -\mathrm{Re}\frac{\partial p}{\partial x} \quad a^2 = \frac{\mu h^2}{\eta}, b^2 = a^2/\in, \quad c^2 = a^2/Da$$

Solving equation (1.11) subject to the boundary conditions (1.15) – (1.16) we obtain the temperature as.

where

1.6 Results and discussion

(i) Mass flow rate

The dimensionaless mass flow rate of the flow of a couple stress fluid through the porous medium bounded by parallel plates is given by

$$M = \int_{-1}^{1} u dy$$

$$= 2\left(\frac{c_1}{a}\right)\left(e^a - e^{-a}\right) + 2\left(\frac{c_3}{a}\right)\left(e^a - e^{-a}\right) + \frac{2Pa^2}{c^2}$$

(ii) Inferences

From equation 1.17 we have calculated velocity as a function of y for different values of couple stress parameter a for fixed Da = 0.1, P = 2, and $\in$ = 0.5 and is shown in figure 1.2. We observe that the velocity increases with the increasing in y in $0 \le y \le 0.5$ and it decreases with the increasing y for $0.5 \le y \le 1$. The velocity attains the maximum value at the central line of the channel. For a given y, we notice that the velocity increases with increasing couple stress parameter a.

The variation of velocity U with y is calculated from equation 1.17 for different values of porosity $\in$ and is shown in figure 1.3. for fixed a = 2, P = 5 and Da = 0.03. We observe that for a given y, the velocity increases with the increasing $\in$. This is due to increase in the porosity of the porous layer.

The variation of velocity U with y is calculated from equation 1.17 for different values of Darcy number Da and is shown in figure 1.4. We observe that for a given y, the velocity increases with increasing Da. This may be due to the increase in the permeability of the porous layer.

The variation of velocity U with y is calculated from equation 1.17 for different values of P and is shown in figure 1.5. For fixed a = 2, $\in$ = 0.2, Da = 0.03, we observe that for a given y, the velocity increases with the increasing P. This may be due to the increase in the Reynolds number of the porous layer.

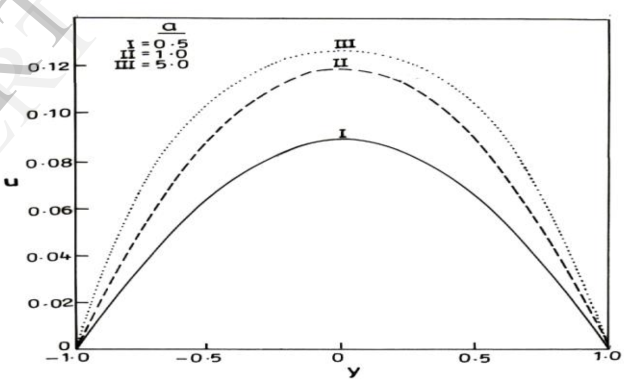
From equation 1.18, we have calculated temperature as a function of y for different values of Darcy number Da, for fixed a = 1.2, $\in$ = 0.01, Pr = 0.7, Ec = 0.01, Q = 5, and is shown in figure 1.6. We observe that for a given Da, the temperature increases with the increasing in y the temperature attains the maximum and minimum values at the lower and upper boundaries of the porous layer. For a given y, we notice that the temperature decreases with the increasing Darcy number Da.

The variation of temperature θ with y is calculated from equation 1.18 for different values of Prandtl number Pr and is shown in figure 1.7. We observe that for a given y, the temperature θ increases with increasing Prandtl number.

The variation of temperature θ with y is calculated from equation 1.18 for different values of Eckert number Ec and is shown in figure 1.8. We observe that for a given y, the emperature increases with the increasing Eckert number.

1.7. Graphs and Tables:

Fig. 1.2 Velocity profiles for different values of couple stress parameter a, with fixed values of $\in = 0.2, Da = 0.03, P = 5,$



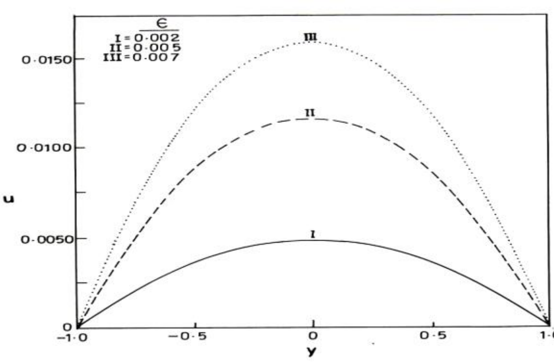


Fig. 1.2 Velocity profiles for different values of couple stress parameter a, with fixed values of $\in = 0.2, Da = 0.03, P = 5,$

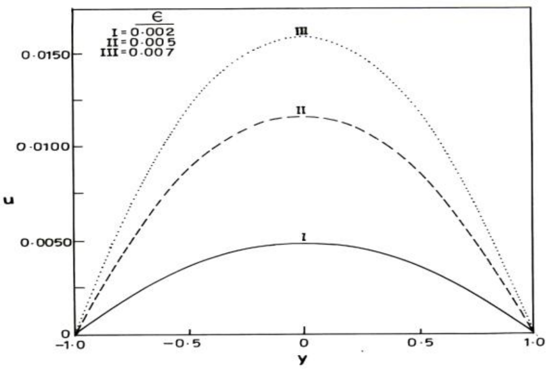


Fig. 1.3: Velocity Profiles for different values of $\in$, with fixed values of $a = 2,\ Da = 0.03,\ P = 5,$

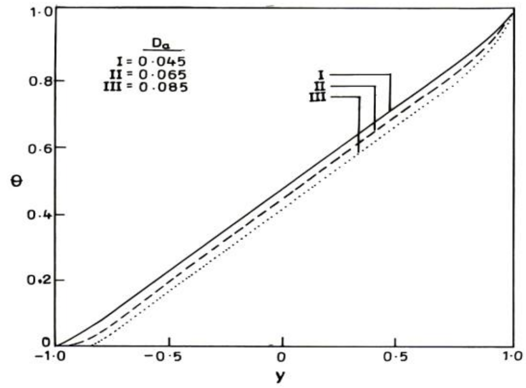


Fig. 1.6 : Temperature profiles for different values of Da, with fixed values of
$a = 1.2,\ \in = 0.001,\ P = 0.7,\ E = 0.1,\ Da = 0.03,\ Q = -5$

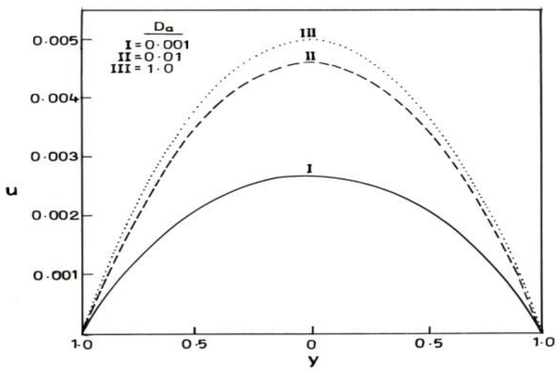


Fig. 1.4: Velocity profiles for different values of Da, with fixed values of $a = 2,\ \in = 0.002,\ P = 5,$

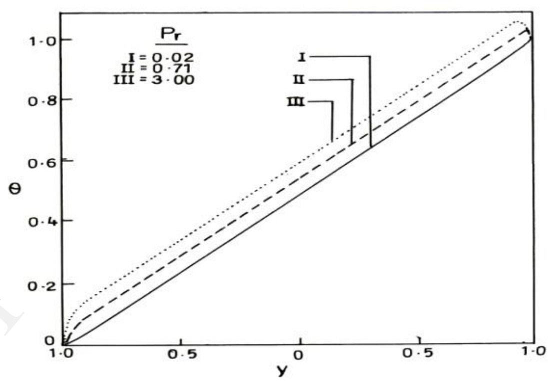


Fig.1.7: Temperature profiles for different values of Pr with Fixed values of
$a = 1.2,\ \in = 0.001,\ E = 1,\ Da = 0.03,\ Q = -5$

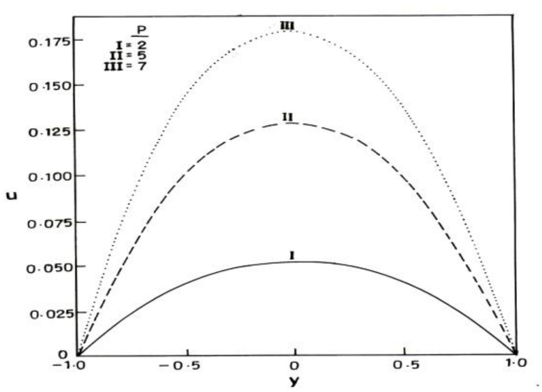


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Fig. 1.5: Velocity profiles for different values of P with fixed values of $a = 2,\ \in = 0.002,\ Da = 0.03$

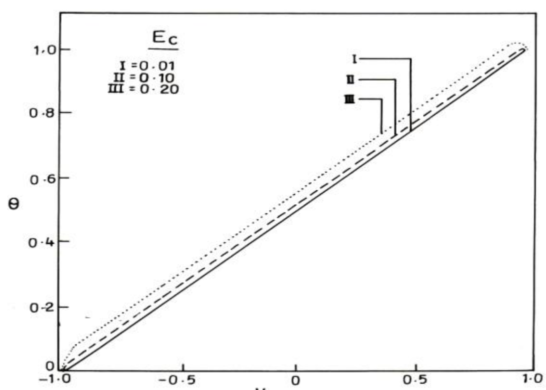

Fig. 1.8: Temperature profiles for different values of Ec with fixed values of
$a = 1.2,\ \in = 0.001,\ Da = 0.03,\ Q = -5$

1.8 Appendix:

$$\theta = -E_c P_r \begin{bmatrix} \frac{s_1}{4a^2}\cosh 2ay - s_1\frac{y^2}{2} + \frac{s_2}{4b^2}\cosh 2by - \\ s_2\frac{y^2}{2} + \frac{s_3}{(a+b)^2}\cosh(a+b)y \\ -\frac{s_3}{(a-b)^2}\cosh(a-b)y \end{bmatrix}$$

$$+F_1 y + F_2$$

$$\theta = \begin{bmatrix} \frac{s_1}{4a^2}\cosh 2a - \frac{s_1}{2} + \frac{s_2}{4b^2}\cosh 2b - \\ \frac{s_2}{2} + \frac{s_3}{(a+b)^2}\cosh(a+b) \\ -\frac{s_3}{(a-b)^2}\cosh(a-b) \end{bmatrix}$$

$$F_1 = \frac{1}{2} \quad \text{and} \quad F_2 = \frac{1}{2} + E_c\ P_r\ H$$

$$s_1 = 4a^2c_1^{\,2} \quad s_2 = 4c_3^{\,2}b^2 \quad s_3 = 4abc_1c_3$$

1.9 References

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