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Flow of a Non-Newtonian Second-Order Fluid Over an Enclosed Torsionally Oscillating Disc in the Presence of the Magnetic Field

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Abstract **- The problem of the flow of an incompressible non-Newtonian second-order fluid over an enclosed torsionally oscillating discs in the presence of the magnetic field has been discussed. The obtained differential equations are highly non-linear and contain upto fifth order derivatives of the flow functions. Hence exact or numerical solutions of the differential equations are not possible subject to the given natural boundary conditions, therefore the regular perturbation technique is applied. The flow functions H, G, L and M are expanded in the powers of the amplitude $\in$ (taken small) of the oscillations. The behaviour of the radial, transverse and axial velocities at different values of Reynolds number, phase difference, magnetic field and second-order parameters has been studied and shown graphically. The results obtained are compared with those for the infinite torsionally oscillating discs by taking the Reynolds number of out-flow R_m and circulatory flow R_L equal to zero. The transverse shearing stress and moment on the lower and upper discs have also been obtained.**

Key Words: Flow; Second-Order Fluid;Enclosed Torsionally Oscillating Disc, Magnetic Field.

1 INTRODUCTION

The phenomenon of flow of the fluid over an enclosed torsionally oscillating disc (enclosed in a cylindrical casing) has important engineering applications. The most common practical application of it is the domestic washing machine and blower of curd etc.. Soo[1)] has considered first the problem of laminar flow over an enclosed rotating disc in case of Newtonian fluid. The torsional oscillations of Newtonian fluids have been discussed by Rosenblat[2)]. He has also discussed the case when the Newtonian fluid is confined between two infinite torsionally oscillating discs[3)]. Sharma & Gupta[4)] have considered a general case of a second-order fluid between two infinite torsionally oscillating discs. Thereafter Sharma & Singh[5)] extended the same problem for the case of porous discs subjected to uniform suction and injection. Hayat[6)] has considered non-Newtonian flows over an oscillating plate with variable suction. Chawla[7)] has considered flow past of a torsionally oscillating plane Riley & Wybrow[8)] have considered the flow induced by the torsional oscillations of an elliptic cylinder. Bluckburn[9)] has considered a study of two-dimensional flow past of an oscillating cylinder. Sadhna Kahre[10)] studied the steady flow between a rotating and porous stationary disc in the presence of transverse magnetic field. B. B. Singh and Anil Kumar[11)] have considered the flow of a second-order fluid due to the rotation of an infinite porous disc near a stationary parallel porous disc. Present paper is extended work of Reshu Agarwal[13)] who has considered Flow of a Non-Newtonian Second-Order Fluid over an Enclosed Torsionally Oscillating Disc.

Due to complexity of the differential equations and tedious calculations of the solutions, no one has tried to solve the most practical problems of enclosed torsionally oscillating discs so far. The authors have considered the present problem of flow of a non-Newtonian second-order fluid over an enclosed torsionally oscillating disc in the presence of the magnetic field and calculated successfully the steady and unsteady part both of the flow functions. The flow functions are expanded in the powers of the amplitude $\in$ (assumed to be small) of the oscillations of the disc. The non-Newtonian effects are exhibited through two dimensionless parameters $\tau_1(=n\mu_2/\mu_1)$ and $\tau_2(=n\mu_3/\mu_1)$, where μ_1, μ_2, μ_3 are coefficient of Newtonian viscosity, elastico-viscosity and cross-viscosity. n being the uniform frequency of the oscillation. The variation of radial, transverse and axial velocities with elastico-viscous parameter τ_1, cross-viscous parameter τ_2, Reynolds number R, magnetic field m_1 at different phase difference τ is shown graphically.

2 FORMULATION OF THE PROBLEM

The constitutive equation of an incompressible second-order fluid as suggested by Colemann and Noll[12)] can be written as:

$$\tau_{ij} = - p\delta_{ij} + 2\mu_1 d_{ij} + 2\mu_2 e_{ij} + 4\mu_3 c_{ij} \qquad \text{---------------- (1)}$$

where

$$d_{ij} = ½ (u_{i,j} + u_{j,i}),$$

$$e_{ij} = ½(a_{i,j} + a_{j,i}) + u^m{}_{,i} u_{m,j},$$

$$c_{ij} = d_{im}\, d^m{}_{j.} \qquad \text{----------- (2)}$$

p is the hydrostatic pressure, τ_{ij} is the stress-tensor, u_i and a_i are the velocity and acceleration vector.
The equation (1) together with the momentum equation for no extraneous force

$$\rho\, (\partial u_i/\partial t + u^m u_{i,m}) = t^m{}_{i,m} \qquad \text{----------------------(3)}$$

and the equation of continuity for incompressible fluid

$u^i_{,i} = 0$ -- (4)

where ρ is the density of the fluid and comma (,) represents covariant differentiation, form the set of governing equations.

In the three dimensional cylindrical set of co-ordinates (r, θ, z) the system consists of a finite disc of radius r_s (coinciding with the plane z = 0) performing rotatory oscillations on the type rΩCosτ of small amplitude ∊, about the perpendicular axis r = 0 with angular velocity Ω in an incompressible second-order fluid forming the part of a cylindrical casing or housing. The top of the casing (coinciding with the plane $z = z_0 < r_s$) may be considered as a stationary disc (stator) placed parallel to and at a distance equal to gap length z_0 from the oscillating disc. The symmetrical radial steady outflow has a small mass rate 'm' of radial outflow ('-m' for net radial inflow). The inlet condition is taken as a simple radial source flow along z-axis starting from radius r_0. A constant magnetic field B_0 is applied normal to the plane of the oscillating disc. The induced magnetic field is neglected.

Assuming (u, v, w) as the velocity components along the cylindrical system of axes (r, θ, z)the boundary conditions of the problem are:

z = 0, u = 0, v = r $\Omega e^{i\tau}$(Real part),

w = 0,

$z = z_0$, u = 0, v = 0, w = 0, --------------------(5)

where the gap z_0 is assumed small in comparison with the disc radius r_s. The velocity components for the axisymmetric flow compatible with the continuity criterion can be taken as [4)].

$U = -\xi H'(\zeta,\tau) + (R_m/R_z)\, M'(\zeta,\tau)/\xi$

$V = \xi G(\zeta,\tau) + (R_L/R_z)L(\zeta,\tau)/\xi,$

$W = 2H(\zeta,\tau).$ -- (6)

where $U = u/\Omega z_0$, $V = v/\Omega z_0$, $W = w/\Omega z_0$, $\xi = r/z_0$, ζ, τ are dimensionless quantities and H(ζ,τ), G(ζ,τ), L(ζ,τ), M′(ζ,τ) are dimensionless function of the dimensionless variables $\zeta = z/z_0$ and τ = nt. $R_m(=m/2\Pi\rho z_0\nu_1)$, R_L $(=L/2\Pi\rho z_0\nu_1)$ are dimensionless number to be called the Reynolds number of net outflow and circulatory flow respectively. $R_z(=\Omega z_0^2/\nu_1)$ be the flow Reynolds number. The small mass rate 'm' of the radial outflow is represented by

$$m = 2\Pi\rho \int_0^{z_0} r\, u\, dz \quad \text{-------------- (7)}$$

Using expression (6), the boundary condition (5) transform for G, L & H into the following form:

$G(0,\tau) = \mathrm{Real}(e^{i\tau})$, $G(1,\tau) = 0$,
$L(0,\tau) = 0$, $L(1,\tau) = 0$,
$H(0,\tau) = 0$, $H(1,\tau) = 0$,
$H'(0,\tau) = 0$, $H'(1,\tau) = 0$.
---------------------------- (8)

The conditions on M on the boundaries are obtainable form the eq.(7) for m as follows:

$M(1,\tau) - M(0,\tau) = 1,$ --------------------------- (9)

which on choosing the discs as streamlines reduces to

$M(1,\tau) = 1,$ $M(0,\tau) = 0$
-- (10)

Using eqs.(1) and expression (6)in equation (3) and neglecting the squares & higher powers of R_m/R_z (assumed small), we have the following equations in dimensionless form:

$- (1/\rho z_0)(\partial p/\partial \xi) = - n\Omega z_0\{\xi\partial H' -(R_m/R_z)(\partial M'/\xi)\}+ \Omega^2 z_0\, \xi(H'^2 - 2HH''-G^2) + \Omega^2 z_0(R_m/R_z)(2HM''/\xi) - \Omega^2 z_0(R_L/R_z)(2LG/\xi)+(\nu_1\Omega/z_0)\{H'''\xi - (R_m/R_z)(M'''/\xi)\}-(2\nu_2/z_0)[(n\Omega/2)\{(R_m/R_z)(\partial M'''/\xi)-\xi\,\partial H'''\} + \Omega^2\xi(H''^2 - HH^{iv}) + (R_m/R_z)(\Omega^2/\xi)(H'''M'+H''M''+H'M'''+HM^{iv}) -(R_L/R_z)(2\Omega^2/\xi)(L'G' + LG'')]-(4\nu_3\Omega^2/z_0)\{(R_m/R_z)(1/2\xi)(H'''M' +H'M''' +H''M'') -(R_L/R_z)(1/2\xi)(2L'G' + LG'') + (\xi/4)(H''^2 - G'^2 - 2H'H''')\}+(\sigma B_0^2\Omega z_0/\rho)\{-\xi H'+ (R_m/R_z)(M'/\xi)\}$ -- -- (11)

$0 = -n\Omega z_0\{\xi\partial G + (R_L/R_z)(\partial L/\xi)\}-2\Omega^2 z_0\xi)(HG'-H'G)-\Omega^2 z_0(R_m/R_z)(2M'G/\xi) -\Omega^2 z_0(R_L/R_z)(2HL'/\xi)+ (\nu_1\Omega/z_0)\{\xi G'' + (R_L/R_z)(L''/\xi)\}+ (2\nu_2/z_0)[(n\Omega/2)\{\xi\partial G''+(R_L/R_z)(\partial L''/\xi)\}+ (R_L/R_z)(\Omega^2/\xi)(H''L' +H'''L+ HL''' +H'L'') +(\Omega^2\xi)(HG'''- H''G')+(R_m/R_z)(2\Omega^2/\xi)(M'G''+M''G')]+(2\nu_3\Omega^2/z_0)\{\xi(H'G'' -H''G')+(R_L/R_z)(1/\xi)(H''L' + H'''L + H'L'')+(R_m/R_z)(1/\xi)(2M''G' + M'G'') - (\sigma B_0^2\Omega z_0/\rho)\{\xi G+(R_L/R_z)(L/\xi)\}$ ---(12)

$-(1/\rho z_0)(\partial p/\partial \zeta) = 2n\Omega z_0\partial H+4\Omega^2 z_0 HH'- 2\nu_1\Omega H''/z_0 - (2\nu_2/z_0)\{n\Omega\partial H''+ 2\Omega^2\xi^2(H''H'''+G'G'')+\Omega^2(22H'H'' + 2HH''') -(R_m/R_z)2\Omega^2(H''M'''+H'''M'') + (R_L/R_z)2\Omega^2(L'G'' + L''G')\}-(2\nu_3\Omega^2/z_0)\{\xi^2(H''H''' + G'G'') + 14H'H'' - (R_m/R_z)(H''M''' + H'''M'') + (R_L/R_z)(L'G'' + L''G')\}$ ------- (13)

where B_0 and σ are intensity of the magnetic field and conductivity of the fluid considered. R $(=nz_0^2/\nu_1)$ is the Reynolds number, $\tau_1(=n\nu_2/\nu_1)$, $\tau_2(=n\nu_3/\nu_1)$ and $\in(=\Omega/n)$ are the dimensionless parameter, $m^2 = \sigma B_0^2 z_0^2/\mu_1$ is the dimensionless magnetic field.

Differentiating (11) w.r.t. ζ and (13) w.r.t. ξ and then eliminating $\partial^2 p/\partial\zeta.\partial\xi$ from the equation thus obtained. We get

$-n\Omega z_0\{\xi\partial H''-(R_m/R_z)\partial M''/\xi\}-2\Omega^2 z_0\xi(HH'''+GG') +(R_m/R_z)(2\Omega^2 z_0/\xi)(H'M''+HM''')-(R_L/R_z)(2\Omega^2 z_0/\xi)(LG'+L'G)-(\nu_1\Omega/z_0)\{(R_m/R_z)(M^{iv}/\xi)-\xi H^{iv}\}-(2\nu_2/z_0)[(n\Omega/2)\{(R_m/R_z)(\partial M^{iv}/\xi) - \xi\partial H^{iv}\}-\Omega^2\xi(2H''H'''+H'H^{iv}+HH^{v}+4G'G'')+(R_m/R_z)(\Omega^2/\xi)(2H'''M''+H^{iv}M'+2H''M'''+2H'M^{iv}+HM^{v})-(R_L/R_z)(2\Omega^2/\xi)(2L'G''+L''G'+LG''')]-(2\nu_3\Omega^2/z_0)\{(R_m/R_z)(1/\xi)(H^{iv}M'+2H'''M''+ 2H''M'''+H'M^{iv}) -(R_L/R_z)(1/\xi)(3L'G''+2L''G'+LG''')-\xi(H'H^{iv} + 3G'G''+2H''H''')\} + (\sigma B_0^2\Omega z_0/\rho)\{-\xi H''+(R_m/R_z)(M''/\xi)\}= 0$ --------------------------- (14)

On equating the coefficients of ξ and 1/ξ from the equation (12) & (14), we get the following equations:

$G'' = R\partial G + 2\in R(HG'-H'G) - \tau_1\partial G'' - 2\in\tau_1(HG'''-H''G') - 2\in\tau_2(H'G''-H''G') + m^2G$ -----------------(15)

$L'' = R\partial L + 2\in R(M'G + HL') - \tau_1\partial L'' - 2\in\tau_1(H''L'+H'''L+HL'''+H'L''+2M'G''+2M''G') - 2\in\tau_2(H''L'+H'''L+H'L''+2M''G'+M'G'') + m^2L$ ------------- (16)

$H^{iv} = R\partial H'' + 2\in R(HH'''+GG') - \tau_1\partial H^{iv} - 2\in\tau_1(H'H^{iv}+HH^{V}+2H''H'''+4G'G'') - 2\in\tau_2\ (H'H^{iv}+2H''H'''+3G'G'') + m^2H''$ ------------------------------ (17)

$M^{iv} = R\partial M'' + 2\in R(H'M''+HM'''-LG'-L'G) - \tau_1\partial M^{iv} - 2\in\tau_1(2H'''M''+H^{iv}M' + 2H''M''' + 2H'M^{iv}+HM^{v}-4L'G''-2L''G'-2LG''') - 2\in\tau_2(H^{iv}M'+2H'''M''+2H''M''' + H'M^{iv}-3L'G''-2L''G'-LG''') + m^2M''$ ----- (18)

3. SOLUTION OF THE PROBLEM

Assuming the relationship $m^2 = \in m_1^2$, equations (15)- (18) becomes

$G'' = R\partial G + 2\in R(HG'-H'G) - \tau_1\partial G'' - 2\in\tau_1(HG'''-H''G') - 2\in\tau_2(H'G''-H''G') + \in m_1^2G$ ------------------(19)

$L'' = R\partial L + 2\in R(M'G + HL') - \tau_1\partial L'' - 2\in\tau_1(H''L'+H'''L+HL'''+H'L''+2M'G''+2M''G') - 2\in\tau_2(H''L'+H'''L+H'L''+2M''G'+M'G'') + \in m_1^2L$ ---- (20)

$H^{iv} = R\partial H'' + 2\in R(HH'''+GG') - \tau_1\partial H^{iv} - 2\in\tau_1(H'H^{iv}+HH^{v}+2H''H'''+4G'G'') - 2\in\tau_2\ (H'H^{iv}+2H''H'''+3G'G'') + \in m_1^2H''$ --- ----------------- (21)

$M^{iv} = R\partial M'' + 2\in R(H'M''+HM'''-LG'-L'G) - \tau_1\partial M^{iv} - 2\in\tau_1(2H'''M''+H^{iv}M' + 2H''M''' + 2H'M^{iv}+HM^{v}-4L'G''-2L''G'-2LG''') - 2\in\tau_2(H^{iv}M'+2H'''M''+2H''M'''+H'M^{iv}-3L'G''-2L''G'-LG''') + \in m_1^2M''$ ------------- (22)

Substituting the expressions

$G(\zeta,\tau) = \Sigma\in^N G_N(\zeta,\tau)$
$L(\zeta,\tau) = \Sigma\in^N L_N(\zeta,\tau)$
$H(\zeta,\tau) = \Sigma\in^N H_N(\zeta,\tau)$
$M(\zeta,\tau) = \Sigma\in^N M_N(\zeta,\tau)$ ------------------------------ (23)

into (19) to (22) neglecting the terms with coefficient of $\in^2$ (assumed negligible small) and equating the terms independent of $\in$ and coefficient of $\in$, we get the following equations:

$G_0'' = R\ \partial G_0/\partial\tau - \tau_1\ \partial G_0''/\partial\tau$ ----- (24)

$G_1'' = R\ \partial G_1/\partial\tau - 2R(H_0'G_0'-H_0G_0') - \tau_1\partial G_1''/\partial\tau - 2\tau_1(H_0G_0'''-H_0''G_0') - 2\tau_2(H_0'G_0''-H_0''G_0') + m_1^2G_0$ --- (25)

$L_0'' = R\ \partial L_0/\partial\tau - \tau_1\ \partial L_0''/\partial\tau$ ------- (26)

$L_1'' = R\partial L_1/\partial\tau - 2R(M_0'G_0 + H_0L_0') - \tau_1\ \partial L_1''/\partial\tau - 2\tau_1(H_0'''L_0+H_0''L_0'+H_0'L_0''+H_0L_0'''+2M_0''G_0'+2M_0'G_0'') - 2\tau_2(H_0'''L_0+H_0''L_0'+H_0'L_0''+2M_0''G_0'+M_0'G_0'') + m_1^2L_0$ ------------(27)

$H_0^{iv} = R\partial H_0''/\partial\tau - \tau_1\ \partial H_0^{iv}/\partial\tau$ -- (28)

$H_1^{iv} = R\partial H_1''/\partial\tau + 2R(H_0H_0''' + G_0G_0') - \tau_1\partial H_1^{iv}/\partial\tau - 2\tau_1(H_0'H_0^{iv} + H_0H_0^{v}+2H_0''H_0''' + 4G_0'G_0'') - 2\tau_2(3G_0'G_0''+H_0'H_0^{iv}+2H_0''H_0''') + m_1^2H_0''$ ------------- (29)

$M_0^{iv} = R\ \partial M_0''/\partial\tau - \tau_1\ \partial M_0^{iv}/\partial\tau$ -------------------- (30)

$M_1^{iv} = R\partial M_1''/\partial\tau + 2R(H_0'M_0''+H_0M_0'''-L_0'G_0-L_0G_0') - \tau_1\partial M_1^{iv}/\partial\tau - 2\tau_1(2H_0'''M_0''+H_0^{iv}M_0'+2H_0''M_0'''-4L_0'G_0''-2L_0''G_0'-2L_0G_0'''+H_0M_0^{v}+2H_0'M_0^{iv}) - 2\tau_2(2H_0'''M_0''+H_0^{iv}M_0'+2H_0''M_0'''-3L_0'G_0''-2L_0''G_0'-L_0G_0'''+H_0'M_0^{iv}) + m_1^2M_0''$ ----------------- (31)

Taking $G_n(\zeta,\tau) = G_{ns}(\zeta) + e^{i\tau}G_{nt}(\zeta)$
$L_n(\zeta,\tau) = L_{ns}(\zeta) + e^{i\tau}L_{nt}(\zeta)$
$H_n(\zeta,\tau) = H_{ns}(\zeta) + e^{2i\tau}H_{nt}(\zeta)$
$M_n(\zeta,\tau) = M_{ns}(\zeta) + e^{2i\tau}M_{nt}(\zeta)$----------------- (32)

Using (23) and (32), the boundary conditions (8) & (10) for N = 0, 1 transforms to

$G_{0s}(0) = 0$, $G_{0t}(0) = 1$, $G_{1s}(0) = 0$, $G_{1t}(0) = 0$,
$G_{0s}(1) = 0$, $G_{0t}(1) = 0$, $G_{1s}(1) = 0$, $G_{1t}(1) = 0$,
$H_{0s}(0) = 0$, $H_{0t}(0) = 0$, $H_{1s}(0) = 0$, $H_{1t}(0) = 0$,
$H_{0s}(1) = 0$, $H_{0t}(1) = 0$, $H_{1s}(1) = 0$, $H_{1t}(1) = 0$,
$H'_{0s}(0) = 0$, $H'_{0t}(0) = 0$, $H'_{1s}(0) = 0$, $H'_{1t}(0) = 0$,
$H'_{0s}(1) = 0$, $H'_{0t}(1) = 0$, $H'_{1s}(1) = 0$, $H'_{1t}(1) = 0$,
$L_{0s}(0) = 0$, $L_{0t}(0) = 0$, $L_{1s}(0) = 0$, $L_{1t}(0) = 0$,
$L_{0s}(1) = 0$, $L_{0t}(1) = 0$, $L_{1s}(1) = 0$, $L_{1t}(1) = 0$,
$M'_{0s}(0) = 0$, $M'_{0t}(0) = 0$, $M'_{1s}(0) = 0$, $M'_{1t}(0) = 0$,
$M'_{0s}(1) = 0$, $M'_{0t}(1) = 0$, $M'_{1s}(1) = 0$, $M'_{1t}(1) = 0$,
$M_{0s}(0) = 0$, $M_{0t}(0) = 0$, $M_{1s}(0) = 0$, $M_{1t}(0) = 0$,
$M_{0s}(1) = 1$, $M_{0t}(1) = 0$, $M_{1s}(1) = 0$, $M_{1t}(1) = 0$.
------ (33)

Applying (32) & (33) in eqs. (24) to (31), we get

$G_{0s}(\zeta) = G_{1s}(\zeta) = 0$,

$G_{0t}(\zeta) = [\sinh\{d(1-\zeta)\}]/\sinh d$,

where $d = \{iR/(1+i\tau_1)\}^{1/2} = [R\{\tau_1+(1+\tau_1^2)^{1/2}\}/\{2(1+\tau_1^2)\}]^{1/2} + i[R\{(1+\tau_1^2)^{1/2}-\tau_1\}/\{2(1+\tau_1^2)\}]^{1/2} = A + iB$,

$G_0(\zeta,\tau) = \mathrm{Real}\{e^{i\tau}\ G_{0t}(\zeta)\}$,
$= [\cos\tau.\{\cosh\{(2-\zeta)A\}.\cos B\zeta - \cosh A\zeta.\cos\{(2-\zeta)B\}\} - \sin\tau.\{\sinh A\zeta.\ \sin\{(2-\zeta)B\} - \sinh\{(2-\zeta)A\}.\sin B\zeta\}]/(\cosh 2A - \cosh 2B)$,

$G_{1t}(\zeta) = [(m_1^2 \sinh d\zeta)/\{2d(1+i\tau_1)\sinh^2 d\}] - [\{m_1^2\ \zeta\ \cosh d(1-\zeta)\}/\{2d(1+i\tau_1)\sinh d\}]$,

$G_1(\zeta,\tau) = \mathrm{Real}\{e^{i\tau}\ G_{1t}(\zeta)\}$,
$= (m_1^2/2)\{(\alpha_{17}-\zeta\alpha_{19}).\cos\tau - (\alpha_{18}-\zeta\alpha_{20}).\sin\tau\}$,

where, $\alpha_1 = \cos B\zeta.\sinh A\zeta$,
$\alpha_2 = \sin B\zeta.\cosh A\zeta$,
$\alpha_3 = (\cos 2B.\cosh 2A - 1)/2$,
$\alpha_4 = (\sin 2B.\sinh 2A)/2$,
$\alpha_5 = \cos B(1-\zeta).\cosh A(1-\zeta)$,
$\alpha_6 = \sin B(1-\zeta).\sinh A(1-\zeta)$,
$\alpha_7 = \cos B.\sinh A$,
$\alpha_8 = \sin B.\cosh A$,
$\alpha_9 = A\alpha_3 - B\alpha_4$,
$\alpha_{10} = B\alpha_3 + A\alpha_4$,
$\alpha_{11} = A\alpha_7 - B\alpha_8$,
$\alpha_{12} = B\alpha_7 + A\alpha_8$,
$\alpha_{13} = \alpha_9 - \tau_1\alpha_{10}$,
$\alpha_{14} = \alpha_{10} + \tau_1\alpha_9$,
$\alpha_{15} = \alpha_{11} - \tau_1\alpha_{12}$,
$\alpha_{16} = \alpha_{12} + \tau_1\alpha_{11}$,
$\alpha_{17} = (\alpha_1\alpha_{13} + \alpha_2\alpha_{14})/(\alpha_{13}^2 + \alpha_{14}^2)$,
$\alpha_{18} = (\alpha_2\alpha_{13} - \alpha_1\alpha_{14})/(\alpha_{13}^2 + \alpha_{14}^2)$,
$\alpha_{19} = (\alpha_5\alpha_{15} + \alpha_6\alpha_{16})/(\alpha_{15}^2 + \alpha_{16}^2)$
$\alpha_{20} = (\alpha_6\alpha_{15} - \alpha_5\alpha_{16})/(\alpha_{15}^2 + \alpha_{16}^2)$,

$G(\zeta,\tau) = G_0(\zeta,\tau) + \varepsilon\ G_1(\zeta,\tau)$.

$L_{0s}(\zeta) = L_{0t}(\zeta) = L_{1s}(\zeta) = 0$,

$L_{1t}(\zeta) = -\{\sinh d\zeta/\sinh d\}[\{(A_1-A_3)/2\}(1/2d^3 - 1/6d) + A_2/2d^2] + \cosh\{d(1-\zeta)\}\ [\{(A_1-A_3)/2\}\{\zeta/2d^3 - \zeta^2/2d + \zeta^3/3d\} + (A_2\zeta/2d^2)] + \sinh\{d(1-\zeta)\}[\{(A_1-A_3)/2\}\{(\zeta^2-\zeta)/2d^2\} + A_2(\zeta^2-\zeta)/2d]$,

where, $A_1 = 12R/\{(1+i\tau_1)\sinh d\}$,
$A_2 = 24(\tau_1+\tau_2)d/\{(1+i\tau_1)\sinh d\}$,
$A_3 = 6d^2(4\tau_1+2\tau_2)/\{(1+i\tau_1)\sinh d\}$,

$L_1(\zeta,\tau) = \mathrm{Real}\{e^{i\tau}\ L_{1t}(\zeta)\}$,
$= (N_7+N_9-N_5).\cos\tau - (N_8+N_{10}-N_6).\sin\tau$,

where, $N_1 = [\{6R-(12\tau_1+6\tau_2)(A^2-B^2)\}(\cos B.\sinh A - \tau_1\sin B.\cosh A) - 2AB(12\tau_1+6\tau_2)\ (\tau_1.\cos B.\sinh A + \sin B.\cosh A)] / [(\cos B.\sinh A - \tau_1\sin B.\cosh A)^2 + (\tau_1.\cos B.\sinh A + \sin B.\cosh A)^2]$,

$N_2 = [\{6R-(12\tau_1+6\tau_2)(A^2-B^2)\}(\tau_1\cos B.\sinh A + \sin B.\cosh A) - 2AB(12\tau_1+6\tau_2)\ (\cos B.\sinh A - \tau_1.\sin B.\cosh A)] / [(\cos B.\sinh A - \tau_1\sin B.\cosh A)^2 + (\tau_1.\cos B.\sinh A + \sin B.\cosh A)^2]$,

$N_3 = [24A(\tau_1+\tau_2)(\cos B.\sinh A-\tau_1\sin B.\cosh A)+24B(\tau_1+\tau_2)$
$(\tau_1.\cos B.\sinh A+\sin B.\cosh A)] / [(\cos B.\sinh A-\tau_1\sin B.\cosh A)^2 +$
$(\tau_1.\cos B.\sinh A+\sin B.\cosh A)^2]$,
$N_4 = [24B(\tau_1+\tau_2)(\cos B.\sinh A-\tau_1\sin B.\cosh A)-24A(\tau_1+\tau_2)$
$(\tau_1.\cos B.\sinh A+\sin B.\cosh A)] / [(\cos B.\sinh A-\tau_1\sin B.\cosh A)^2 +$
$(\tau_1.\cos B.\sinh A+\sin B.\cosh A)^2]$,
$I_1 = (\cos B\zeta.\sinh A\zeta.\cos B.\sinh A + \sin B\zeta.\cosh A\zeta.\sin B.\cosh A)/$
$(\sin^2 B+\sinh^2 A)$,
$I_2 = (\sin B\zeta.\cosh A\zeta.\cos B.\sinh A-\cos B\zeta.\sinh A\zeta.\sin B.\cosh A)/$
$(\sin^2 B+\sinh^2 A)$,
$I_3 = [3(A^2+B^2)(A^3-3AB^2)-A\{(A^3-3AB^2)^2+(3A^2B-B^3)^2\}] / [6(A^2+B^2)$
$\{(A^3-3AB^2)^2+(3A^2B-B^3)^2\}]$,
$I_4 = [B(A^3-3AB^2)^2+(3A^2B-B^3)^2\} -3(A^2+B^2)(3A^2B-B^3)] / [6(A^2+B^2)$
$\{(A^3-3AB^2)^2+(3A^2B-B^3)^2\}]$,
$I_5 = \{(A^2-B^2)N_3+2N_4AB\}/ [2\{(A^2-B^2)^2+4A^2B^2\}]$,
$I_6 = \{(A^2-B^2)N_4-2N_3AB\}/ [2\{(A^2-B^2)^2+4A^2B^2\}]$,
$I_7 = [3\zeta(A^2+B^2)(A^3-3AB^2)-(3A\zeta^2-2A\zeta^3)\{(A^3-3AB^2)^2+(3A^2B-B^3)^2\}] /$
$[6(A^2+B^2)\{(A^3-3AB^2)^2+(3AB^2-B^3)^2\}]$,
$I_8 = [-3\zeta(A^2+B^2)(3A^2B-B^3)+(3B\zeta^2-2B\zeta^3)\{(A^3-3AB^2)^2+(3A^2B-B^3)^2\}] /$
$[6(A^2+B^2)\{(A^3-3AB^2)^2+(3AB^2-B^3)^2\}]$,
$I_9 = [\zeta\{N_3(A^2-B^2)+2ABN_4\}] / [2\{(A^2-B^2)^2+4A^2B^2\}]$,
$I_{10} = [\zeta\{N_4(A^2-B^2)-2ABN_3\}] / [2\{(A^2-B^2)^2+4A^2B^2\}]$,
$I_{11} = (A^2-B^2)/[(A^2-B^2)^2+4A^2B^2]$,
$I_{12} = 2AB/[(A^2-B^2)^2+4A^2B^2]$,
$I_{13} = (AN_3+BN_4)/(A^2+B^2)$,
$I_{14} = (AN_4-BN_3)/(A^2+B^2)$,
$N_5 = (N_1I_1I_3+N_2I_1I_4+I_1I_5-N_1I_2I_4+N_2I_2I_3-I_2I_6)$,
$N_6 = (N_1I_2I_3+N_2I_2I_4+I_2I_5+N_1I_1I_4-N_2I_1I_3+I_1I_6)$,
$N_7 = (N_1I_7+N_2I_8+I_9)\cos B(1-\zeta).\cosh A(1-\zeta)-(N_1I_8-N_2I_7+I_{10})\sin B(1-\zeta).\sinh A(1-\zeta)$,
$N_8 = (N_1I_8-N_2I_7+I_{10})\cos B(1-\zeta).\cosh A(1-\zeta)+(N_1I_7+N_2I_8+I_9)\sin B(1-\zeta).\sinh A(1-\zeta)$,
$N_9 = [(N_1I_{11}-N_2I_{12}+I_{13})\cos B(1-\zeta).\sinh A(1-\zeta)+(N_2I_{11}+N_1I_{12}-I_{14})\sin B(1-\zeta).\cosh A(1-\zeta)](\zeta^2-\zeta/2)$,
$N_{10} = [(N_1I_{11}-N_2I_{12}+I_{13})\sin B(1-\zeta).\cosh A(1-\zeta)-(N_2I_{11}+N_1I_{12}-I_{14})\cos B(1-\zeta).\sinh A(1-\zeta)](\zeta^2-\zeta/2)$,

$L(\zeta,\tau) = L_0(\zeta,\tau) + \varepsilon L_1(\zeta,\tau) = \varepsilon L_1(\zeta,\tau)$.

$H_{0s}(\zeta) = H_{0t}(\zeta) = H_{1t}(\zeta) = 0$,
$H_{1t}(\zeta) = c_1e^{f\zeta}/f^2 + c_2e^{-f\zeta}/f^2 + A_4\sinh 2d(1-\zeta)/\{8d^2(4d^2-f^2)\} + c_3\zeta + c_4$,
Where,
$c_1 = [\{c_2(1-e^{-f})/(1-e^f)\} + A_4f(\cosh 2d-1\}]/\{4d(4d^2-f^2)(1-e^f)$,

$c_2 = A_4f[f(1-e^f)(2d.\cosh 2d-\sinh 2d) - 2d(1+f-e^f)(\cosh 2d-1)]/\{8d^2(4d^2-f^2)\ (4-2e^{-f}-2e^f-fe^{-f}+fe^f)$,
$c_3 = (1/f)[c_2 - c_1 + \{fA_4\cosh 2d/\{4d(4d^2-f^2)\}]$,
$c_4 = -(1/f^2)[c_1 + c_2 + \{A_4f^2\sinh 2d/\{8d^2(4d^2-f^2)\}]$,
$A_4 = [(8\tau_1 + 6\tau_2)d^3 - 2Rd]/\{(1+2i\tau_1)\sinh^2 d\}$,
$f = (2iR/(1+2i\tau_1\}^{1/2}$
$= [2R\{2\tau_1+(1+4\tau_1^2)^{1/2}\}/\{2(1+4\tau_1^2)\}]^{1/2}$
$+ [2R\{(1+4\tau_1^2)-2\tau_1\}/\{2(1+4\tau_1^2)\}]^{1/2}$
$= C + iD$,
$H_1(\zeta,\tau) = \text{Real}\{e^{2i\tau}H_{1t}(\zeta)\}$,
$= (J_1+J_3+J_5+J_7).\cos 2\tau - (J_2+J_4+J_6+J_8).\sin 2\tau$,
where, $X_1 = (8\tau_1+6\tau_2)(A^3-3AB^2)-2RA$,
$X_2 = (8\tau_1+6\tau_2)(3A^2B-B^3)-2RB$,
$Y_1 = (\cos 2B.\cosh 2A-1-2\tau_1\sin 2B.\sinh 2A)/2$.
$Y_2 = \{2\tau_1(\cos 2B.\cosh 2A-1)+\sin 2B.\sinh 2A\}/2$.
$Q1 = (X_1Y_1+X_2Y_2)/(Y_1^2+Y_2^2)$,
$Q_2 = (X_2Y_1-X_1Y_2)/(Y_1^2+Y_2^2)$,
$W_1 = Q_1C-Q_2D$,
$W_2 = Q_2C+Q_1D$,
$W_3 = C(1-e^C\cos D)+De^C\sin D$,
$W_4 = D(1-e^C\cos D)-Ce^C\sin D$,
$W_5 = 2A\cos 2B.\cosh 2A-2B\sin 2B.\sinh 2A$,
$W_6 = 2B\cos 2B.\cosh 2A+2A\sin 2B.\sinh 2A$,
$W_7 = \cos 2B.\sinh 2A$,
$W_8 = \sin 2B.\cosh 2A$,
$W_9 = 2A(\cos 2B.\cosh 2A-1)-2B\sin 2B.\sinh 2A$,
$W_{10} = 2B(\cos 2B.\cosh 2A-1)+2A\sin 2B.\sinh 2A$,
$W_{11} = 1+C-e^C\cos D$,
$W_{12} = D-e^C\sin D$,
$W_{13} = W_3(W_5-W_7)-W_4(W_6-W_8)-W_9W_{11}+W_{10}W_{12}$,
$W_{14} = W_4(W_5-W_7)+W_3(W_6-W_8)-W_{10}W_{11}-W_9W_{12}$,
$W_{15} = W_1W_{13}-W_2W_{14}$,
$W_{16} = W_2W_{13}+W_1W_{14}$,
$W_{17} = 4A^2-4B^2-C^2+D^2$,
$W_{18} = 8AB-2CD$,
$W_{19} = W_{17}(8A^2-8B^2)-16ABW_{18}$,
$W_{20} = W_{18}(8A^2-8B^2)+16ABW_{17}$,
$W_{21} = 4-e^{-C}\{(C+2).\cos D + D.\sin D\}+e^C\{(C-2).\cos D-D\sin D\}$,
$W_{22} = e^C\{(C-2).\sin D + D.\cos D\}-e^{-C}\{D\cos D-(C+2).\sin D\}$,
$W_{23} = W_{19}W_{21}-W_{20}W_{22}$,
$W_{24} = W_{20}W_{21}+W_{19}W_{22}$,
$Q_3 = (W_{15}W_{23}+W_{16}W_{24})/(W_{23}^2+W_{24}^2)$,
$Q_4 = (W_{16}W_{23}-W_{15}W_{24})/(W_{23}^2+W_{24}^2)$,
$X_3 = Q_3(1-e^{-C}\cos D)-Q_4e^{-C}\sin D$,
$X_4 = Q_4(1-e^{-C}\cos D)+Q_3e^{-C}\sin D$,
$X_5 = (1-e^C\cos D)$,
$X_6 = -e^C\sin D$,
$X_7 = (X_3X_5+X_4X_6)/(X_5^2+X_6^2)$;
$X_8 = (X_4X_5-X_3X_6)/(X_5^2+X_6^2)$;
$X_9 = W_1W_9-W_2W_{10}$,
$X_{10} = W_2W_9+W_1W_{10}$,
$X_{11} = W_{19}X_5-W_{20}X_6$,
$X_{12} = W_{20}X_5+W_{19}X_6$,
$X_{13} = (X_9X_{11}+X_{10}X_{12})/(X_{11}^2+X_{12}^2)$,
$X_{14} = (X_{10}X_{11}-X_9X_{12})/(X_{11}^2+X_{12}^2)$,
$Y_3 = \{C(Q_3-Q_5)+D(Q_4-Q_6)\}/(C^2+D^2)$,
$Y_4 = \{C(Q_4-Q_6)-D(Q_3-Q_5)\}/(C^2+D^2)$,
$Y_5 = Q_1\cos 2B.\cosh 2A-Q_2\sin 2B.\sinh 2A$,
$Y_6 = Q_2\cos 2B.\cosh 2A+Q_1\sin 2B.\sinh 2A$,
$Y_7 = 4AW_{17}-4BW_{18}$,
$Y_8 = 4BW_{17}+4AW_{18}$,
$Y_9 = (Y_5Y_7+Y_6Y_8)/(Y_7^2+Y_8^2)$,
$Y_{10} = (Y_6Y_7-Y_5Y_8)/(Y_7^2+Y_8^2)$,
$Q_5 = X_7+X_{13}$,
$Q_6 = X_8+X_{14}$,
$Q_7 = Y_3+Y_9$,
$Q_8 = Y_4+Y_{10}$,
$Y_{11} = [(Q_3+Q_5)(C^2-D^2)+2CD(Q_4+Q_6])/[(C^2-D^2)^2+4C^2D^2]$,
$Y_{12} = [(Q_4+Q_6)(C^2-D^2)-2CD(Q_3+Q_5])/[(C^2-D^2)^2+4C^2D^2]$,
$Y_{13} = [W_{19}(Q_1W_7-Q_2W_8)+W_{20}(Q_2W_7+Q_1W_8)] /(W_{19}^2+W_{20}^2)$,
$Y14 = [-W_{20}(Q_1W_7-Q_2W_8)+W_{19}(Q_2W_7+Q_1W_8)] /(W_{19}^2+W_{20}^2)$,
$Q_9 = -(Y_{11}+Y_{13})$,
$Q_{10} = -(Y_{12}+Y_{14})$,
$O_1 = e^{C\zeta}(Q_5\cos D\zeta-Q_6\sin D\zeta)$,
$O_2 = e^{C\zeta}(Q_6\cos D\zeta+Q_5\sin D\zeta)$,
$O_3 = e^{-C\zeta}(Q_3\cos D\zeta+Q_4\sin D\zeta)$,
$O_4 = e^{-C\zeta}(Q_4\cos D\zeta-Q_3\sin D\zeta)$,
$J_1 = [O_1(C^2-D^2)+2CDO_2]/[(C^2-D^2)^2+4C^2D^2]$,
$J_2 = [O_2(C^2-D^2)-2CDO_1]/[(C^2-D^2)^2+4C^2D^2]$,
$J_3 = [O_3(C^2-D^2)+2CDO_4]/[(C^2-D^2)^2+4C^2D^2]$,
$J_4 = [O_4(C^2-D^2)-2CDO_3]/[(C^2-D^2)^2+4C^2D^2]$,
$J_5 = Q_7\zeta+Q_9$,
$J_6 = Q_8\zeta+Q_{10}$,
$Y_{15} = \cos\{2B(1-\zeta)\}.\sinh\{2A(1-\zeta)\}$,

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$Y_{16} = \sin\{2B(1-\zeta)\}.\cosh\{2A(1-\zeta)\}$,

$J_7 = [W_{19}(Q_1Y_{15}-Q_2Y_{16})+W_{20}(Q_2Y_{15}+Q_1Y_{16})]/ \{W_{19}^2+W_{20}^2\}$,

$J_8 = [-W_{20}(Q_1Y_{15}-Q_2Y_{16})+W_{19}(Q_2Y_{15}+Q_1Y_{16})]/\{W_{19}^2+W_{20}^2\}$,

$H(\zeta,\tau) = H_0(\zeta,\tau) + \varepsilon H_1(\zeta,\tau) = \varepsilon H_1(\zeta,\tau)$.

$.M_0(\zeta,\tau) = M_{0s}(\zeta) = - 2\zeta^3 + 3\zeta^2$,

$M_{0t}(\zeta) = M_{1t}(\zeta) = 0$,

$M_1(\zeta,\tau) = M_{1s}(\zeta) = - (m_1^2/20)(2\zeta^5-5\zeta^4+4\zeta^3-\zeta^2)$,

$M(\zeta,\tau) = M_0(\zeta,\tau) + \varepsilon M_1(\zeta,\tau)$.

4.RESULTS AND CONCLUSION

For $R_m = R_L = 0$, the results are in good agreement with those obtained by Sharma and Gupta[4)] (for $s_1 = s_2 = 1$ and $B_0 = 0$) and Sharma and Singh[5)] (for $s_1= s_2=1$ and suction parameter = 0).

The variation of the radial velocity with ζ for different values of elastico-viscous parameter τ_1 = -0.5, -0.4, -0.3; when cross-viscous parameter $\tau_2 = 2$, $m_1 = 2$, R = 0.5, $R_m = 0.05$, $R_L = 0.049$, $R_z = 2$, $\in = 0.02$, $\xi = 5$ at phase difference $\tau = 2\pi/3$, $\pi/3$ and 0 is shown in fig(1), fig(4) and fig(7) respectively. In all these figures, the behaviour of the radial velocity is being represented through the approximate parabolic curve with vertex upward. It is evident from fig (1) that the radial velocity decreases with an increase in τ_1 before the common point of intersection which is just before the line $\zeta = 0.5$ while it increases with an increase in τ_1 after this common point of intersection and from fig (4) & fig (7), the radial velocity increases with an increase in τ_1 before the common point of intersection and decreases with an increase in τ_1 after this common point of intersection. The value of the radial velocity is same approximately in the middle of the gap-length for all the values of elastico-viscous and phase difference parameters. In fig (1) the point of maxima of radial velocity is little beyond the half of the gap-length whenever in fig(4) it is in the middle of the gap-length and in fig(7) it is being shifted a little towards the oscillating disc from the middle of the gap-length.

The variation of the transverse velocity with ζ for different values of elastico-viscosity parameter τ_1 = -0.5, -0.4, -0.3; when cross-viscous parameter $\tau_2 = 2$, $m_1 = 2$, R = 0.5, $R_m = 0.05$, $R_L = 0.049$, $R_z = 2$, $\in = 0.02$, $\xi = 5$ at phase difference $\tau = 2\pi/3$, $\pi/3$ and 0 is shown in fig(2), fig(5) and fig(8) respectively. In fig (2), the transverse velocity increases linearly and in fig(5) & fig(8), it decreases linearly throughout the gap-length. It is also evident from fig (2) and fig (5) that the transverse velocity increases with increase in τ_1 and from fig(8), the transverse velocity decreases with increase in τ_1 throughout the gap-length.

The variation of the axial velocity with ζ for different values of elastico-viscous parameter τ_1 = -0.5, -0.4, -0.3; when cross-viscous parameter $\tau_2 = 2$, $m_1 = 2$, R = 0.5, $R_m = 0.05$, $R_L = 0.049$, $R_z = 2$, $\in = 0.02$, $\xi = 5$ at phase difference $\tau = 2\pi/3$, $\pi/3$ and 0 is shown in fig(3), fig(6) and fig(9) respectively. In fig(3), the axial velocity increases in the first half and then decreases in the second half of the gap-length and forming the bell shape curve with point of maxima at $\zeta = 0.5$. It is also evident that the axial velocity increases with increase in τ_1 throughout the gap-length. In fig (6) the behaviour of the axial velocity is being represented through the bell shape curve with point of maxima at $\zeta = 0.5$ for τ_1 = -0.5 & -0.4 whenever point of minima at $\zeta = 0.5$ for $\tau_1 = -0.3$. It is also evident that the axial velocity decreases with increase in τ_1 and values of the axial velocity remains negative throughout the gap-length for $\tau_1 = -0.3$. In fig(9), the behaviour of the axial velocity is being represented through the bell shape curve with of minima at $\zeta = 0.5$. It is also evident that the axial velocity decreases with an increase in τ_1 and remains negative throughout the gap-length.

The variation of the radial velocity with ζ for different values of cross-viscous parameter τ_2 = 30, 20, 6; when elastico-viscous parameter $\tau_1 = -2$, $m_1 = 2$, R = 0.5,$R_m = 0.05$, $R_L = 0.049$, $R_z = 2$, $\in = 0.02$, $\xi = 5$ at phase difference $\tau = 2\pi/3$, $\pi/3$ and 0 is shown in fig(10), fig(13) and fig(16) respectively. It is evident from fig (10) and fig(16) that the radial velocity increases with increase in τ_2 before the common point of intersection and it decreases with an increase in τ_2 after the common point of intersection while in fig (13), the radial velocity decreases with increase in τ_2 before the common point of intersection (lying approximately the middle of the gap-length) and increases with an increase in τ_2 after the common point of intersection. The value of the radial velocity is same approximately in the middle of the gap-length for all the values of cross-viscous and phase difference parameters. In fig (10) the point of maxima of the radial velocity is approximately in the middle of the gap-length whenever in fig(13) it is little beyond the half of the gap-length and in fig(16) it is being shifted a little towards the oscillating disc from the middle of the gap-length.

The variation of the transverse velocity with ζ for different values of cross-viscous parameter τ_2 = 30, 20, 6; when cross-viscous parameter $\tau_1 = -2$, $m_1 = 2$, R = 0.5, R_m = 0.05, $R_L = 0.049$, $R_z = 2$, $\in = 0.02$, $\xi = 5$ at phase difference $\tau = 2\pi/3$, $\pi/3$ and 0 is shown in fig(11), fig(14) and fig(17) respectively. In fig(11), the transverse velocity increases linearly and in fig(14) & fig(17), it decreases linearly throughout the gap-length. It is also evident from fig(11) & fig(14) that the transverse velocity slightly increases with increase in τ_2 in the first half and decreases with increase in τ_2 in the second half of the gap-length whenever in fig(17) the transverse velocity slightly decreases with increase in τ_2 in the first half and increases with increase in τ_2 in the second half of the gap-length. Because of their slightly increment or decrement, these figures are being overlapped.

The variation of the axial velocity with ζ at different values of cross-viscous parameter τ_2 = 30, 20, 6; when elastico-viscous parameter $\tau_1 = -2$, $m_1 = 2$, R = 0.5, $R_m = 0.05$, $R_L = 0.049$, $R_z = 2$, $\in = 0.02$, $\xi = 5$, phase difference $\tau = 2\pi/3$, $\pi/3$ and 0 is shown in fig(12), fig(15) and fig(18) respectively. In above three figures, axial velocity increases in the first half and then decreases in the second half throughout the gap-length. It is also evident from figure (12), the axial velocity decreases with increase

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in τ_2 and from fig (15) & fig (18) the axial velocity increases with increase in τ_2. the behaviour of the axial velocity is being represented through the normal curve symmetric about $\zeta = 0.5$ line.

The variation of the radial velocity, transverse velocity and axial velocity with ζ at different Reynolds number R = 1, 0.5, 0.1; when cross-viscous parameter $\tau_2 = 10$, elastico-viscous parameter $\tau_1 = -2$, $m_1 = 2$, $R_m = 0.05$, $R_L = 0.049$, $R_z = 2$, $\in = 0.02$, $\xi = 5$, phase difference $\tau = \pi/3$ is shown in fig(19), fig(20) and fig(21) respectively. It is evident from fig (19) that the radial velocity decreases with an increase in Reynolds number R before the common point of intersection while increases with an increase in Reynolds number R after the common point of intersection. In fig (20), transverse velocity decreases linearly throughout the gap-length. It is also evident from this figure that transverse velocity increases with increase in Reynolds number R. In fig (21), the axial velocity increases in the first half and decreases in the second half of the gap-length. It is also evident from this figure that axial velocity increases with increase in Reynolds number R.

The variation of the radial velocity and transverse velocity with ζ at different magnetic field m_1 = 1, 30, 40; when cross-viscous parameter $\tau_2 = 10$, elastico-viscous parameter $\tau_1 = -2$, Reynolds number R = 0.5, $R_m = 0.05$, $R_L = 0.049$, $R_z = 2$, $\in = 0.02$, $\xi = 5$, phase difference $\tau = \pi/3$ is shown in fig(22) and fig(23) respectively. It is seen from this figure that the radial velocity increases with increase in m_1 upto $\zeta = 0.28$, then it decreases with increase in m_1 upto $\zeta = 0.75$ and then it increases with increase in m_1 upto $\zeta = 1$.

In fig (23), the transverse velocity decreases linearly for $m_1 = 1$, it increases upto $\zeta = 0.28$ and decreases thereafter for m_1 = 30, 40. it is also seen from this figure that transverse velocity increases with increase in m_1.

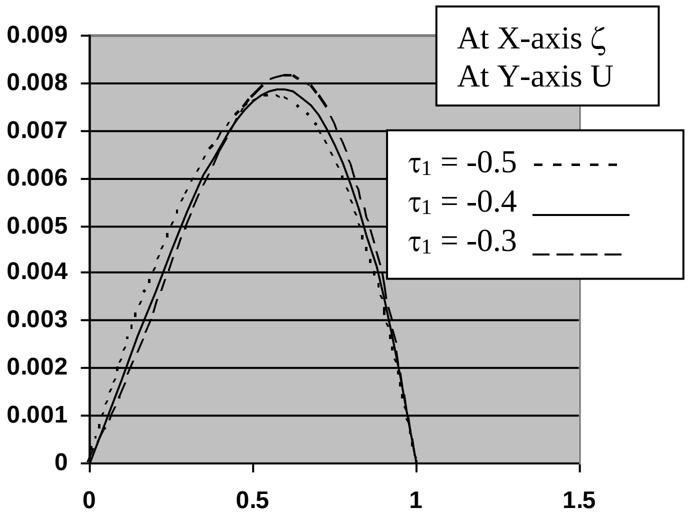
There is no magnetic field term in axial velocity so variation of axial velocity at different m_1 is not possible.

The transverse shearing stress on the lower and upper discs respectively is obtained as ;

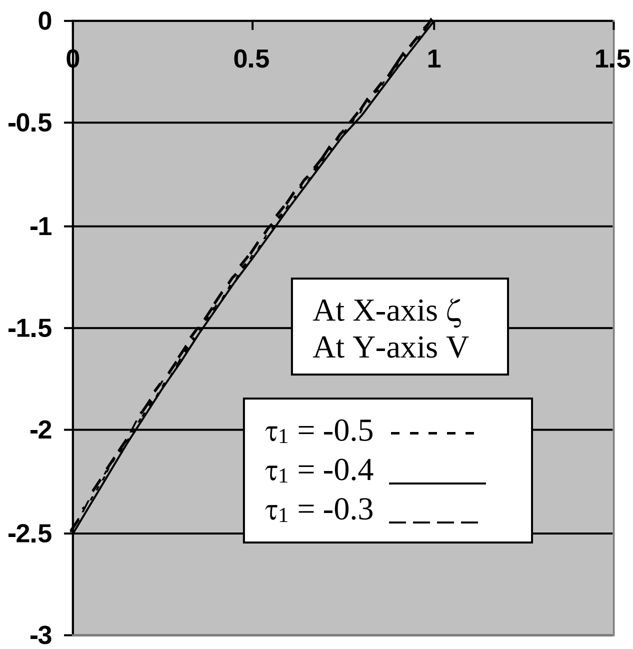
$(\tau_{\theta z})_{z=0} = \mu_1[-(\xi/z_0)\{1/(\cosh 2A - \cos 2B)\}\{(A \sinh 2A + B\sin 2B).\cos\tau +$
$(A \sinh 2B - B\sin 2A).\sin\tau\} + (R_L/R_z)(1/\xi z_0) \in \{N_7' + N_9' - N_5'\}_{\zeta=0}.\cos\tau$
$- (N_8' + N_{10}' - N_6')_{\zeta=0}.\sin\tau]$

and

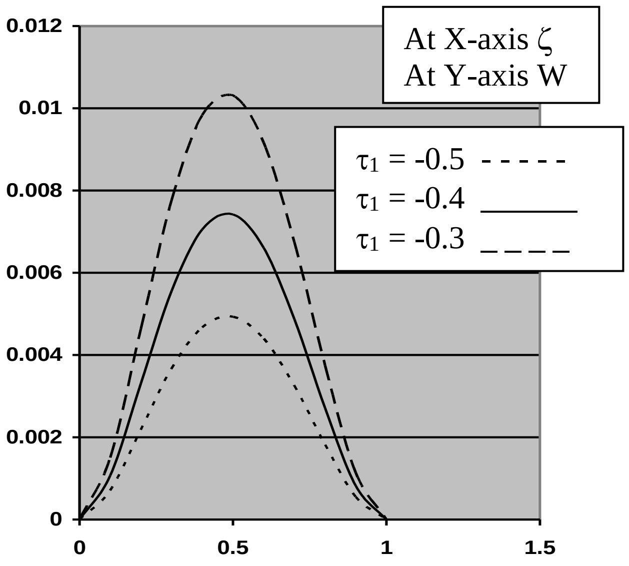
$(\tau_{\theta z})_{z=z0} = \mu_1[-(2\xi/z_0)\{1/(\cosh 2A - \cos 2B)\}\{(A \sinh A.\cos B + B\sin B \cosh A).\cos\tau + (A \cosh A.\sin B - B\sinh A \cos B).\sin\tau\} + (R_L/R_z)(1/\xi z_0) \in \{N_7' + N_9' - N_5'\}_{\zeta=1}.\cos\tau - (N_8' + N_{10}' - N_6')_{\zeta=1}.\sin\tau]$.



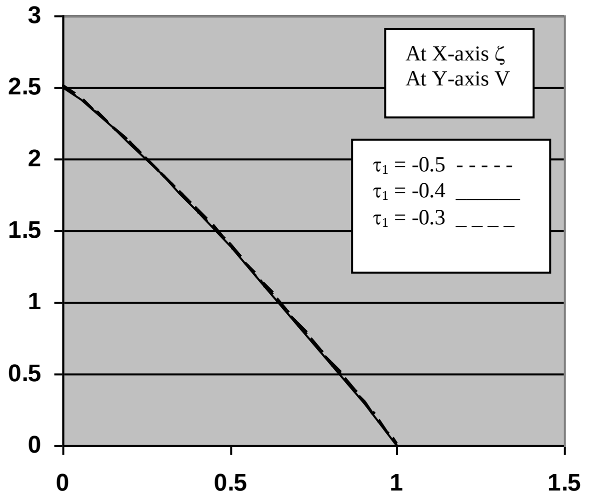
Fig(1) variation of radial velocity U with ζ for different elastico-viscous parameter τ_1 at $\tau = 2\pi/3$



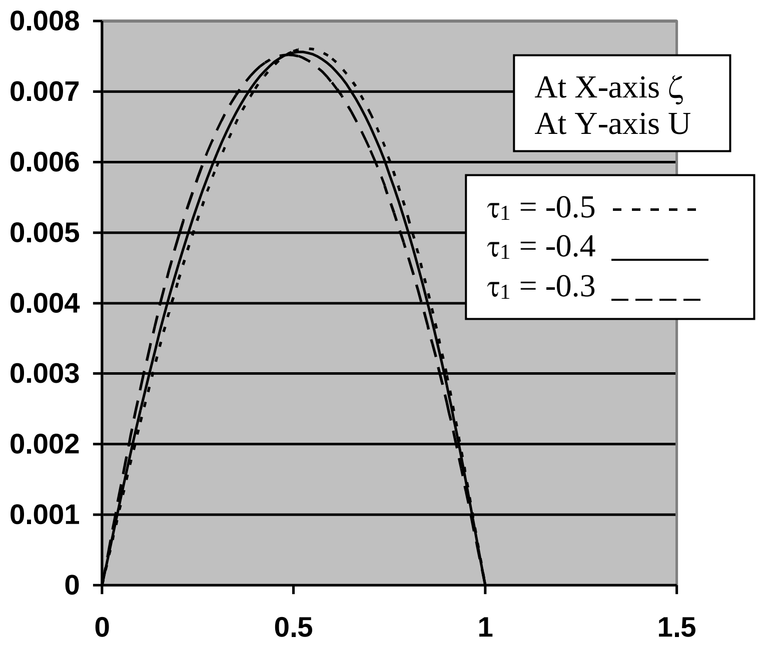
Fig(2) variation of transverse velocity V with ζ for different elastico-viscous parameter τ_1 at $\tau = 2\pi/3$



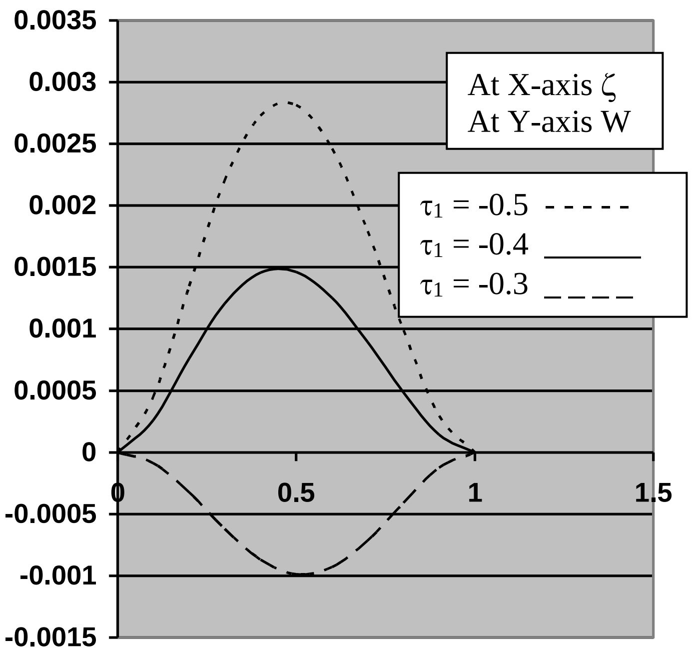
Fig(3) variation of axial velocity W with ζ for different elastico-viscous parameter τ_1 at $\tau = 2\pi/3$



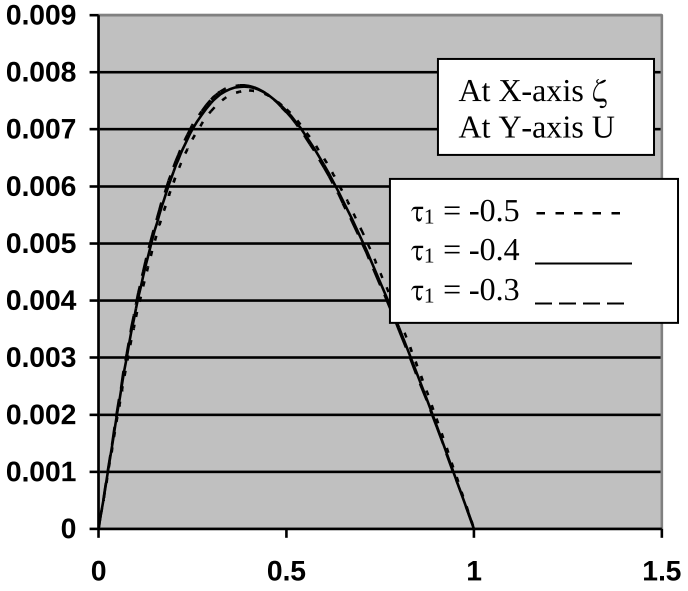
Fig(5) variation of transverse velocity V with ζ for different elastico-viscous parameter τ_1 at $\tau = \pi/3$



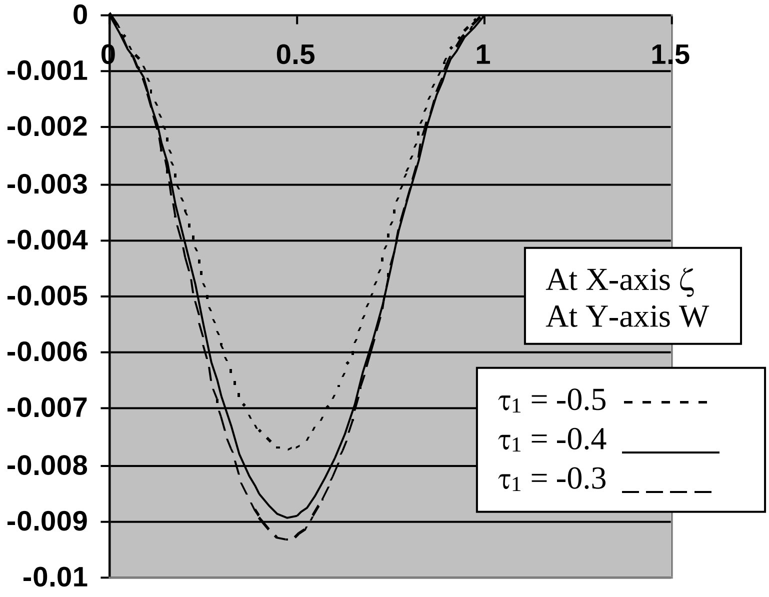
Fig(4) variation of radial velocity U with ζ for different elastico-viscous parameter τ_1 at $\tau = \pi/3$



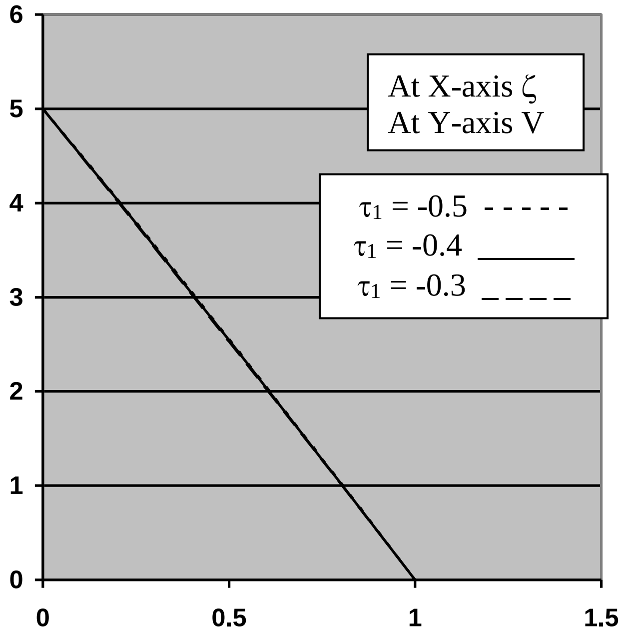
Fig(6) variation of axial velocity W with ζ for different elastico-viscous parameter τ_1 at $\tau = \pi/3$



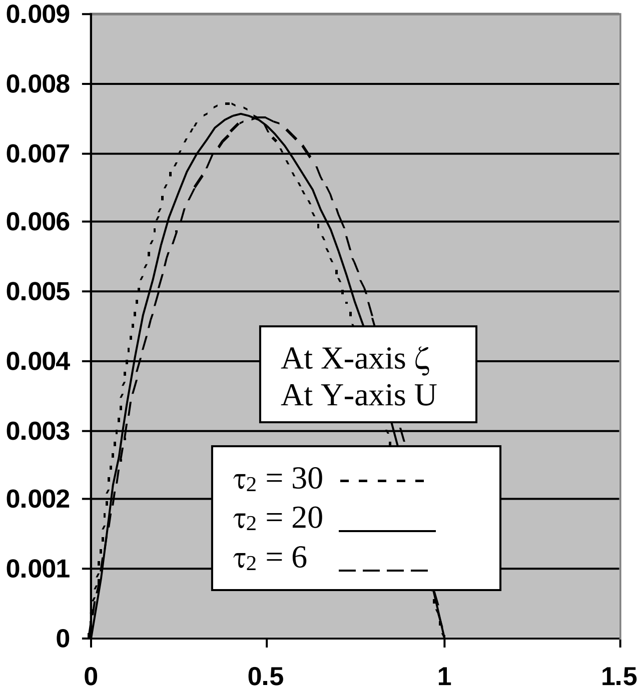
Fig(7) variation of radial velocity U with ζ for different elastico-viscous parameter τ_1 at $\tau = 0$



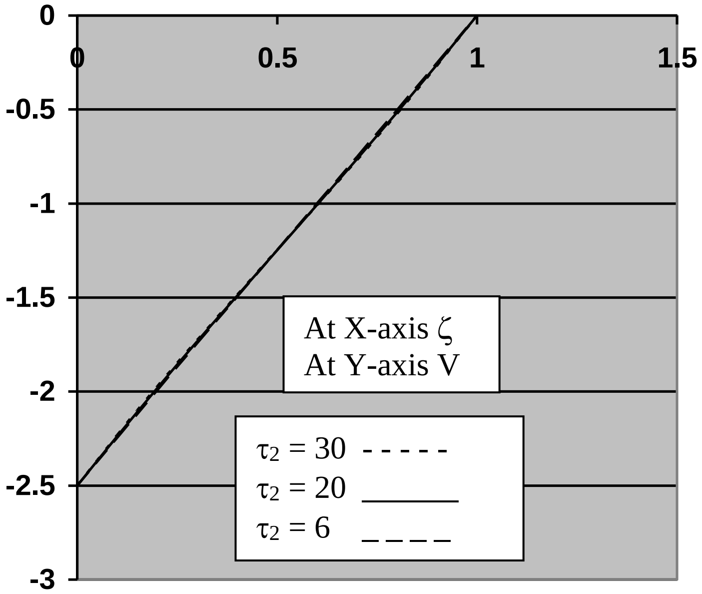
Fig(9) variation of axial velocity W with ζ for different elastico-viscous parameter τ_1 at $\tau = 0$



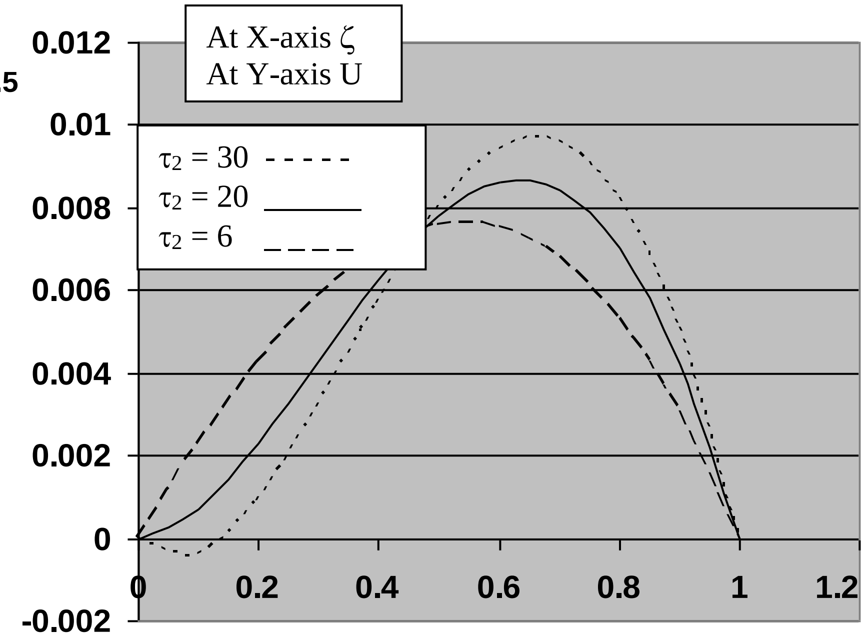
Fig(8) variation of transverse velocity V with ζ for different elastico-viscous parameter τ_1 at $\tau = 0$



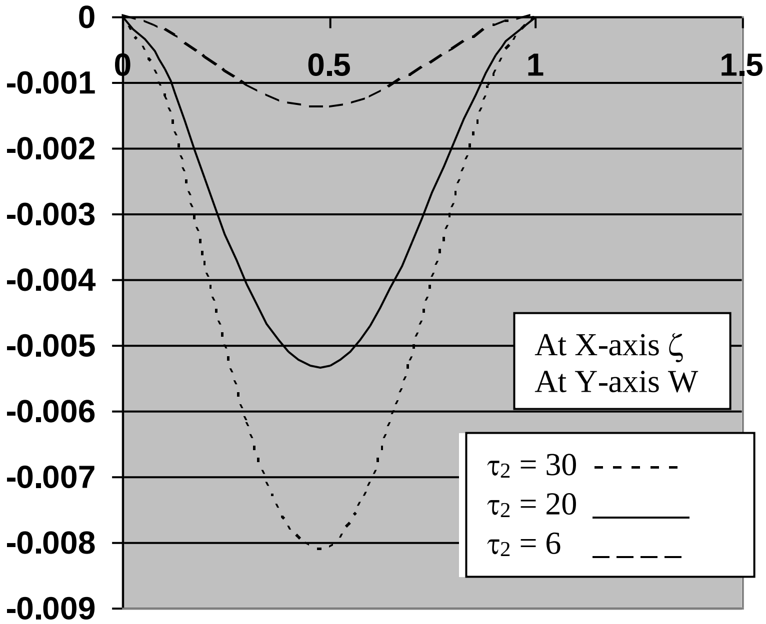
Fig(10) variation of radial velocity U with ζ for different cross-viscous parameter τ_2 at $\tau = 2\pi/3$



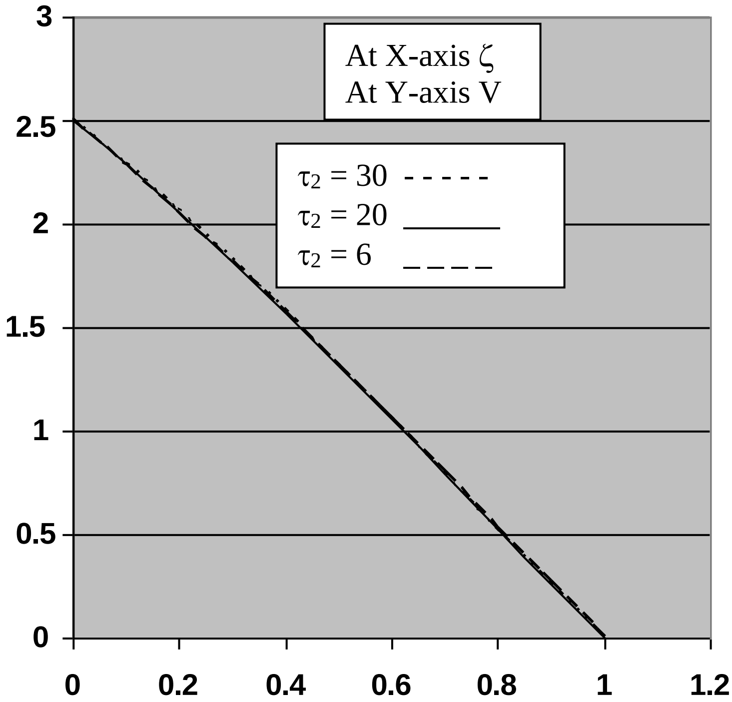
Fig(11) variation of transverse velocity V with ζ for different cross-viscous parameter τ at τ = 2π/3



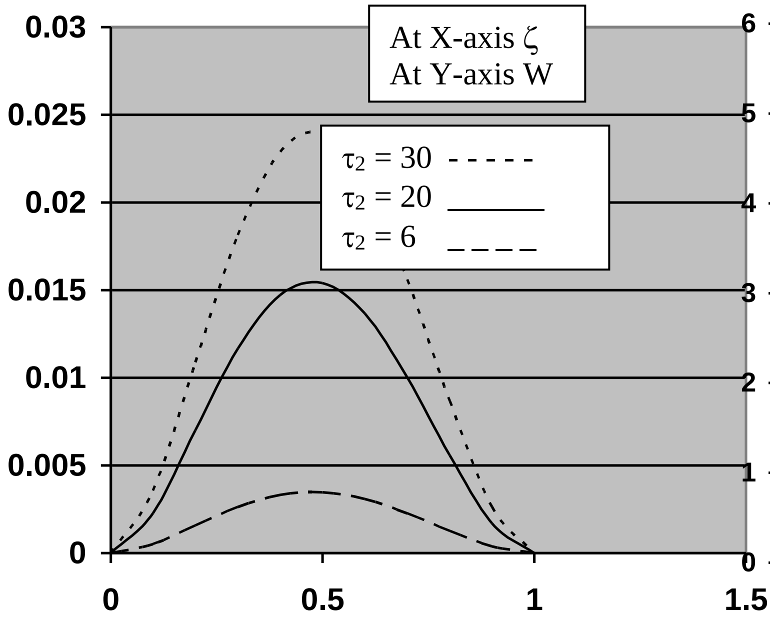
Fig(13) variation of radial velocity U with ζ for different cross-viscous parameter τ_2 at τ = π/3



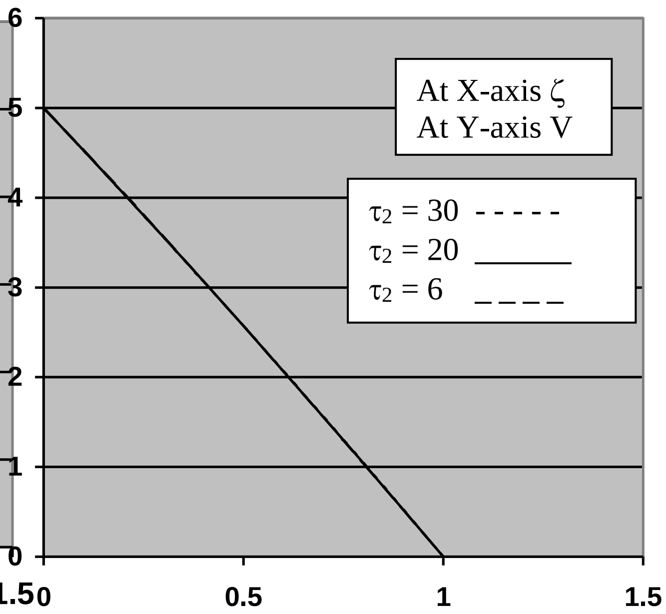
Fig(12) variation of axial velocity W with ζ for different cross-viscous parameter τ_2 at τ = 2π/3



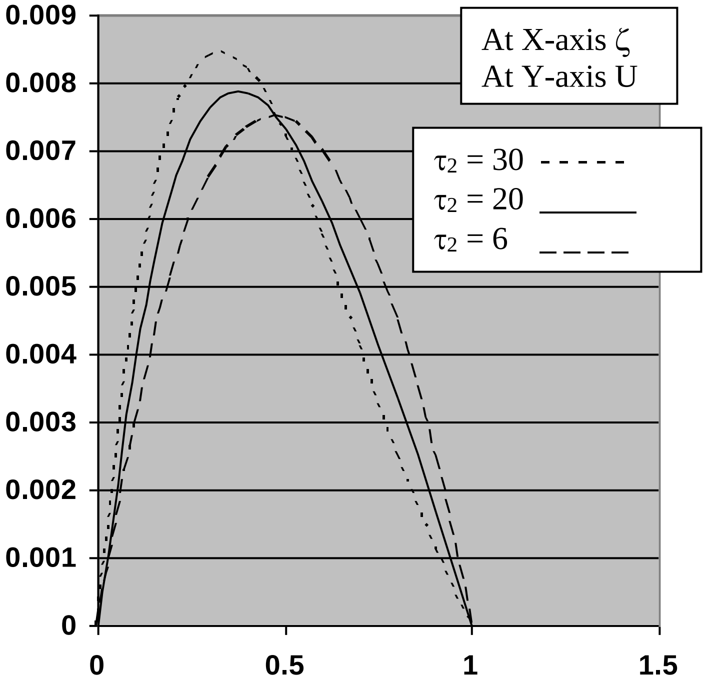
Fig(14) variation of transverse velocity V with ζ for different cross-viscous parameter τ_2 at τ = π/3



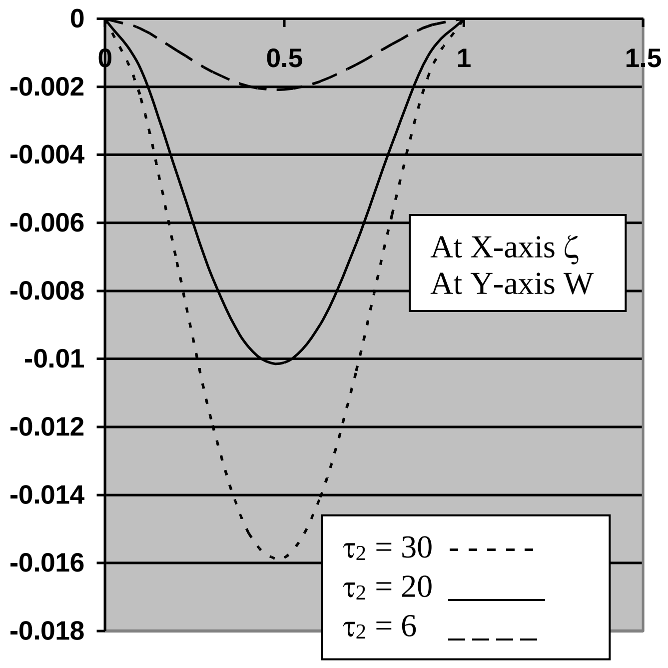
Fig(15) variation of axial velocity W with ζ for different cross-viscous parameter τ_2 at $\tau = \pi/3$



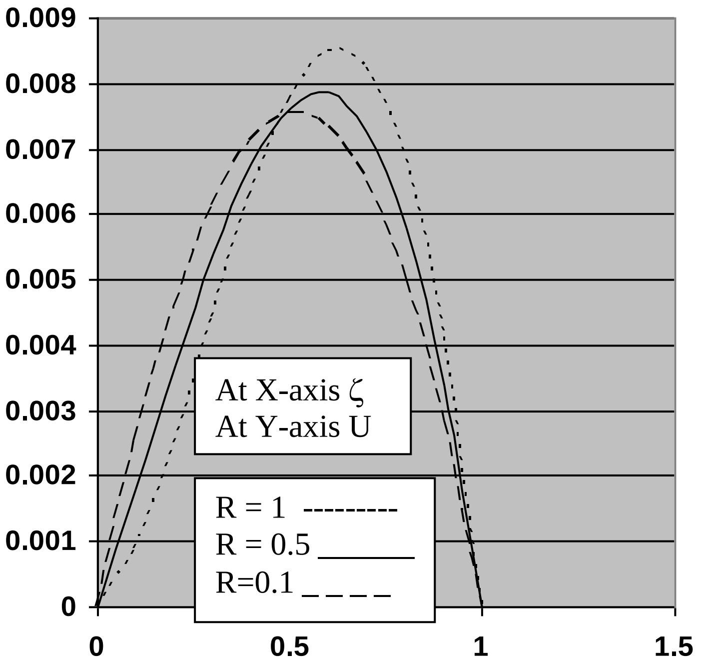
Fig(17) variation of transverse velocity V with ζ for different cross-viscous parameter τ_2 at $\tau = 0$



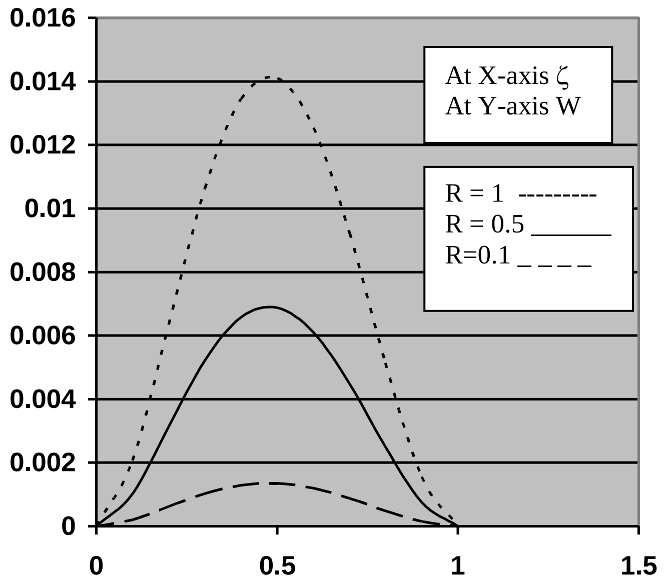
Fig(16) variation of radial velocity U with ζ for different cross-viscous parameter τ_2 at $\tau = 0$



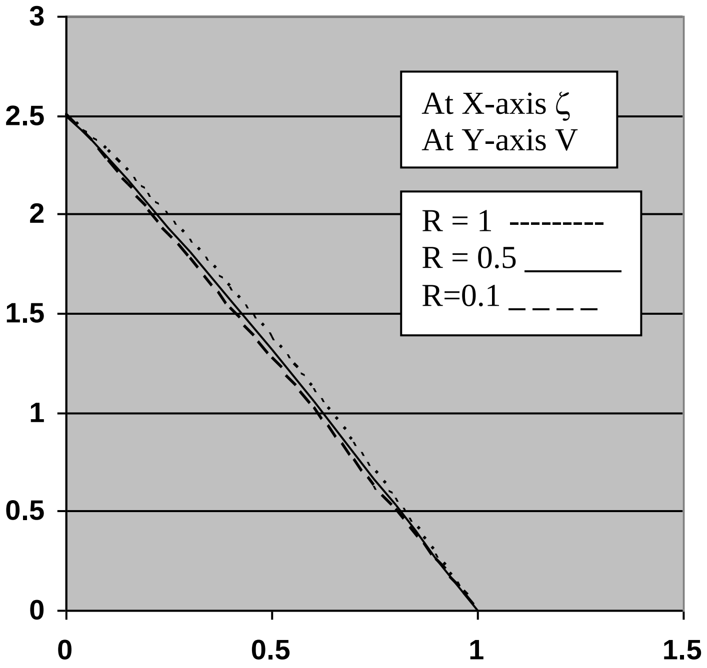
Fig(18) variation of axial velocity W with ζ for different cross-viscous parameter τ_2 at $\tau = 0$



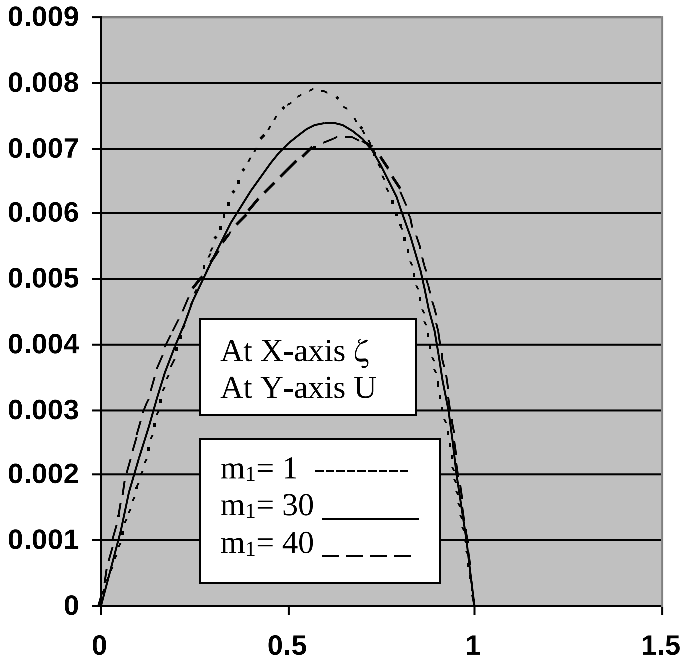
Fig(19) variation of radial velocity U with ζ for different Reynolds number R at $\tau = \pi/3$



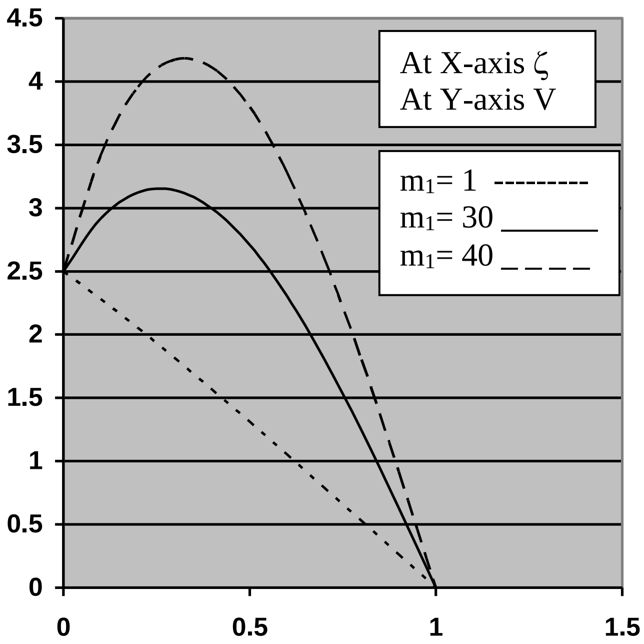
Fig(21) variation of axial velocity W with ζ for different Reynolds number R at $\tau = \pi/3$



Fig(20) variation of transverse velocity V with ζ for different Reynolds number R at $\tau = \pi/3$



Fig(22) variation of radial velocity U with ζ for different m_1 at $\tau = \pi/3$



Fig(23) variation of transverse velocity V with ζ for different m_1 at $\tau = \pi/3$

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