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Flow Through Porous Medium Between Two Co-Axial Cylinders With Rotation And Linear Motion

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Abstract

In the present chapter, we examined the flow of a Newtonian fluid passing through complex porous medium between two coaxial cylinders which are rotating about the axis of the flow and moving parallel to the axis of the cylinders with different velocities. Flow under different situations is studied. In each case, the results are obtained when the permeability of the medium is very large and is very small. The classical Darcy's effect is found very near to the axis of the cylinders and the non-Darcian effect is found near the boundaries of the flow. The effect of the permeability coefficient of the porous medium is examined in each case.

Key words and phrases: Newtonian fluid, Co-axial cylinders, Porous medium, Permeability.

1. Introduction

Due to applications in solid mechanics, transpiration cooling, food preservation, cosmetic Industry, blood flow and artificial dialysis, the problem of flows through porous medium has received a great attention in technological as well as in biophysical fields. The problem of flow under pressure gradient in the angular region between cylinders has important applications in hydrology. Many researchers Sharma and Gupta [1] worked on the subject. Rotating fluid flows analysis is of interest because of their applications in different branches of engineering and meteorology. Rott and Lawellen [2] have given an extensive study of rotating flows and their various applications. Raptis and Perdikis [3] have discussed the oscillating flow in the presence of free convective flow through a porous medium. Beghel *et al*. [4] have examined the two dimensional unsteady free convection flow of a viscous incompressible fluid through a rotating porous medium. Pattabhi Ramacharyulu [5] examined the steady laminar flow of a viscous liquid through annulus whose walls rotate and move with constant velocities. Narasimha Charyulu and Pattabhi Ramacharyulu [6] have investigated the flow of a viscous liquid through porous region contained between two cylinders by applying the generalized Brinkman's law [7].

2. Formulation and solution of the problem

To investigate the problem of flow of Newtonian fluid through porous medium contained between two co-axial cylinders which are rotating slowly about the axis of the cylinders and moving slowly parallel to the axis of the cylinders. We consider the cylindrical co-ordinate system (r, θ, z) such that the z-axis lies along the length of common axis of the cylinder and r in the radial direction. The physical quantities are independent of θ due to symmetry of the flow and they are independent of z as the cylinders are considered to be infinite in length.

The cylinders (r = a and r = b) are assumed to rotate with angular velocities, Ω_a and Ω_b are moving with linear velocities $w_{a,}$, w_b parallel to the z-axis respectively.

The equation of motion of a viscous liquid through a porous medium as proposed by Brinkman is

$$0 = -\nabla P - \left(\frac{\mu}{k}\right)\vec{V} + \mu\nabla^2\vec{V}$$

(2.1)

together with the equation of continuity

$$\operatorname{Div} \vec{V} = 0 \tag{2.2}$$

where P is the pressure, $\vec{V}$ is the velocity field, μ is the coefficient of viscosity of the fluid and k is the permeability constant of the medium.

The choice of the velocity $\vec{V}[0, v(r), w(r)]$ satisfies the equation of continuity (2.2) and the pressure

$$P = c - Gz + \int_0^r \tfrac{1}{r}[v(r)]^2\,dr \tag{2.3}$$

is taken to balance the centrifugal force generated by the velocity component v(r), where $G = -\frac{\partial P}{\partial z}$ is a constant and c is the constant of integration.

Equation (2.1) gives

$$\frac{d^2v}{dr^2} + \frac{1}{r}\frac{dv}{dr} - \left(r^2 + \frac{1}{k}\right)v = 0 \tag{2.4}$$

$$\frac{d^2w}{dr^2} + \frac{1}{r}\frac{dw}{dr} - \frac{1}{k}w = -\left(\frac{G}{\mu}\right) \tag{2.5}$$

With the boundary conditions

$$w(r) = w_a, \quad v(r) = a\Omega_a \quad \text{at } r = a$$
$$w(r) = w_b, \quad v(r) = b\Omega_b \quad \text{at } r = b \tag{2.6}$$

Solving equations (2.4) and (2.5) with the boundary conditions (2.6), we get

$$v(r) = \frac{a\Omega_a T_1(r,b) - b\Omega_b T_1(r,a)}{T_1(a,b)} \tag{2.7}$$

$$w(r) = \frac{w_a T_0(r,b) - w_b T_0(r,a) + ka^*[T_0(r,b) - T_0(r,a) - T_0(a,b)]}{T_0(a,b)} \tag{2.8}$$

With

$$T_i(x,y) = I_i\left(\frac{x}{\sqrt{k}}\right)K_i\left(\frac{y}{\sqrt{k}}\right) - I_i\left(\frac{y}{\sqrt{k}}\right)K_i\left(\frac{x}{\sqrt{k}}\right) \tag{2.9}$$

$$a^* = -\frac{G}{\mu} \qquad i = 0, 1$$

where I_i, K_i are modified Bessel functions.

2.1 When the permeability of the medium is very small (i.e., k⟶ 0, 1/k⟶ ∞)

Equation (2.9) becomes,

$$T_i(x,y) = \frac{1}{2}\sqrt{\frac{k}{xy}}\left[\exp\{(x-y)/\sqrt{k}\} - \exp\{-(x-y)/\sqrt{k}\}\right] \tag{2.10}$$

and we get

$$v(r) = \frac{1}{\sqrt{r}}\left[a^{3/2}\Omega_a \exp\{-(r-a)/\sqrt{k}\} + b^{3/2}\Omega_b \exp\{-(b-r)/\sqrt{k}\}\right] \tag{2.11}$$

$$w(r) = \frac{1}{\sqrt{r}}\left[a^{1/2}w_a \exp\{-(r-a)/\sqrt{k}\} - b^{1/2}w_b \exp\{-(b-r)/\sqrt{k}\}\right]$$
$$+ ka^*\left\langle \frac{1}{\sqrt{r}}\left[a^{1/2}\exp\{-(r-a)/\sqrt{k}\} + b^{1/2}\exp\{-(b-r)/\sqrt{k}\}\right] - 1\right\rangle \tag{2.12}$$

Moment acting on the cylinders r = a and r = b respectively

$$M_a = -2\pi a^2\left[\Omega_a\left(\frac{3}{2} + \frac{a^2}{\sqrt{k}}\right) - \sqrt{\frac{b}{a}}\Omega_b \exp\{(b-a)/\sqrt{k}\}\left(\frac{1}{\sqrt{k}} - \frac{3b}{a}\right)\right] \tag{2.13}$$

$$M_b = -2\pi b^2\left[\sqrt{\frac{a}{b}}\Omega_a \exp\{(b-a)/\sqrt{k}\}\left(\frac{1}{\sqrt{k}} - \frac{3b}{a}\right) - \Omega_b\left(\frac{b^2}{\sqrt{k}} - \frac{3}{2}\right)\right] \tag{2.14}$$

Drag on the cylinders r = a and r = b respectively

$$D_a = 2\pi\mu\left(\frac{a}{\sqrt{k}} + \frac{1}{2}\right)\left\langle ka^*\left[\sqrt{\frac{b}{a}}\exp\{-(b-a)/\sqrt{k}\} - 1\right] + w_b\sqrt{\frac{b}{a}}\exp\{-(b-a)/\sqrt{k}\} - w_a\right\rangle \tag{2.15}$$

$$D_b = 2\pi\mu\left(\frac{b}{\sqrt{k}} + \frac{1}{2}\right)\left\langle ka^*\left[1 - \sqrt{\frac{a}{b}}\exp\{-(b-a)/\sqrt{k}\}\right] - w_a\sqrt{\frac{b}{a}}\exp\{-(b-a)/\sqrt{k}\} + w_b\right\rangle \tag{2.16}$$

2.2 When the permeability k⟶ ∞, 1/k⟶ 0 (i.e., in the clear region)

$$v(r) = \frac{(b^2\Omega_b - a^2\Omega_a)}{b^2 - a^2}r + \frac{(\Omega_a - \Omega_b)}{b^2 - a^2}\frac{a^2b^2}{r} \tag{2.17}$$

$$w(r) = \frac{(bw_b - aw_a)}{b^2 - a^2}r - \frac{(bw_a - aw_b)}{b^2 - a^2}\frac{ab}{r}$$
$$+ \frac{G}{4\mu}\left[a^2 - r^2 + (b^2 - a^2)\frac{\log(r/a)}{\log(b/a)}\right] \tag{2.18}$$

3. SPECIAL CASES

3.1 Case (i). Flow through porous region between cylinders which are rotating with no motion parallel to the axis of the tube (i.e., $w_a = w_b = 0$)

The velocity components are given by

$$v(r) = \frac{a\Omega_a T_1(r,b) - b\Omega_b T_1(r,a)}{T_1(a,b)} \tag{3.1}$$

$$w(r) = \frac{ka^*[T_0(r,b) - T_0(r,a) - T_0(a,b)]}{T_0(a,b)} \tag{3.2}$$

with

$$T_i(x,y) = I_i\left(\frac{x}{\sqrt{k}}\right)K_i\left(\frac{y}{\sqrt{k}}\right) - I_i\left(\frac{y}{\sqrt{k}}\right)K_i\left(\frac{x}{\sqrt{k}}\right)$$

$$a^* = -\frac{G}{\mu} \qquad i = 0, 1$$

where I_i, K_i are modified Bessel functions.

3.1.1 When the permeability of the medium is very small (i.e., k⟶ 0, 1/k⟶ ∞)

The velocity components are given by

$$v(r) = \frac{1}{\sqrt{r}}\left\{a^{3/2}\Omega_a \exp\left[-(r-a)/\sqrt{k}\right] + b^{3/2}\Omega_b \exp\left[-(b-r)/\sqrt{k}\right]\right\} \tag{3.3}$$

$$w(r) = ka^*\left\langle \frac{1}{\sqrt{r}}\left\{a^{1/2}\exp\left[-(r-a)/\sqrt{k}\right] + b^{1/2}\exp\left[-(b-r)/\sqrt{k}\right]\right\} - 1\right\rangle \tag{3.4}$$

Moment acting on the cylinders r = a and r = b respectively

$$M_a = -2\pi a^2 \left[\Omega_a \left(\frac{3}{2} + \frac{a^2}{\sqrt{k}} \right) - \sqrt{\frac{b}{a}} \Omega_b \exp\left\{ (b-a)/\sqrt{k} \right\} \left(\frac{1}{\sqrt{k}} - \frac{3b}{a} \right) \right] \tag{3.5}$$

$$M_b = -2\pi b^2 \left[\sqrt{\frac{a}{b}} \Omega_a \exp\left\{ (b-a)/\sqrt{k} \right\} \left(\frac{1}{\sqrt{k}} - \frac{3b}{a} \right) - \Omega_b \left(\frac{b^2}{\sqrt{k}} - \frac{3}{2} \right) \right] \tag{3.6}$$

Drag on the cylinders r = a and r = b respectively

$$D_a = 2\pi\mu \left(\frac{a}{\sqrt{k}} + \frac{1}{2} \right) \left\langle ka^* \left[\sqrt{\frac{b}{a}} \exp\left\{ -(b-a)/\sqrt{k} \right\} - 1 \right] \right\rangle \tag{3.7}$$

$$D_b = 2\pi\mu \left(\frac{b}{\sqrt{k}} + \frac{1}{2} \right) \left\langle ka^* \left[1 - \sqrt{\frac{a}{b}} \exp\left\{ -(b-a)/\sqrt{k} \right\} \right] \right\rangle \tag{3.8}$$

3.1.2 When the permeability k→ ∞, 1/k→ 0 (i.e., in the clear region)

The velocity components are

$$v(r) = \frac{(b^2\Omega_b - a^2\Omega_a)}{b^2 - a} r + \frac{(\Omega_a - \Omega_b)}{b^2 - a^2} \frac{a^2 b^2}{r} \tag{3.9}$$

$$w(r) = \frac{G}{4\mu} \left[a^2 - r^2 + (b^2 - a^2) \frac{\log(r/a)}{\log(b/a)} \right] \tag{3.10}$$

These results coincide with the results of Narasimha Charyulu and Pattabhi Ramacharyulu [24].

3.2 Case (ii). Flow through porous region between cylinders which are moving with no rotation along the axis (i.e. $\Omega_a = \Omega_b = 0$)

The velocity components are

$$v(r) = 0 \tag{3.11}$$

$$w(r) = \frac{w_a T_0(r,b) - w_b T_0(r,a) + ka^* [T_0(r,b) - T_0(r,a) - T_0(a,b)]}{T_0(a,b)} \tag{3.12}$$

with

$$T_i(x,y) = I_i\left(\frac{x}{\sqrt{k}} \right) K_i\left(\frac{y}{\sqrt{k}} \right) - I_i\left(\frac{y}{\sqrt{k}} \right) K_i\left(\frac{x}{\sqrt{k}} \right) \quad i = 0, 1$$

$$a^* = -\frac{G}{\mu}$$

where I_i, K_i are modified Bessel functions

3.2.1 When k→ 0, 1/k→ ∞, then

$$v(r) = 0 \tag{3.13}$$

$$w(r) = \frac{1}{\sqrt{r}} \left[a^{1/2} w_a \exp\left\{ -(r-a)/\sqrt{k} \right\} - b^{1/2} w_b \exp\left\{ -(b-r)/\sqrt{k} \right\} \right]$$

$$+ ka^* \left\langle \frac{1}{\sqrt{r}} \left[a^{1/2} \exp\left\{ -(r-a)/\sqrt{k} \right\} + b^{1/2} \exp\left\{ -(b-r)/\sqrt{k} \right\} \right] - 1 \right\rangle \tag{3.14}$$

3.2.2 When k→ ∞, 1/k→ 0, then

$$v(r) = 0 \tag{3.15}$$

$$w(r) = \frac{(bw_b - aw_a)}{b^2 - a^2} r - \frac{(bw_a - aw_b)}{b^2 - a^2} \frac{ab}{r}$$

$$+ \frac{G}{4\mu} \left[a^2 - r^2 + (b^2 - a^2) \frac{\log(r/a)}{\log(b/a)} \right] \tag{3.16}$$

3.3 Case (iii). Flow through porous region between cylinders with outer cylinder fixed and inner cylinder is rotating and moving along the axis of the tube (i.e., $\Omega_b = w_b = 0$)

In this case, the velocity components are

$$v(r) = \frac{a\Omega_a T_1(r,b)}{T_1(a,b)} \tag{3.17}$$

$$w(r) = \frac{w_a T_0(r,b) + ka^* [T_0(r,b) - T_0(r,a) - T_0(a,b)]}{T_0(a,b)} \tag{3.18}$$

with

$$T_i(x,y) = I_i\left(\frac{x}{\sqrt{k}} \right) K_i\left(\frac{y}{\sqrt{k}} \right) - I_i\left(\frac{y}{\sqrt{k}} \right) K_i\left(\frac{x}{\sqrt{k}} \right) \quad i = 0, 1$$

$$a^* = -\frac{G}{\mu}$$

where I_i, K_i are modified Bessel functions

3.3.1 When k→ 0, 1/k→ ∞, then

The velocity components are

$$v(r) = \frac{1}{\sqrt{r}} \left[a^{3/2} \Omega_a \exp\left\{ -(r-a)/\sqrt{k} \right\} \right] \tag{3.19}$$

$$w(r) = \frac{1}{\sqrt{r}} \left[a^{1/2} w_a \exp\left\{ -(r-a)/\sqrt{k} \right\} \right]$$

$$+ ka^* \left\langle \frac{1}{\sqrt{r}} \left[a^{1/2} \exp\left\{ -(r-a)/\sqrt{k} \right\} + b^{1/2} \exp\left\{ -(b-r)/\sqrt{k} \right\} \right] - 1 \right\rangle \tag{3.20}$$

3.3.1 When k→ ∞, 1/k→ 0, then

$$v(r) = \frac{a^2 \Omega_a r}{b^2 - a^2} \left(\frac{b^2}{r^2} - 1 \right) \tag{3.21}$$

$$w(r) = \frac{-w_a a r}{b^2 - a^2} - \frac{w_a a b^2}{(b^2 - a^2) r}$$

$$+ \frac{G}{4\mu} \left[a^2 - r^2 + (b^2 - a^2) \frac{\log(r/a)}{\log(b/a)} \right] \tag{3.22}$$

3.4 Case (iv). Flow when inner cylinder is fixed and outer cylinder is rotating and moving along the axis of the tube (i.e., $\Omega_a = w_a = 0$)

Then,

$$v(r) = \frac{-b\Omega_b T_1(r,a)}{T_1(a,b)} \tag{3.23}$$

$$w(r) = \frac{-w_b T_0(r,a) + ka^* [T_0(r,b) - T_0(r,a) - T_0(a,b)]}{T_0(a,b)} \tag{3.24}$$

with

$$T_i(x,y) = I_i\left(\frac{x}{\sqrt{k}} \right) K_i\left(\frac{y}{\sqrt{k}} \right) - I_i\left(\frac{y}{\sqrt{k}} \right) K_i\left(\frac{x}{\sqrt{k}} \right)$$

$$a^* = -\frac{G}{\mu} \qquad i = 0, 1$$

where I_i, K_i are modified Bessel functions

3.4.1 When k→ 0, 1/k→ ∞, then

The velocity components are

$$v(r) = \frac{1}{\sqrt{r}}\left[b^{3/2}\Omega_b \exp\left\{-(b-r)/\sqrt{k}\right\}\right] \tag{3.25}$$

$$w(r) = \frac{-1}{\sqrt{r}}\left[b^{1/2}w_b \exp\left\{-(b-r)/\sqrt{k}\right\}\right] + ka^*\left\langle \frac{1}{\sqrt{r}}\left[a^{1/2}\exp\left\{-(r-a)/\sqrt{k}\right\} + b^{1/2}\exp\left\{-(b-r)/\sqrt{k}\right\}\right] - 1\right\rangle \tag{3.26}$$

3.4.2 When k→ ∞, 1/k→ 0, then

$$v(r) = \frac{b^2\Omega_b r}{b^2 - a^2}\left(1 - \frac{a^2}{r^2}\right) \tag{3.27}$$

$$w(r) = \frac{bw_b r}{b^2 - a^2} + \frac{w_b a^2 b}{(b^2 - a^2)r} + \frac{G}{4\mu}\left[a^2 - r^2 + (b^2 - a^2)\frac{\log(r/a)}{\log(b/a)}\right] \tag{3.28}$$

3.5 Case (v). Flow when inner and outer cylinders are rotating and only outer cylinder is moving along the axis of the tube (i.e. w_a = 0)

$$v(r) = \frac{a\Omega_a T_1(r,b) - b\Omega_b T_1(r,a)}{T_1(a,b)} \tag{3.29}$$

$$w(r) = \frac{-w_b T_0(r,a) + ka^*[T_0(r,b) - T_0(r,a) - T_0(a,b)]}{T_0(a,b)} \tag{3.30}$$

with

$$T_i(x,y) = I_i\left(\frac{x}{\sqrt{k}}\right)K_i\left(\frac{y}{\sqrt{k}}\right) - I_i\left(\frac{y}{\sqrt{k}}\right)K_i\left(\frac{x}{\sqrt{k}}\right)$$

$$a^* = -\frac{G}{\mu} \qquad i = 0, 1$$

where I_i, K_i are modified Bessel functions

3.5.1 When k→ 0, 1/k→ ∞, then

$$v(r) = \frac{1}{\sqrt{r}}\left[a^{3/2}\Omega_a \exp\left\{-(r-a)/\sqrt{k}\right\} + b^{3/2}\Omega_b \exp\left\{-(b-r)/\sqrt{k}\right\}\right] \tag{3.31}$$

$$w(r) = \frac{-1}{\sqrt{r}}\left[b^{1/2}w_b \exp\left\{-(b-r)/\sqrt{k}\right\}\right] + ka^*\left\langle \frac{1}{\sqrt{r}}\left[a^{1/2}\exp\left\{-(r-a)/\sqrt{k}\right\} + b^{1/2}\exp\left\{-(b-r)/\sqrt{k}\right\}\right] - 1\right\rangle \tag{3.32}$$

3.5.2 When k→ ∞, 1/k→ 0, then

$$v(r) = \frac{(b^2\Omega_b - a^2\Omega_a)}{b^2 - a^2}r + \frac{(\Omega_a - \Omega_b)}{b^2 - a^2}\frac{a^2b^2}{r} \tag{3.33}$$

$$w(r) = \frac{bw_b r}{b^2 - a^2} + \frac{w_b a^2 b}{(b^2 - a^2)r} + \frac{G}{4\mu}\left[a^2 - r^2 + (b^2 - a^2)\frac{\log(r/a)}{\log(b/a)}\right] \tag{3.34}$$

3.6 Case (vi). Flow through porous region between cylinders which are rotating and only inner cylinder is moving (i.e., $w_b = 0$))

$$v(r) = \frac{a\Omega_a T_1(r,b) - b\Omega_b T_1(r,a)}{T_1(a,b)} \tag{3.35}$$

$$w(r) = \frac{w_a T_0(r,b) + ka^*[T_0(r,b) - T_0(r,a) - T_0(a,b)]}{T_0(a,b)} \tag{3.36}$$

with

$$T_i(x,y) = I_i\left(\frac{x}{\sqrt{k}}\right)K_i\left(\frac{y}{\sqrt{k}}\right) - I_i\left(\frac{y}{\sqrt{k}}\right)K_i\left(\frac{x}{\sqrt{k}}\right)$$

$$a^* = -\frac{G}{\mu} \qquad i = 0, 1$$

where I_i, K_i are modified Bessel functions.

3.6.1 When k→ 0, 1/k→ ∞, then

$$v(r) = \frac{1}{\sqrt{r}}\left[a^{3/2}\Omega_a \exp\left\{-(r-a)/\sqrt{k}\right\} + b^{3/2}\Omega_b \exp\left\{-(b-r)/\sqrt{k}\right\}\right] \tag{3.37}$$

$$w(r) = \frac{1}{\sqrt{r}}\left[a^{1/2}w_a \exp\left\{-(r-a)/\sqrt{k}\right\}\right] + ka^*\left\langle \frac{1}{\sqrt{r}}\left[a^{1/2}\exp\left\{-(r-a)/\sqrt{k}\right\} + b^{1/2}\exp\left\{-(b-r)/\sqrt{k}\right\}\right] - 1\right\rangle \tag{3.38}$$

3.5.2 When k→ ∞, 1/k→ 0, then

$$v(r) = \frac{(b^2\Omega_b - a^2\Omega_a)}{b^2 - a^2}r + \frac{(\Omega_a - \Omega_b)}{b^2 - a^2}\frac{a^2b^2}{r} \tag{3.39}$$

$$w(r) = \frac{-w_a ar}{b^2 - a^2} - \frac{w_a ab^2}{(b^2 - a^2)r} + \frac{G}{4\mu}\left[a^2 - r^2 + (b^2 - a^2)\frac{\log(r/a)}{\log(b/a)}\right] \tag{3.40}$$

3.7 Case (vii). Flow through porous region between cylinders which are moving with same velocity and rotating along the axis of the tube (i.e. $w_a = w_b = w$)

$$v(r) = \frac{a\Omega_a T_1(r,b) - b\Omega_b T_1(r,a)}{T_1(a,b)} \tag{3.41}$$

$$w(r) = \frac{w[T_0(r,b) - T_0(r,a)] + ka^*[T_0(r,b) - T_0(r,a) - T_0(a,b)]}{T_0(a,b)} \tag{3.42}$$

with

$$T_i(x,y) = I_i\left(\frac{x}{\sqrt{k}}\right)K_i\left(\frac{y}{\sqrt{k}}\right) - I_i\left(\frac{y}{\sqrt{k}}\right)K_i\left(\frac{x}{\sqrt{k}}\right)$$

$$a^* = -\frac{G}{\mu} \qquad i = 0, 1$$

where I_i, K_i are modified Bessel functions

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3.7.1 When k→ 0, 1/k→ ∞, then

$$v(r)=\frac{1}{\sqrt{r}}\left[a^{3/2}\Omega_a\exp\left\{-(r-a)/\sqrt{k}\right\}+b^{3/2}\Omega_b\exp\left\{-(b-r)/\sqrt{k}\right\}\right] \tag{3.43}$$

$$w(r)=\frac{w}{\sqrt{r}}\left[a^{1/2}\exp\left\{-(r-a)/\sqrt{k}\right\}-b^{1/2}\exp\left\{-(b-r)/\sqrt{k}\right\}\right]+ka^{*}\left\langle\frac{1}{\sqrt{r}}\left[a^{1/2}\exp\left\{-(r-a)/\sqrt{k}\right\}+b^{1/2}\exp\left\{-(b-r)/\sqrt{k}\right\}\right]-1\right\rangle \tag{3.44}$$

3.7.2 When k→ ∞, 1/k→ 0, then

$$v(r)=\frac{(b^2\Omega_b-a^2\Omega_a)}{b^2-a^2}r+\frac{(\Omega_a-\Omega_b)}{b^2-a^2}\frac{a^2b^2}{r} \tag{3.45}$$

$$w(r)=\frac{(r-ab)w}{b+a}+\frac{G}{4\mu}\left[a^2-r^2+(b^2-a^2)\frac{\log(r/a)}{\log(b/a)}\right] \tag{3.46}$$

3.8 Case (viii). Flow when inner and outer cylinders are moving and only outer cylinder is rotating along the axis of the tube (i.e. $\Omega_a=0$)

$$v(r)=\frac{-b\Omega_bT_1(r,a)}{T_1(a,b)} \tag{3.47}$$

$$w(r)=\frac{w_aT_0(r,b)-w_bT_0(r,a)+ka^{*}[T_0(r,b)-T_0(r,a)-T_0(a,b)]}{T_0(a,b)} \tag{3.48}$$

with

$$T_i(x,y)=I_i\left(\frac{x}{\sqrt{k}}\right)K_i\left(\frac{y}{\sqrt{k}}\right)-I_i\left(\frac{y}{\sqrt{k}}\right)K_i\left(\frac{x}{\sqrt{k}}\right)\qquad i=0,1$$

$$a^{*}=-\frac{G}{\mu}.$$

where I_i, K_i are modified Bessel functions

3.8.1 When k→ 0, 1/k→ ∞, then

$$v(r)=\frac{1}{\sqrt{r}}\left[b^{3/2}\Omega_b\exp\left\{-(b-r)/\sqrt{k}\right\}\right] \tag{3.49}$$

$$w(r)=\frac{1}{\sqrt{r}}\left[a^{1/2}w_a\exp\left\{-(r-a)/\sqrt{k}\right\}-b^{1/2}w_b\exp\left\{-(b-r)/\sqrt{k}\right\}\right]+ka^{*}\left\langle\frac{1}{\sqrt{r}}\left[a^{1/2}\exp\left\{-(r-a)/\sqrt{k}\right\}+b^{1/2}\exp\left\{-(b-r)/\sqrt{k}\right\}\right]-1\right\rangle \tag{3.50}$$

3.8.2 When k→ ∞, 1/k→ 0, then

$$v(r)=\frac{b^2\Omega_br}{b^2-a^2}\left(1-\frac{a^2}{r^2}\right) \tag{3.51}$$

$$w(r)=\frac{(bw_b-aw_a)}{b^2-a^2}r-\frac{(bw_a-aw_b)}{b^2-a^2}\frac{ab}{r}+\frac{G}{4\mu}\left[a^2-r^2+(b^2-a^2)\frac{\log(r/a)}{\log(b/a)}\right] \tag{3.52}$$

3.9 Case (ix). Flow through porous region between cylinders which are moving and only inner cylinder is rotating (i.e. $\Omega_b=0$)

$$v(r)=\frac{a\Omega_aT_1(r,b)}{T_1(a,b)} \tag{3.53}$$

$$w(r)=\frac{w_aT_0(r,b)-w_bT_0(r,a)+ka^{*}[T_0(r,b)-T_0(r,a)-T_0(a,b)]}{T_0(a,b)} \tag{3.54}$$

with

$$T_i(x,y)=I_i\left(\frac{x}{\sqrt{k}}\right)K_i\left(\frac{y}{\sqrt{k}}\right)-I_i\left(\frac{y}{\sqrt{k}}\right)K_i\left(\frac{x}{\sqrt{k}}\right)\qquad i=0,1$$

$$a^{*}=-\frac{G}{\mu}.$$

where I_i, K_i are modified Bessel functions.

3.9.1 When k→ 0, 1/k→ ∞, then

$$v(r)=\frac{1}{\sqrt{r}}\left[a^{3/2}\Omega_a\exp\left\{-(r-a)/\sqrt{k}\right\}\right] \tag{3.55}$$

$$w(r)=\frac{1}{\sqrt{r}}\left[a^{1/2}w_a\exp\left\{-(r-a)/\sqrt{k}\right\}-b^{1/2}w_b\exp\left\{-(b-r)/\sqrt{k}\right\}\right]+ka^{*}\left\langle\frac{1}{\sqrt{r}}\left[a^{1/2}\exp\left\{-(r-a)/\sqrt{k}\right\}+b^{1/2}\exp\left\{-(b-r)/\sqrt{k}\right\}\right]-1\right\rangle \tag{3.56}$$

3.9.2 When k→ ∞, 1/k→ 0, then

$$v(r)=\frac{a^2\Omega_ar}{b^2-a^2}\left(\frac{b^2}{r^2}-1\right) \tag{3.57}$$

$$w(r)=\frac{(bw_b-aw_a)}{b^2-a^2}r-\frac{(bw_a-aw_b)}{b^2-a^2}\frac{ab}{r}+\frac{G}{4\mu}\left[a^2-r^2+(b^2-a^2)\frac{\log(r/a)}{\log(b/a)}\right] \tag{3.58}$$

3.10 Case (x). Flow when the cylinders are rotating with same angular velocities and moving parallel to the axis of the tube (i.e. $\Omega_a=\Omega_b=\Omega$)

$$v(r)=\frac{\Omega[aT_1(r,b)-bT_1(r,a)]}{T_1(a,b)} \tag{3.59}$$

$$w(r)=\frac{w_aT_0(r,b)-w_bT_0(r,a)+ka^{*}[T_0(r,b)-T_0(r,a)-T_0(a,b)]}{T_0(a,b)} \tag{3.60}$$

with

$$T_i(x,y)=I_i\left(\frac{x}{\sqrt{k}}\right)K_i\left(\frac{y}{\sqrt{k}}\right)-I_i\left(\frac{y}{\sqrt{k}}\right)K_i\left(\frac{x}{\sqrt{k}}\right)\qquad i=0,1$$

$$a^{*}=-\frac{G}{\mu}.$$

where I_i, K_i are modified Bessel functions.

3.10.1 When k→ 0, 1/k→ ∞, then

$$v(r)=\frac{\Omega}{\sqrt{r}}\left[a^{3/2}\exp\left\{-(r-a)/\sqrt{k}\right\}+b^{3/2}\exp\left\{-(b-r)/\sqrt{k}\right\}\right] \tag{3.61}$$

$$w(r)=\frac{1}{\sqrt{r}}\left[a^{1/2}w_a\exp\left\{-(r-a)/\sqrt{k}\right\}-b^{1/2}w_b\exp\left\{-(b-r)/\sqrt{k}\right\}\right]+ka^{*}\left\langle\frac{1}{\sqrt{r}}\left[a^{1/2}\exp\left\{-(r-a)/\sqrt{k}\right\}+b^{1/2}\exp\left\{-(b-r)/\sqrt{k}\right\}\right]-1\right\rangle \tag{3.62}$$

3.10.2 When k→ ∞, 1/k→ 0, then

$$v(r) = \Omega r \quad (3.63)$$

$$w(r) = \frac{(bw_b - aw_a)}{b^2 - a^2} r - \frac{(bw_a - aw_b)}{b^2 - a^2}\frac{ab}{r} + \frac{G}{4\mu}\left[a^2 - r^2 + (b^2 - a^2)\frac{\log(r/a)}{\log(b/a)}\right] \quad (3.64)$$

3.11 Case (xi). Flow when the pressure gradient is absent (i.e. G = 0)

We take the pressure gradient to be absent, then G = 0. In this case, the velocity components are given by

$$v(r) = \frac{a\Omega_a T_1(r,b) - b\Omega_b T_1(r,a)}{T_1(a,b)} \quad (3.65)$$

$$w(r) = \frac{w_a T_0(r,b) - w_b T_0(r,a)]}{T_0(a,b)} \quad (3.66)$$

with

$$T_i(x,y) = I_i\left(\frac{x}{\sqrt{k}}\right)K_i\left(\frac{y}{\sqrt{k}}\right) - I_i\left(\frac{y}{\sqrt{k}}\right)K_i\left(\frac{x}{\sqrt{k}}\right) \qquad i = 0, 1$$

$$a^* = -\frac{G}{\mu}.$$

where I_i, K_i are modified Bessel functions.

3.11.1 When k→ 0, 1/k→ ∞, then

$$v(r) = \frac{1}{\sqrt{r}}\left[a^{3/2}\Omega_a \exp\left\{-(r-a)/\sqrt{k}\right\} + b^{3/2}\Omega_b \exp\left\{-(b-r)/\sqrt{k}\right\}\right] \quad (3.67)$$

$$w(r) = \frac{1}{\sqrt{r}}\left[a^{1/2} w_a \exp\left\{-(r-a)/\sqrt{k}\right\} - b^{1/2} w_b \exp\left\{-(b-r)/\sqrt{k}\right\}\right] \quad (3.68)$$

3.11.2 When k→ ∞, 1/k→ 0, then

$$v(r) = \frac{(b^2\Omega_b - a^2\Omega_a)}{b^2 - a^2} r + \frac{(\Omega_a - \Omega_b)}{b^2 - a^2}\frac{a^2 b^2}{r} \quad (3.69)$$

$$w(r) = \frac{(bw_b - aw_a)}{b^2 - a^2} r - \frac{(bw_a - aw_b)}{b^2 - a^2}\frac{ab}{r} \quad (3.70)$$

3.12 Case (xii). Flow between two infinite parallel plates

Flow through the porous region bounded by two infinite parallel plates y = ± h can be derived by taking a ~ b = 2h, as the distance between the two plates with respect to the Cartesian co-ordinate system (x, y, z). The velocity components are given by

$$v(r) = 0 \quad (3.70)$$

$$w(r) = \frac{ka^*[T_0(r,b) - T_0(r,a) - T_0(a,b)]}{T_1(a,b)} = ka^*\left[\frac{\sqrt{b}\sinh(r-a) + \sqrt{a}\sinh(r-b)}{\sqrt{r}\sinh(a-b)} - 1\right] \quad (3.71)$$

If r = b + h + y, then we get

$$w(y) = ka^*\left[\frac{\sqrt{k}\cosh(y)}{\cosh(h/\sqrt{k})} - 1\right] \quad (3.72)$$

3.12.1 When the permeability of the medium is very small (i.e., k→ 0, 1/k→ ∞)

Then,

$$w(y) = ka^*\left[\exp\left(-(h-y)/\sqrt{k}\right) - 1\right] \quad (3.73)$$

3.12.2 When the permeability k→ ∞, 1/k→ 0 (i.e., in the clear region)

$$w(y) = \frac{a^* h^2}{2}\left(\frac{y^2}{h^2} - 1\right) \quad (3.74)$$

4. Results and Discussions

4.1 Radial velocity

When there is a rotation as well as linear motion and in the case when there is a rotation and there is no linear motion, the velocity profiles for different permeabilities show parabolic nature. The increase in the permeability makes the velocity to increase [Figure 1(a) and Figure 2(a)]. When there is no rotation but only linear motion results the disappearance of parabolic profiles [Figure 3].

From Figure 4(a) and Figure 5(a), it is observed that the velocity profiles are reversed. The velocity at the rotating and moving cylinder is higher than the velocity of the fluid at the cylinder which is having no rotation and no linear motion. It is also observed that increasing the permeability increases the velocity at different points. From Figure 6(a) and Figure 7(a), the radial velocities profiles are not affected whether the outer cylinder or inner cylinder is moving are at rest. From Figure 9(a) and Figure 10 (a), when the cylinders are in linear motion and one of the cylinders in rotation, the velocity profiles are reversed such that the velocity will be higher at the rotation cylinder and lower at the non-rotating cylinder.

4.2 Linear velocity

Figure 2(b) shows even in the absence of linear motion of the cylinders, the rotation of the cylinder results the fluid flow along the axis.

In the absence of rotation of the cylinder the linear velocity profiles shows different flow profiles as shown in the figure.3. From Figure 4(b) and Figure 5(b), the velocity profiles are in reverse direction when one of the cylinders is rotating and moving while other is fixed.

In general, it is observed that from equations (2.11) and (2.12) when $\delta_1 = r - a$, $\delta_2 = b - r$, becomes very large, then

$$v(r) = 0$$

and $w(r) = -ka^*$ = a constant.

These equations show that as the distance from the axis to the wall increases, the velocity w(r) = a constant. The classical Darcy effect is felt in a core very near to the axis of the cylinder. v(r) = 0 shows that there exist a thin layer between the cylinders far away from the boundaries of the cylinder, where the velocity v(r) is zero.

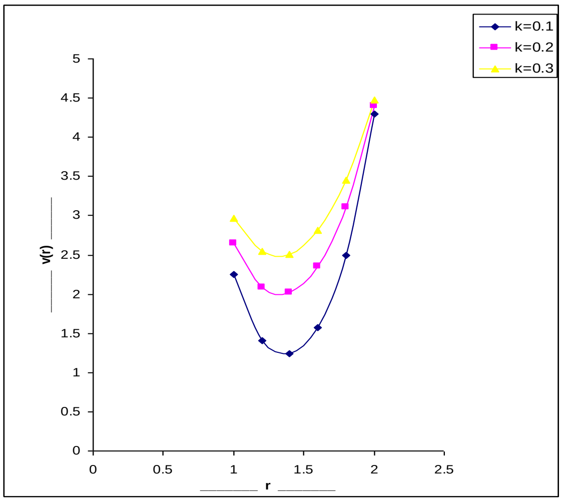

Figure 1(a). Velocity profiles v(r) of the fluid for different values of k

$(a = 1, b = 2, \Omega_a = 2, \Omega_b = 3, w_a = 2, w_b = 3)$

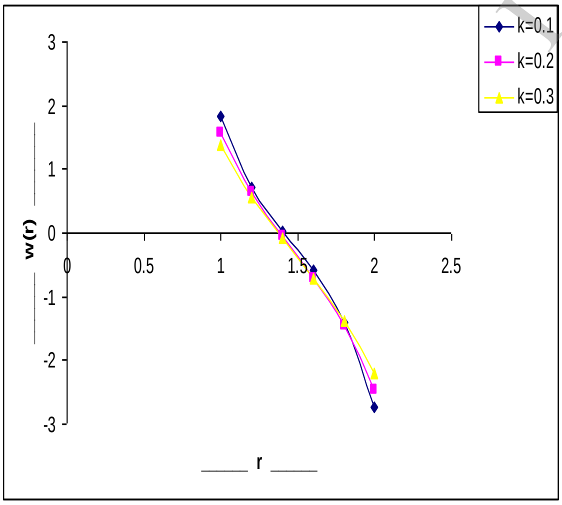

Figure 1(b). Velocity profiles w(r) of the fluid for different values of k

$(a = 1, b = 2, \Omega_a = 2, \Omega_b = 3, w_a = 2, w_b = 3)$

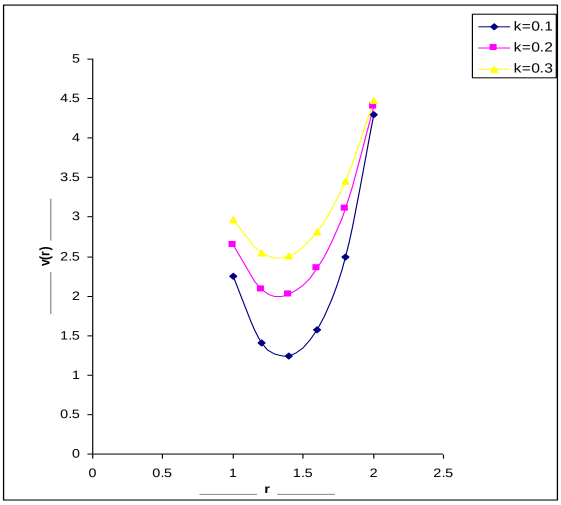

Figure 2(a). Velocity profiles v(r) of the fluid for different values of k

$(a = 1, b = 2, \Omega_a = 2, \Omega_b = 3, w_a = w_b = 0)$

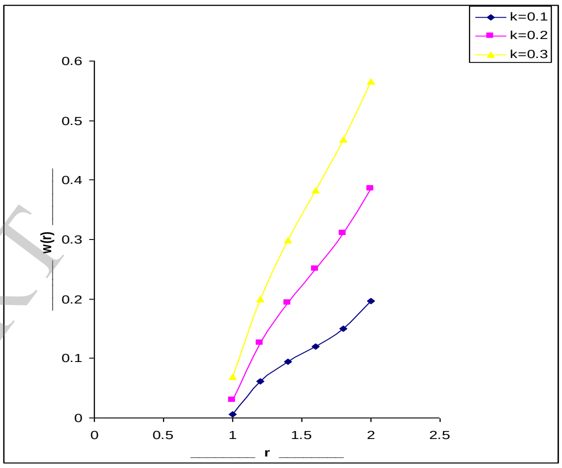

Figure 2(b): Velocity profiles w(r) of the fluid for different values of k

$(a = 1, b = 2, \Omega_a = 2, \Omega_b = 3, w_a = w_b = 0)$

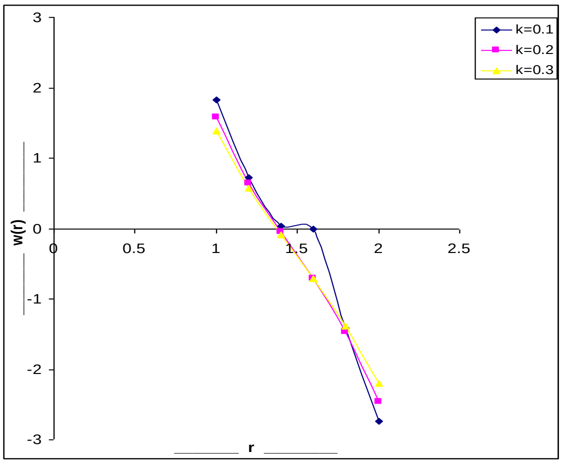

Figure 3. Velocity profiles of the fluid for different values of k

$(a = 1, b = 2, \Omega_a = 0, \Omega_b = 0, w_a = 2, w_b = 3)$

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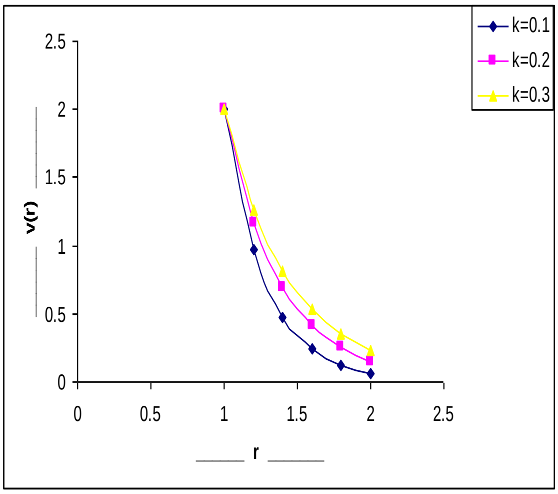

Figure 4(a). Velocity profiles of the fluid for different values of k

$(a=1, b=2, \Omega_a=2, \Omega_b=0, w_a=2, w_b=0)$

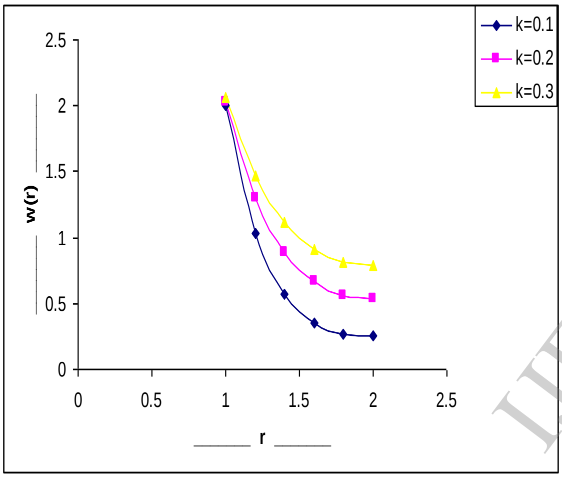

Figure 4(b). Velocity profiles of the fluid for different values of k

$(a=1, b=2, \Omega_a=2, \Omega_b=0, w_a=2, w_b=0)$

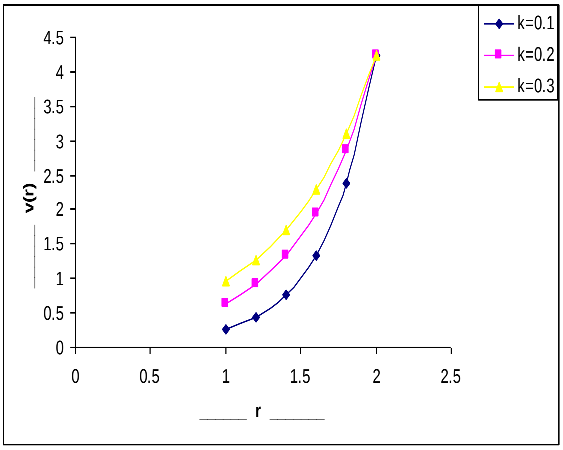

Figure 5(a). Velocity profiles of the fluid for different values of k

$(a=1, b=2, \Omega_a=0, \Omega_b=3, w_a=0, w_b=2)$

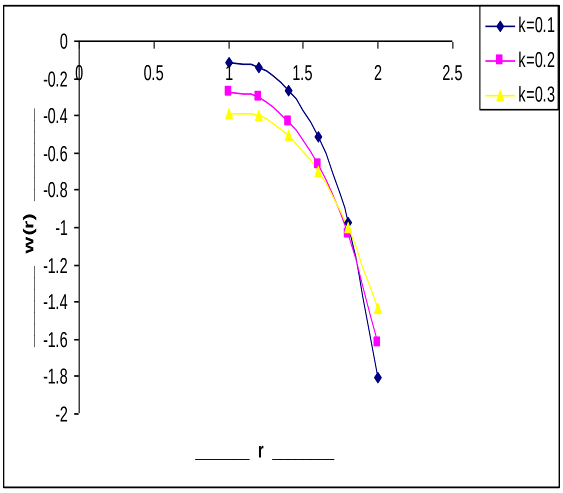

Figure 5(b). Velocity profiles of the fluid for different values of k

$(a=1, b=2, \Omega_a=0, \Omega_b=3, w_a=0, w_b=2)$

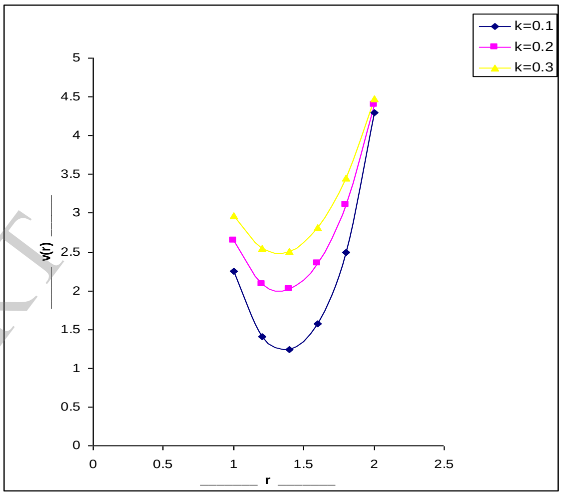

Figure 6(a). Velocity profiles v(r) of the fluid for different values of k

$(a=1, b=2, \Omega_a=2, \Omega_b=3, w_a=0, w_b=3)$

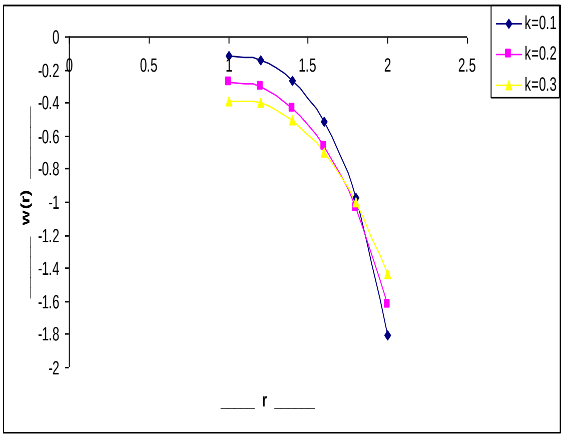

Figure 6(b). Velocity profiles of the fluid for different values of k

$(a=1, b=2, \Omega_a=2, \Omega_b=3, w_a=0, w_b=2)$

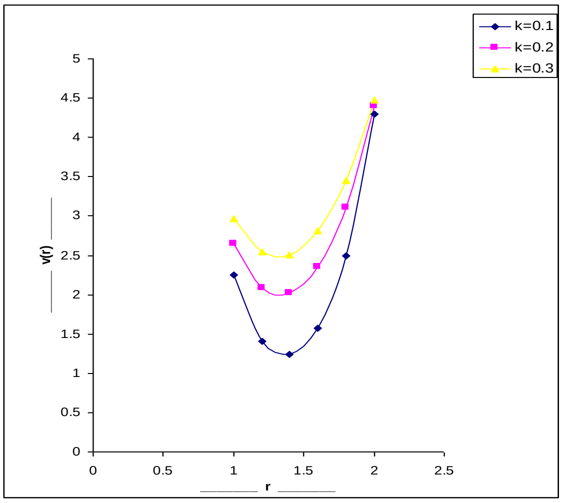

Figure 7(a). Velocity profiles v(r) of the fluid for different values of k

$(a=1, b=2, \Omega_a=2, \Omega_b=3, w_a=2, w_b=0)$

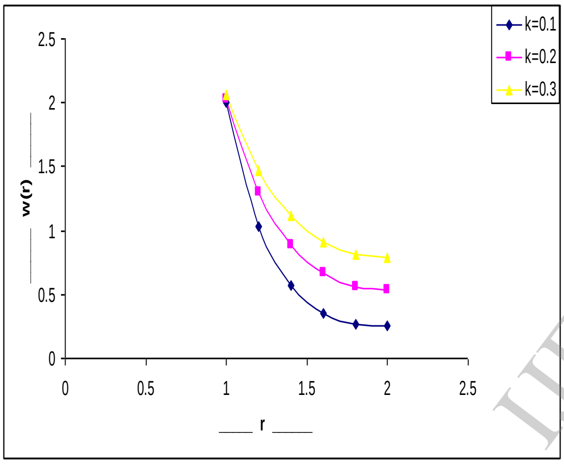

Figure 7(b). Velocity profiles of the fluid for different values of k

$(a=1, b=2, \Omega_a=2, \Omega_b=3, w_a=2, w_b=0)$

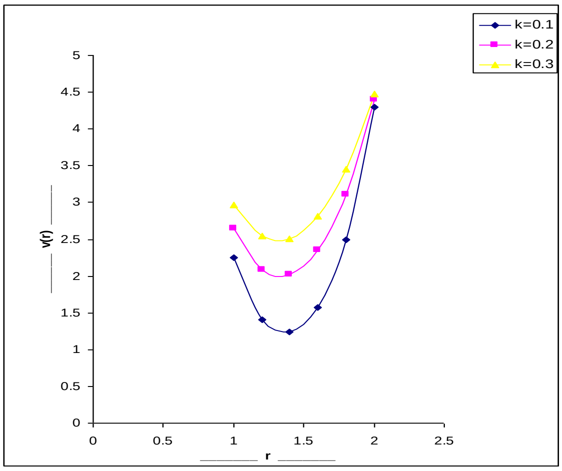

Figure 8(a). Velocity profiles v(r) of the fluid for different values of k

$(a=1, b=2, \Omega_a=2, \Omega_b=3, w_a=w_b=2)$

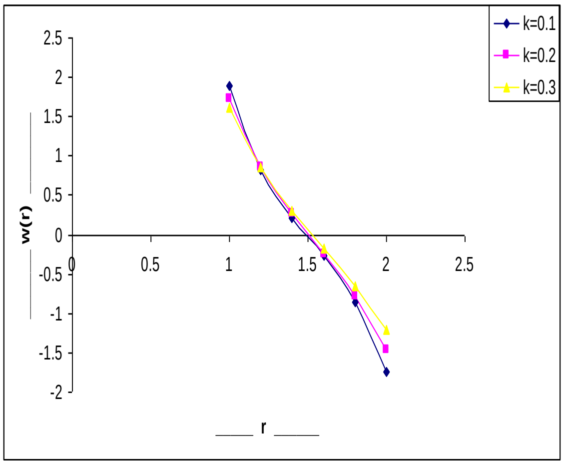

Figure 8(b). Velocity profiles of the fluid for different values of k

$(a=1, b=2, \Omega_a=2, \Omega_b=3, w_a=w_b=2)$

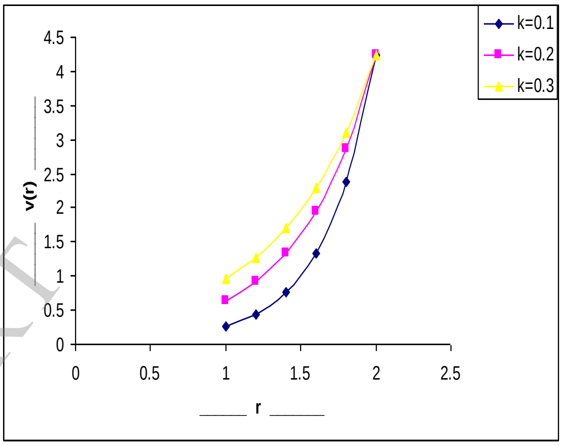

Figure 9(a). Velocity profiles of the fluid for different values of k

$(a=1, b=2, \Omega_a=0, \Omega_b=3, w_a=2, w_b=3)$

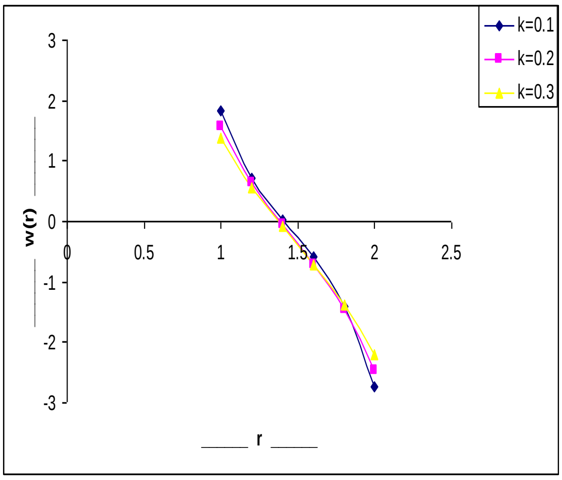

Figure 9(b). Velocity profiles w(r) of the fluid for different values of k

$(a=1, b=2, \Omega_a=0, \Omega_b=3, w_a=2, w_b=3)$

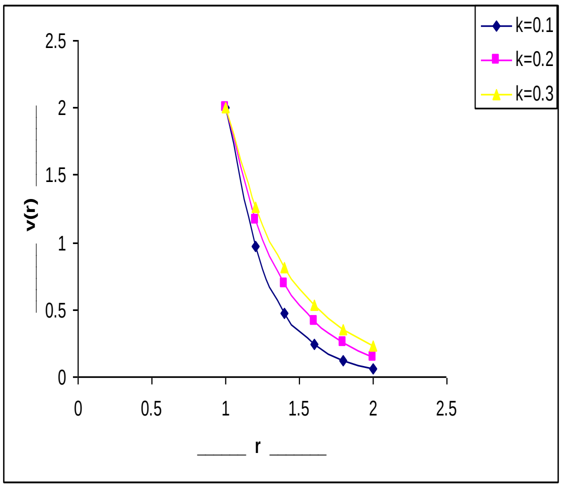

Figure 10(a). Velocity profiles of the fluid for different values of k

$(a = 1, b = 2, \Omega_a = 2, \Omega_b = 0, w_a = 2, w_b = 3)$

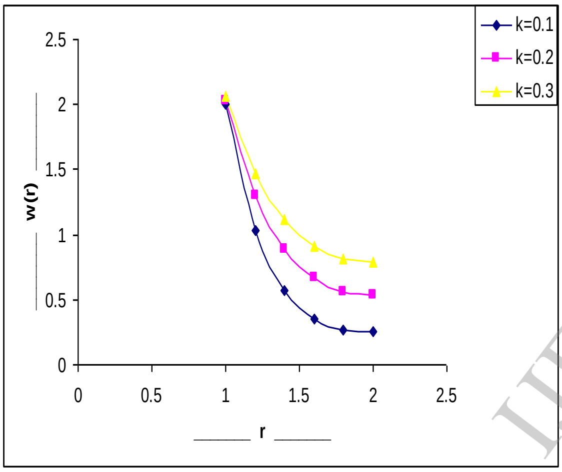

Figure 10(b). Velocity profiles of the fluid for different values of k

$(a = 1, b = 2, \Omega_a = 2, \Omega_b = 0, w_a = 2, w_b = 3)$

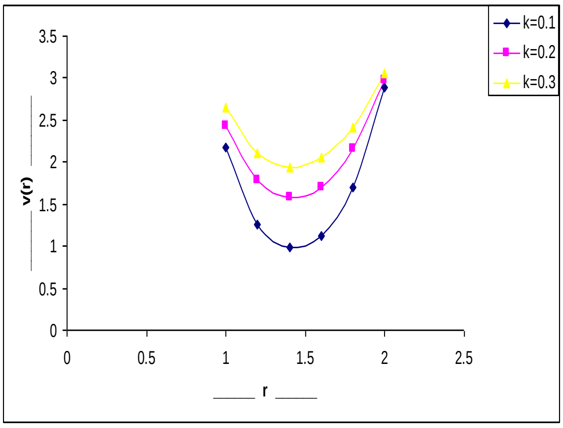

Figure 11(a). Velocity profiles of the fluid for different values of k

$(a = 1, b = 2, \Omega_a = \Omega_b = 2, w_a = 3, w_b = 4)$.

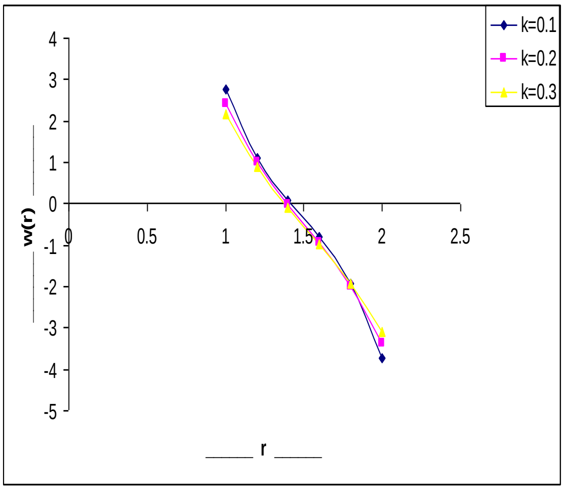

Figure 11(b). Velocity profiles of the fluid for different values of k

$(a = 1, b = 2, \Omega_a = \Omega_b = 2, w_a = 3, w_b = 4)$

5. Conclusions

In radial velocity the increase in permeability makes the velocity to increase. Even in the absence of linear motion of cylinders, the rotation of the cylinder results the fluid along the axis and as the distance from the axis to the wall increases, the velocity becomes constant. The non-Darcian effect is observed near the boundaries and the Darcian effect is observed in the porous medium away from the boundaries.

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