

# Forward and Inverse Kinematic Singularity analysis of Eight bar 3 – RRR Planar Parallel Manipulator

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**Abstract** - An eight bar 3-RRR planar parallel manipulator with forward and inverse kinematic singularities are analyzed. The determination of singularities in the workspace of 3- RRR planar parallel manipulator with eight bar mechanism is an important role in machining process of several parallel manipulators for generating symmetric coupler curves. The mathematical equations are derived for the proposed approach.

**Keywords** - Singularities; 3-dof; Forward Kinematic Analysis; Inverse Kinematic Analysis; Eight Bar Mechanism

## I. INTRODUCTION

A planar kinematic eight bar manipulator with binary and ternary limbs are selected. The selected architecture has three degrees of freedom. The design of eight bar planar parallel manipulator and its coupler curves are more complex due to their kinematic structure. This paper presents forward and inverse kinematic analysis of eight bar mechanism to determine the singularities of a work space, because the singularities are more attractive in several researches. The mathematical equations are derived by using loop closer technique.

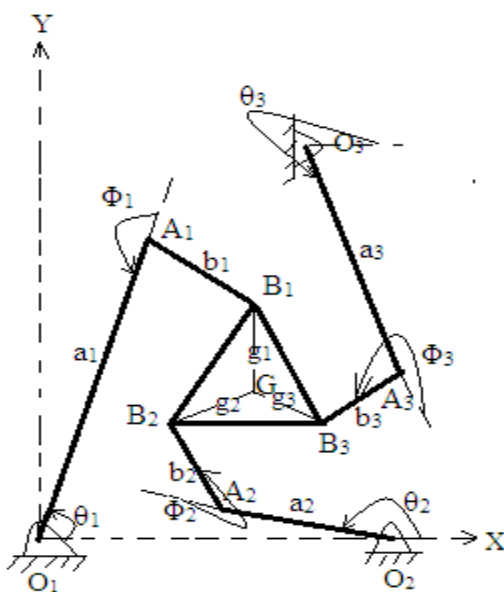


Fig (1) Eight bar 3-RRR planar parallel manipulator

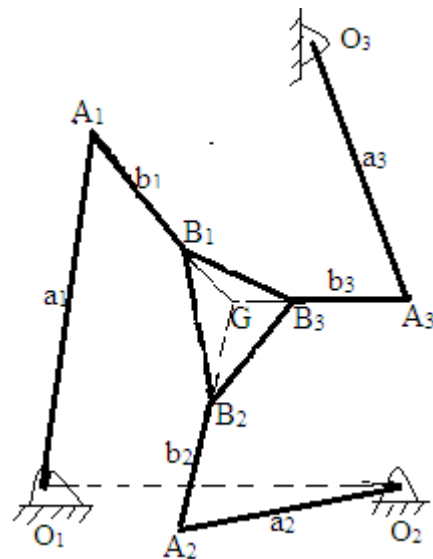


Fig (2) Forward Kinematic Singularity of Eight bar 3-RRR planar parallel manipulator (All unconstrained limbs intersect at common point G)

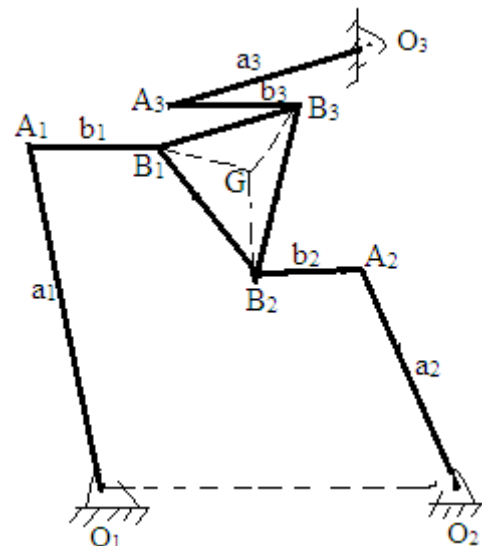


Fig (3) Forward Kinematic Singularity of Eight bar 3-RRR planar parallel manipulator (All unconstrained limbs parallel to each other)

## II. KINEMATIC ARCHITECTURE OF EIGHT BAR PLANAR PARALLEL MANIPULATOR

Kinematic architecture of eight bar planar parallel manipulator consists of a movable triangular platform  $B_1B_2B_3$  which is connected with unconstrained three binary limbs  $B_iA_i|_{i=1,2,3}$  and a fixed base  $O_1O_2O_3$  which is connected with three constrained limbs  $O_iA_i|_{i=1,2,3}$ , the constrained limbs are actuated by rotary actuators which are actuated by rotary variable differential transducers. The angular positions of input limbs  $O_iA_i|_{i=1,2,3}$  are measured with respect to positive X- axis which represents the angular positions of three successive constrained limbs, which are denoted by  $\theta_i|_{i=1,2,3}$  for initial orientation of three constrained limbs and for different orientations it is denoted as  $\theta_{ij}|_{i=1,2,3 \text{ and } j=1,2,3,4 \text{ etc'}}$  similarly the initial inclinations of three unconstrained limbs  $A_iB_i|_{i=1,2,3}$  with respect to  $O_iA_i|_{i=1,2,3}$  are denoted as  $\Phi_i|_{i=1,2,3}$  and for different orientations of three unconstrained limbs are denoted as  $\Phi_{ij}|_{i=1,2,3 \text{ and } j=1,2,3,4 \text{ etc'}}$

## III. JACOBIAN OF EIGHT BAR 3- RRR PLANAR PARALLEL MANIPULATOR

The degree of freedom of the selected moving platform is three. The translational and rotational coordinates of the

$$\begin{bmatrix} b_{x1} & b_{y1} & g_{x1}b_{y1} - g_{y1}b_{x1} \\ b_{x2} & b_{y2} & g_{x2}b_{y2} - g_{y2}b_{x2} \\ b_{x3} & b_{y3} & g_{x3}b_{y3} - g_{y3}b_{x3} \end{bmatrix} \begin{bmatrix} \dot{x}_G & \dot{y}_G & \dot{\psi}_G \end{bmatrix}^T = \begin{bmatrix} a_{x1}b_{y1} - a_{y1}b_{x1} & 0 & 0 \\ 0 & a_{x2}b_{y2} - a_{y2}b_{x2} & 0 \\ 0 & 0 & a_{x3}b_{y3} - a_{y3}b_{x3} \end{bmatrix} \begin{bmatrix} \dot{\theta}_1 & \dot{\theta}_2 & \dot{\theta}_3 \end{bmatrix}^T$$

## IV. FORWARD KINEMATIC SINGULARITY ANALYSIS OF EIGHT BAR 3 - RRR PLANAR PARALLEL MANIPULATOR

The forward kinematic singularities occur if  $\begin{bmatrix} b_{x1} & b_{y1} & g_{x1}b_{y1} - g_{y1}b_{x1} \\ b_{x2} & b_{y2} & g_{x2}b_{y2} - g_{y2}b_{x2} \\ b_{x3} & b_{y3} & g_{x3}b_{y3} - g_{y3}b_{x3} \end{bmatrix} = 0$ .

By the inspection of the above determinant, the forward kinematic singularities are possible, if  $[g_{xi}b_{yi} - g_{yi}b_{xi}]|_{i=1,2,3} = 0$ . By the inspection of "Fig (2)," the limbs  $A_iB_i|_{i=1,2,3}$  are in the line with  $GB_i|_{i=1,2,3}$  that means, all unconstrained limbs intersect at common point G. At this position the moving platform  $GB_i|_{i=1,2,3}$  require

moving platform centroid is denoted as  $x_G, y_G$  and  $\psi_G$ . The input rotational vector of three constrained limbs are  $[\theta_1 \ \theta_2 \ \theta_3]^T$  and the output translational and rotational vector of moving platform centroid as  $[x_G \ y_G \ \psi_G]^T$ . The loop closer equation for each limb can be expressed as

$$[\overrightarrow{O_iG} + \overrightarrow{GB_i}]|_{i=1,2,3} = [\overrightarrow{O_iA_i} + \overrightarrow{A_iB_i}]|_{i=1,2,3} \quad (1)$$

The velocity vector loop equation can be obtained by making derivative of "(1)" with respect to time, then the velocity vector loop equations as

$$V_G + \dot{\psi}_G(k * g_i)|_{i=1,2,3} = [\dot{\theta}_i(k * a_i) + (\dot{\theta}_i + \dot{\Phi}_i)(k * b_i)]|_{i=1,2,3} \quad (2)$$

Where  $V_G$  is the velocity of the centroid of the moving platform: 'k' is the unit vector pointing in positive Z- direction and  $\dot{\Phi}_i|_{i=1,2,3}$  is the passive variables. In "(2)" the passive variables can be eliminated by making dot product of above equation by  $b_i|_{i=1,2,3}$  then "(2)" can be written as

$$[b_i V_G + \dot{\psi}_i k \cdot (g_i * b_i)]|_{i=1,2,3} = \dot{\theta}_i \cdot k(a_i * b_i)|_{i=1,2,3} \quad (3)$$

That means

infinitesimal rotation about point of contact G. In this case the degree of freedom of entire structure is converted into four, that means the moving platform gains one dof and it cannot withstand any external moment about point G. Therefore the rotary actuators are locked. Similarly in "Fig (3)," the three vectors  $A_iB_i|_{i=1,2,3}$  are parallel to each other then another type of forward kinematic singularities will occur. At this position moving platform require infinitesimal translational motion along the translational direction. Therefore again the actuators are locked.

## V. INVERSE KINEMATIC SINGULARITY ANALYSIS OF EIGHT BAR 3 - RRR PLANAR PARALLEL MANIPULATOR

Inverse kinematic singularities will occur if

$$\begin{vmatrix} a_{x1}b_{y1} - a_{y1}b_{x1} & 0 & 0 \\ 0 & a_{x2}b_{y2} - a_{y2}b_{x2} & 0 \\ 0 & 0 & a_{x3}b_{y3} - a_{y3}b_{x3} \end{vmatrix} = 0$$

By the inspection of the above determinant the inverse kinematic singularities are possible in one of the diagonal elements are zero, that means  $[a_{xi}b_{yi} - a_{yi}b_{xi}]_{i=1 \text{ or } 2 \text{ or } 3} = 0$ . This type of problem will be raised if one of the limbs  $A_iB_i|_{i=1,2,3}$  is fully stretched out or folded completely back. In this case the manipulator losses 1 or 2 or 3 dof, this depends on whether  $A_1B_1$  or  $A_2B_2$  or  $A_3B_3$  or combination of these limbs are fully stretched out or folded completely back. Then the result is zero, there is no output motion of moving platform for an infinitesimal rotation of input limbs.

### CONCLUSION

This paper presents singularities in the workspace of 3 – RRR planar parallel manipulator with eight bar mechanism. The main contribution of this work is the possible singularity expressions one analyzed by forward and inverse kinematic motion equations which are derived by using closed loop technique. The forward kinematic singularities are possible if the unconstrained limbs are parallel to each other or all unconstrained limbs must meet at same point. Similarly the inverse kinematic singularities are possible one of the set of constrained and unconstrained limbs position is on the same line of action. The future work can be extended by generating different cognate linkages to generate symmetric coupler curves for eight bar 3- RRR planar parallel manipulators.

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