International Journal of Engineering Research & Technology (IJERT)
ISSN: 2278-0181
Vol. 3 Issue 2, February - 2014

Fractal and Multifractal Analysis of Synthetic and Real RR Time Series of Healthy Subjects

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***Abstract*— In this paper are presented the current results of scientific research of the synthetic and real RR time series for healthy subjects. The synthetic RR intervals are generated by algorithm [8] and the real RR intervals are obtained from 24-hour digital Holter ECG records of normal subjects. The two kinds of signals are evaluated by using Rescaled range analysis (R/S) and Detrended Fluctuation Analysis (DFA) methods for fractal analysis and MF-DFA method for multifractal analysis. The obtained results show that the investigated signals have fractal and multifractal properties. The results are implemented upon the Matlab platform.**

I. Introduction

The recent scientific research of wide range physiological signals as electrocardiograms (ECG), electroencephalograms (ECC) and more show that they have a fractal structure [1, 2, 3]. These signals normally are non-linear and non-stationary, as the significant part of the information is coded in the dynamics of their fluctuation. Through implementation of the conventional analytical methods, based on the statistical parameters as mean values, standard deviations and harmonically analysis of the energetic spectrum of the signals, part of the important signal characteristics are missed [4, 5]. From the recent scientific research worldwide follow that scale invariance of the fluctuations can be easy identified for healthy and pathological cases via implementation of methods for fractal and multifractal analysis [6, 7]. Through these methods is determine that fluctuation of the physiological signals possess hidden information in the form of self-similarity, scale structure, and multifractality. The fractal and multifractal analysis of the fluctuations is useful not only for getting the comprehensive information for physiological signals of the patients, but give a possibility for foresight, prognosis and prevention of the pathological statuses. The prevention in the medicine is important not only for the human, but for the community as whole.

Electrocardiography is non-invasive method for analysis of the functional status of the cardiovascular activity. By the electrocardiogram is measured the electrical activity of the heart. The variation of time intervals between heart beats (Heart Rate Variability -HRV) is the method for evaluation of cardiological intervals. HRV is used to diagnose and estimate of alterations in heart rate by measuring the variation of RR intervals and it can be used to provide an assessment of cardiovascular diseases. The heart rate is not stationary signal and it is under the influence of sympathetic and parasympathetic activity of the autonomic nervous system, as part of the central nervous system.

In this study, the main objectives are:

- To generate the RR time series. For the generation is used algorithm described in publication [8].
- Comparative analysis between synthetic and real RR time series, obtained from 24-hour Holter ECG recordings of healthy subjects by applying fractal and multifractal methods.

II. Subjects

The article analyzes these two kinds of signals:
- Synthetic signals, consisting of about 100 000 data points, corresponding to a 24-hour recordings of ECG RR intervals . Signals are generated using the algorithm described in publication [8]. This algorithm is based on the Gaussian distributions and represents a modification of McSarry algorithm [9] in which the Inverse Fourier Transform is replaced by the Inverse Wavelet Transform.
- Real 24-hour digital Holter ECG records of RR intervals of 6 healthy adults (3 men and 3 women aged 35 to 62 years). The data are taken from the cardiology department of Multiprofile District Hospital for Active Treatment "Dr. Stefan Cherkezov" AD - Veliko Tarnovo, Bulgaria.

Figures 1 and 2 show 5- minute series of simulated and real RR intervals. Average RR interval is approximately 1 second, and the duration of the cardiac interval ranges between 0.8 and 1.2 sec.

III. Fractal analysis

A. *R/S statistical method*

The rescaled range (R/S) method is suggested by British hydrologist Hurst [10]. The R/S for the time series X(n) is asymptotically given by a power law:

$$\frac{R(n)}{S(n)} \propto n^H \qquad (1)$$

Where:

- R(n) is the range which is the difference between the minimum and maximum accumulated values;

- S(n) is the standard deviation estimated from the observed data X(n);

- H is the Hurst exponent.

To estimate the Hurst exponent is plotted R(n)/S(n) versus n in log-log axes. The slope of the regression line approximates the Hurst exponent. The values of the Hurst exponent range between 0 and 1. Based on the Hurst exponent value, the following classifications of time series can be realized:

- $H=0.5$ indicates a random series;
- $0<H<0.5$ – the data in the signal are anti-correlated;
- $0.5<H<1$ – the data in the signal are long-range correlated.

B. *DFA method*

The method DFA (Detrended Fluctuation Analysis) is suitable for the study of non-stationary signals [11, 12]. This method allows to determine the correlation properties of the signal. Fluctuations (F) of the signal can be expressed as a function of time intervals (n) by the formula:

$$F(n) \propto n^{\alpha}. \qquad (1)$$

Where α is a scalable parameter, which depends on the correlation properties of the signal. By changing the parameter n can be studied how to change the fluctuations of the signal. Linear behavior of the dependence F (n) is an indicator of the presence of a scalable behavior of the signal. From the slope of the straight line is determined the value of the parameter α. For uncorrelated signals, the value of this parameter is within the range (0, 0.5), where α > 0.5-it is an indication for the presence of correlation. When $\alpha = 1$, the signal is 1 / f – noise, while $\alpha = 1.5$ – usually Brownian motion.

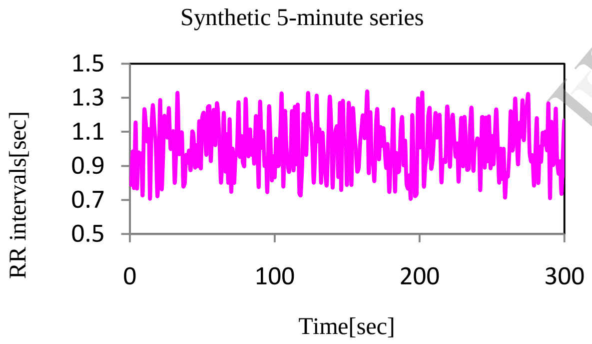

Figure 1: Generated 5-minute RR time series

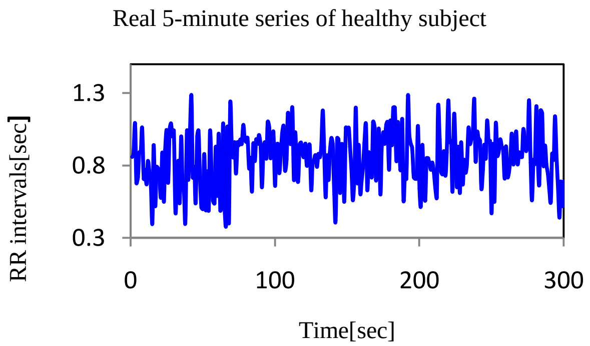

Figure 2: Real 5-minute RR time series

IV. MULTIFRACTAL ANALYSIS

With R/S and DFA methods can determine only one parameter (Hurst exponent-H, scaling exponent-α) characterizing the sampled signal. This fact shows that these methods are more appropriate for study of monofractal signals. Monofractal signals are homogeneous because they have the same scaling properties for the entire signal. These signals are characterized locally by a single singularity exponent h_0 and by a single global exponent $H \equiv h_0$, which suggests that they are stationary. The following definition of multifractal is used in the scientific literature [13]:

Definition: A stochastic process Z(t) is called multifractal if it has stationary increments and satisfies:

$$E\left(|Z(t)|^{q}\right) = c(q)t^{\tau(q)+1}, \quad \text{for all } t \in T, q \in Q. \qquad (2)$$

Where:

- T and Q are intervals on the real line;
- c(q) is independent of t;
- $\tau(q)$ is scaling function.

As result of the mentioned definition follow that $\tau(q)$ is a concave function. If $\tau(q)$ is linear in q, then the signal is monofractal, otherwise it is multifractal. If the signal is fractal (self-similar) with Hurst exponent, then $\tau(q) = qH-1$.

The multifractal detrended fluctuation analysis (MF-DFA) is generalization of DFA to detect multifractal properties of time series [14]. The aim of this method is to determine the behavior of the q dependent fluctuation functions $F_q(n)$ with regard to time scale n, for different values of q. If the time series is of long-range correlation, then:

$$F_q(n) \propto n^{h(q)}. \qquad (3)$$

Where h(q) is a scalable parameter, which is obtained as slope of the linear regression of Log $F_q(n)$ versus Log(n). For stationary time series, h(2) is identical to the Hurst exponent H, and therefore h(q) is called the generalized Hurst exponent.

A. *Generalized Hurst exponent*

This parameter determines whether the time series is monofractal or multifractal. For monofractal time series h(q) is independent of q, while for multifractal series small and large fluctuations scale differently and this parameter is a decreasing function of q.

B. *Multifractal scaling, Partition function and Legendre multifractal spectrum*

The method for estimation of scaling function $\tau(q)$ is based on the absolute moments of the process and it calculates $\tau(q)$ directly from the observation data. For a time series $Z=\{Z_i, i=1,2,\ldots,N\}$ the partition function $S_m^Z(q)$ can be given by [14]:

$$S_m^Z(q) = \sum_{k=1}^{N/m} (\overline{Z}_k^{(m)})^q. \qquad (3)$$

Where:

$$\overline{Z}_k^{(m)} = \sum_{i=1}^{m} Z_{(k-1)m+i}. \qquad (4)$$

The ratio N/m is the number of blocks that cover the observation data, and m is the size of blocks.

From the definition of multifractality, follow if the logarithm of partition functions $\log S_m^Z(q)$ is linearly depending on logarithm of m, the data exhibits multifractal scaling. The slope of the straight line can be obtained using the linear regression and it is denoted by scaling function $\tau(q)$ [14]:

$$\log S_m^Z(q) \approx \tau(q).\log(m) + const. \quad (5)$$

An important tool in multifractal theory is the Legendre transform. The relationship among multifractal scaling function $\tau(q)$, Legendre multifractal spectrum $f(\alpha)$ and Hölder exponent α are following [13]:

$$\alpha(q) = \frac{d\tau(q)}{dq}. \quad (6)$$

$$f(\alpha) = \min_{\alpha}(\alpha q - \tau(q)). \quad (7)$$

The spectrum width on multifractality degree is $\Delta\alpha = \alpha_{max} - \alpha_{min}$, this quantity is a measure of the range of fractal exponents in the time series, so if $\Delta\alpha$ is large, the signal is multifractal.

The dimension function D(q) and $f(\alpha)$ are related through a Legendre transform. The generalized dimension is defined by the following equation [14]:

$$D(q) = \lim_{m \to 0} \frac{1}{q-1} \frac{\ln(S(q,m))}{\ln(m)} \quad (8)$$

D(0) is the capacity dimension; D(1) is the information dimension, and D(2) is the correlation dimension. The observation process is called monofractal if D(q) is constant for all values of q, otherwise is called multifractal.

C. The algorithm for the multifractal Legendre spectrum calculation

The algorithm for the multifractal Legendre spectrum calculation consists of following steps:

Step 1: Initialization of the input parameters:
- N – length of the investigated data;
- m – the size of the non-overlapped blocks;
- (m_begin, m_end) – the range of deviation of the size of blocks;
- m_step – the step of the deviation of the size of blocks;
- (q_min, q_max) – range of the deviation of the parameter q;
- q_step – step of the deviation of the parameter q.

Step 2: The value of the parameter q is equal to the left border (q_min) of its deviation. The value of this parameter is incremented by the value q_step. The iterations are repeated, when the value of q achieve q_max.

Step 3: The input data are divided into non-overlapped blocks, the size of which during the first iteration is equal to the left border of the interval of the scale deviation m (m_begim). The size of the blocks m is incremented by the value m_step. The iterations are repeated, while the size of the block achieves value m_end.

Step 4: For each bock is calculated the sum, using the formula (4).

Step 5: From equation (3) is calculated the partition function $S_m^Z(q)$.

Step 6: The scaling function $\tau(q)$ is calculated by the linear regression for every parameter q.

Step 7: The multifractal Legendre spectrum is calculated by the formulas (6) and (7).

V. Results and Discussion

In the paper are used two methods: R/S and DFA for fractal analysis of two RR-time series: synthetic and real RR-time series of healthy subjects.

The results of the R/S method applied to the two studied signals to determine the value of the Hurst exponent are shown in Figure 3. The determined values of the Hurst exponent are:
- H=0.5978 for synthetic RR-time series;
- H=0.6048 for real RR-time series for healthy subject.

The obtained results show that both RR-time series are correlated, i.e. they are fractal time series.

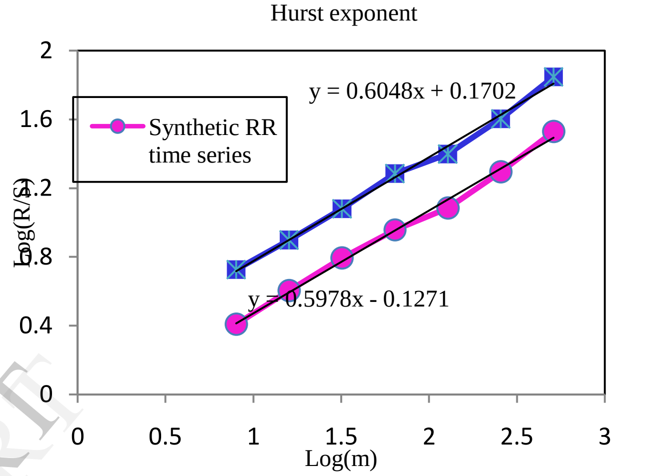

Figure 3: Hurst exponent of synthetic and real RR time series of healthy subject

By DFA method are calculated the scaling exponents of both investigated RR-time series by plotting detrended fluctuations function F(n) against the box size (n). Figure 4 shows the plots of F(n) against n for synthetic and real RR-time series of healthy subject. The scaling exponent for the real RR-time series is 0.63102 and for synthetic RR-time series is 0.61015. In both case the scaling exponent is $0.5<\alpha<1.0$ and thus show that investigated time series have long-range correlation.

Figure 5 shows the generalized Hurst exponent of synthetic and real RR time series of healthy subject. Hurst exponents h are not a constant and hence the two RR-time series have multifractal properties.

Figure 6 shows the relation of scaling function $\tau(q)$ on q of synthetic and real RR time series of healthy subject. The graphics are not linear; hence the two signals have multifractal behavior.

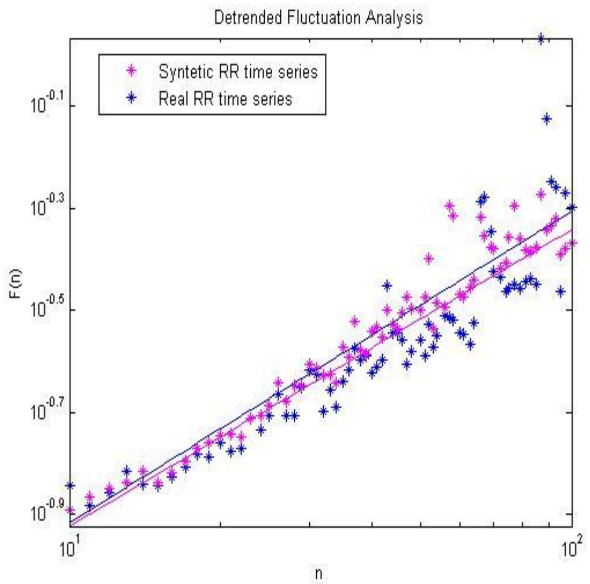

Figure 4: Results of DFA method applied to synthetic and real RR time series of healthy subject

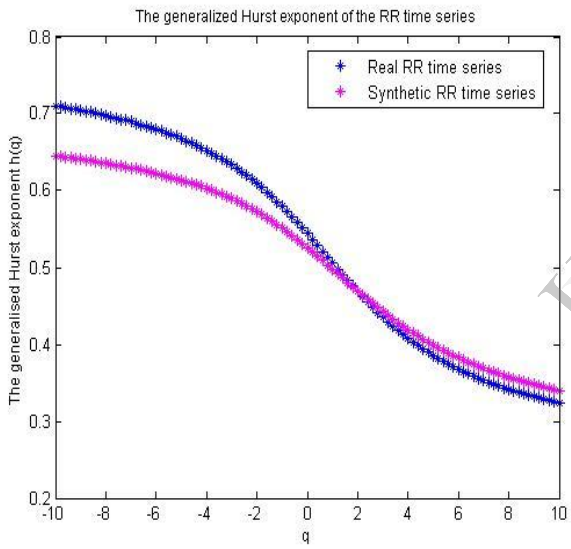

Figure 5: The generalized Hurst exponent of synthetic and real RR time series of healthy subject

Figure 7 shows the multifractal spectrum of synthetic and real RR time series of healthy subject. The two curves have multifractal behaviour, due to the wide range of local values of the Hölder exponent α ($\Delta\alpha=\alpha_{max}-\alpha_{min}$). The range of values of the Hölder exponent α for real RR time series is Δα=0.5162, and for synthetic RR time series is Δα=0.5249. The values of α, f(α) and Δα for different values of q of synthetic and real RR series are reported in Table 1.

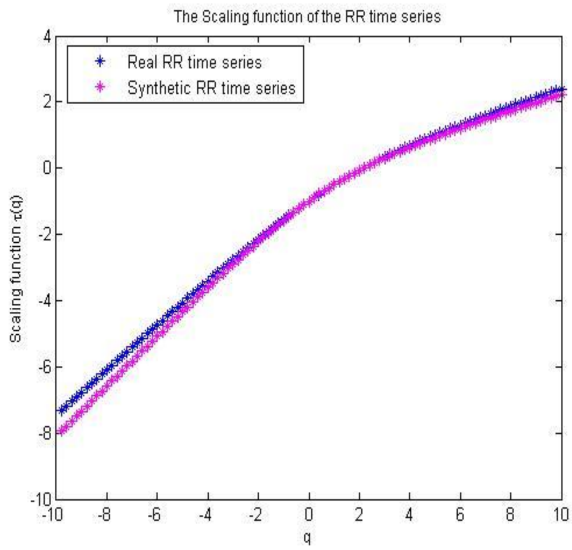

Figure 6: The scaling function of synthetic and real RR time series of healthy subject

Table1: The values of α, f(α) and Δα for different values of q of synthetic and real RR series

q	Synthetic RR series			Real RR series		
	α	f(α)	Δα	α	f(α)	Δα
q=0	0.54	1	0.525	0.54	1	0.516
q=10	0.24	0.19		0.24	0.243	
q=-10	0.77	0.29		0.771	0.464	

Figure 8 and Figure 9 show the multifractal spectrum and the scaling function of real RR time series for all healthy subjects.

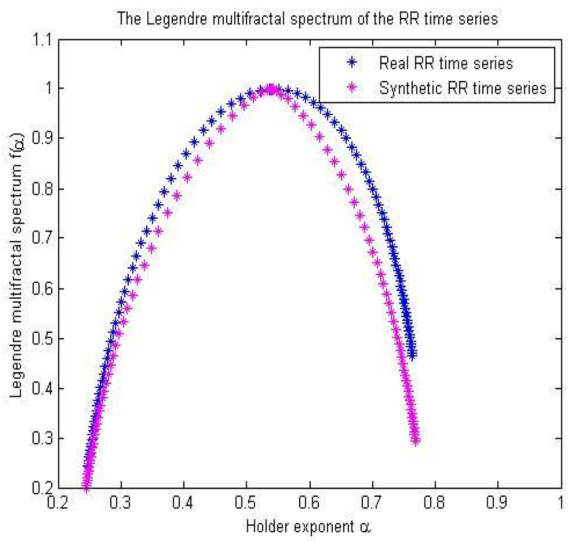

Figure 7: The multifractal spectrum of synthetic and real RR time series of healthy subject

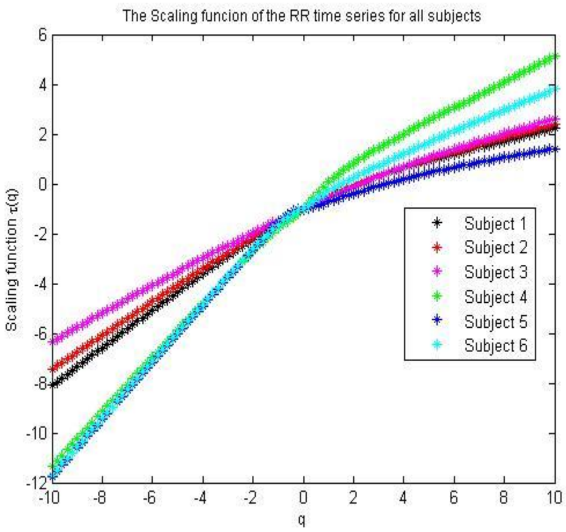

Figure 8: The scaling function of real RR time series for all healthy subjects

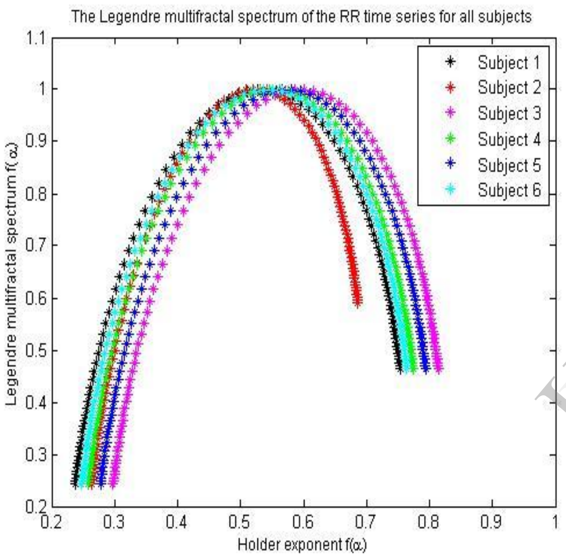

Figure 9: The multifractal spectrum of the real RR time series for all healthy subjects

VI. CONCLUSION

In the recent years, the nonlinear analysis of Electrocardiogram signals using fractal and multifractal theory has been proved to be effective.

In this paper, a detailed analysis of the fractal and multifractal properties of the synthetic and real RR time series for healthy subjects are investigated. Both signals are evaluated by using R/S and DFA methods for fractal analysis and MF-DFA method for multifractal analysis. The obtained results from all these methods show that the investigated signals have fractal and multifractal properties.

The existence of the long-range correlations and multifractal properties in synthetic RR interval time series verifies that algorithm [8] generates data for healthy subjects.

From a practical point of view, the fractal and the multifractal methods may have potential application for ambulatory monitoring of subjects. Fractal and multifractal analysis of the RR time series are suitable non invasive methods of diagnostics, forecast and prevention of the pathological statuses.

ACKNOWLEDGEMENT

This research was carried out as part of the project “Improving the efficiency and quality of education and research potential in the field of system-engineering and robotics” № BG051PO001-3.3.06-0002, founded by European Science Fund.

REFERENCES

[1] Ivanov, P., Z. Chen, K. Hu, H. Stanley, *Multiscale aspects of cardiac control.* Physica A 344 (3–4), 2004, pp 685–704.

[2] Ivanov, P., L. Amaral, A. Goldberger, S. Halvin, M. Rosenblum, H. Stanley, Z. Struzik, *From 1/f noise to multifractal cascades in heartbeat dynamics.* Chaos, Vol 11, No. 3, 2001, pp 641-652.

[3] Ivanov, P., L. Amaral, A. Goldberger, S. Halvin, M. Rosenblum, H. Stanley, Z. Struzik, *Multifractality in human heartbeat dynamics.* Macmillan Magazines Ltd, 1999, pp 461-465.

[4] Dimitrova, M., S. Lozanova, L. Lahtchev, Ch. Roumenin, *New interface technologies for cloud healthcare services.* Comptes rendus de l’Acad´emie bulgare des Sciences, Tome 65, No.1, 2012, pp 83-88.

[5] Lee, K., J. Kim, B. Vidakovic. *Regularity of Irregularity: Testing for Monofractatlity by Multifractal Tools.* International Journal of Mathematics and Computer Science, 5(2010), No. 2, pp 65-85.

[6] Stanley, H.E., L.A.N. Amaral, A.L.Goldberger, S.Havlin, P.Ch.Ivanov, C.-K.Peng, *Statistical physics and physiology: Monofractal and multifractal approaches.* Elsevier, Physica A 270, 1999, pp. 309-324.

[7] Wang, J., Y.Ning, Y.Chen, *Multifractal analysis of electronic cardiogram taken from healthy and unhealthy adult subjects.* Physica A 323, 2003, pp. 561-568.

[8] Georgieva-Tsaneva G., M. Gospodinov , E. Gospodinova, *Simulation of Heart Rate Variability Data with Methods of Wavelet Transform.* Proc of 13th International Conference on Computer Systems and Technologies-CompSysTech’12, Ruse, Bulgaria, June 22-23, 2012, ACM Press, ACM ICPS Vol. 630, pp.306-312.

[9] McSharry P.E., G. Clifford, L. Tarassenko, L.A. Smith. *A dynamical model for generating synthetic electrocardiogram signals.* IEEE Trans. Biomed. Eng.,Vol.50, No 3, 2003, pp.289-294.

[10] Hurst, H.E., R. Black and Y.M.Sinaika, *Long-term Storage in Reservoirs: An experimental Study,* Constable, London, 1965.

[11] Peng, C.-K., S.Havlin, H.E.Stanley, A.L.Goldberger, *Quantification of scaling exponents and crossover phenomena in nonstationary heartbeat time series.* CHAOS, vol. 5, No 1, 1995, pp. 82-87.

[12] Yazawa, T., K. Tanaka and T. Katsuyama, *The Scaling Exponent Calculated by the Detrended Fluctuation Analysis, Distinguishes the Injured Sick Hearts Against Normal Healthy Hearts.* Proceedings of the World Congress on Engineering and Computer Science 2008, WCECS 2008, October 22-24, 2008, San Francisco, USA.

[13] Sheluhin, O., *Multifractals. Infocommunication applications* (In Russian). Goryachaya Linia Publishers, Russia, 2011.

[14] Kantelhardt, J.W. et al. *Multifractal detrended fluctuation analysis of nonstationary time series.* Pysica A 316,, 2002, pp. 87-114.