Fractional Order PID Controller for Level Control in a Spherical Tank

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***Abstract*— Fractional order mathematical phenomena can be used to describe and model a real object more accurately than the classical integer methods. It is a generalisation of Classical integer order control theory with differentiation and integration of non-integer orders. The main reason for using the integer-order models was the absence of solution methods for fractional differential equations. The PID (Proportional integral derivative) controllers are most popular due to their simple structure and effective and simple tuning methods. In a Fractional Order PID (FOPID) controller, there are two more parameters to tune than the conventional PID controller making it more flexible, and better performance can be expected.**

A spherical tank occupies less space for a definite volume and is capable of draining all the stored material. Therefore it is a popular storage tank in most of the industries. Due to different rates of inlet and outlet streams the height of stored liquid changes with time. The level of liquid varies nonlinearly as the area of cross section varies with height. A better control of the nonlinear plant is possible by using a FOPID controller. This is shown by a comparison of performance with an ordinary integer order controller in SIMULINK.

Keywords*—*F ractional order, Integer order, Spherical tank

I. INTRODUCTION

A controller is needed to get desired performances for stable systems and to stabilize the unstable processes first, and obtain the desired performances in the case of unstable systems, in the presence of various types of disturbances. System identification with less modeling errors, optimum controller selection and proper tuning will ensure the objective of the processes.

Spherical tanks are widely used in many process industries as storage tanks of cryogenic liquids, fuels and other liquids. They have the least space requirement and can drain liquids completely. Control of liquid level in a spherical tank is difficult due to the nonlinearity exhibited because of the change in area of cross section with height of liquid. Even though most industrial processes are nonlinear and highly complex, linear models are often sufficient to approximate a process around a single operating point. Combined multiple linear models of different operating regions can model a non-linear process completely. The model can be described as a piece wise linear model in each particular range.[1]

Most of the nonlinear systems exhibit many challenging control problems due to their non-linear shape, non-linear dynamic behavior, uncertain time varying parameters, constraints on manipulated variables, interaction between manipulated and controlled variables, unmeasured and frequent disturbances, dead time on input and measurements etc.

A Proportional Integral and Derivative (PID) controller is a three term controller which is most popular in the automatic control field. This is because of their simple structure, near optimal performance, robustness and ease in implementation.

Improving and optimizing the controller performance is a major concern in control engineering. A continuous effort is being made to improve the control quality performance of PID controllers by contemporary researchers. The objective here is to improve the control performance of a controller using fractional order PID control.

II. FRACTIONAL CALCULUS

Fractional order calculus is an area in mathematics which deals with derivatives and integrals of non-integer orders. Differ-integral operator is denoted by ${}_aD^{\alpha}_t$. It is the combination of differentiation and integration operation commonly used in fractional calculus.

$$ {}_aD_t^{\alpha} = \begin{cases} \dfrac{d^{\alpha}}{dt^{\alpha}} & \alpha > 0 \\ 1 & \alpha = 0 \\ \int_a^t (d\tau)^{-\alpha} & \alpha < 0 \end{cases} \tag{1} $$

where α ϵ R.

The most common definitions of the fractional differ-integral is the Riemann-Liouville and Caputo definition.

A. *Riemann-Liouville definition*

$$ {}_aD_t^{\alpha} = \frac{1}{\Gamma(m-\alpha)} \frac{d}{dt}^{m} \int_a^t \frac{f(\tau)}{(t-\tau)^{\alpha-m-1}} d\tau \qquad \boxed{(2)} $$

where m-1 < α < m, m ϵ N, α ϵ R+ and Γ is Euler's gamma function.

B. *Caputo definition*

$$ {}_aD_t^{\alpha} = \frac{1}{\Gamma(m-\alpha)} \int_0^t \frac{f^m(\tau)}{(t-\tau)^{\alpha-m-1}} d\tau \tag{3} $$

where m-1 < α < m, m ϵ N

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C. *Fractional order Models*

Laplace transform of the fractional operator is

$$L[D^{\alpha} f(t)] = s^{\alpha} F(s) \tag{4}$$

A fractional-order continuous-time dynamic system be expressed by a fractional differential equation given by

$$\begin{aligned} H(\mathscr{D}^{\alpha_0 \alpha_1 \cdots \alpha_n})\, y(t) &= G(\mathscr{D}^{\beta_0 \beta_1 \cdots \beta_n})\, u(t), \\ H(\mathscr{D}^{\alpha_0 \alpha_1 \ldots \alpha_n}) &= \sum_{k=0}^{n} a_k \mathscr{D}^{\alpha_k}, \\ G(\mathscr{D}^{\beta_0 \beta_1 \ldots \beta_n}) &= \sum_{k=0}^{m} b_k \mathscr{D}^{\beta_k}, \end{aligned} \tag{5}$$

where $a_k, b_k \in R$.

$$a_n \mathscr{D}^{\alpha_n} y(t) + a_{n-1} \mathscr{D}^{\alpha_{n-1}} y(t) + \cdots + a_0 \mathscr{D}^{\alpha_0} y(t) = b_m \mathscr{D}^{\beta_m} u(t) + b_{m-1} \mathscr{D}^{\beta_{m-1}} u(t) + \cdots + b_0 \mathscr{D}^{\beta_0} u(t) \tag{6}$$

can be expressed as a fractional order transfer function G(s).

$$G(s) = \frac{Y(s)}{U(s)} = \frac{b_m s^{\beta_m} + b_{m-1} s^{\beta_{m-1}} + \cdots + b_0 s^{\beta_0}}{a_n s^{\alpha_n} + a_{n-1} s^{\alpha_{n-1}} + \cdots + a_0 s^{\alpha_0}} \tag{7}$$

III. LEVEL SYSTEM

The parameter to be controlled is chosen as the level inside a spherical tank. The schematic of the experimental setup is shown below.

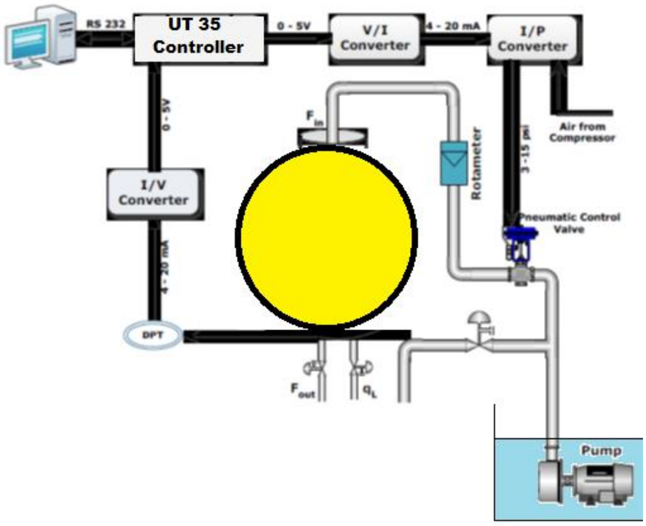


Figure 1 : Schematic of Experimental setup

The spherical tank is made up of Polycarbonate material and has a maximum height and radius of 0.15 meter. Water from the reservoir tank of size 1250mm x 450mm x 450mm is pumped through a Tullu submersible pump having a discharge of 750 lph that flows through a rotameter to the spherical tank. The level of the tank at any instant is measured by a capacitive Differential pressure transmitter. It has a measurement range of (0-400) mm which corresponds to the output range of (4-20) mA. This output is compared with the desired set value of level which will be scaled by the control law as (4 -20) mA. The error signal is also in the range of (4-20) mA which is then amplified based on the controller specifications.

The output of the controller is used to vary the inflow rate of the spherical tank with the use of an Electro pneumatic converter which converts the output of (4 -20) mA in to a pneumatic signal of (3-15) psi so that the Equal percentage control valve will be able to throttle the inflow rate.

Figure 2: Experimental setup

Figure 2. shows the experimental setup of the spherical tank. The pneumatic control valve uses air to close, adjusts the flow of the water pumped to the spherical tank from the water reservoir.

The level of water in the tank is measured by means of a differential pressure transmitter and is transmitted in the form of (4-20) mA to the interfacing module and hence to the PC. After computing the control algorithm in the UT35 controller, control signal is transmitted to I/P converter in the form of current signal (4-20) mA, which passes the air signal to the pneumatic control valve. The pneumatic control valve is actuated by this signal to produce the required flow of water in to the tank. There is a continuous flow of water in and out of the tank.

IV. SYSTEM IDENTIFICATION

The goal of identification is to infer a dynamic system model based upon data, measured during an experiment. In general, it is necessary to obtain a relationship between system inputs and outputs under external disturbances, in order to determine and predict the system behavior.

The general procedure of system identification is

- Design the experiment.
- Collect transient response data in the time-domain by applying a set of predetermined input signals (step)
- Record the dataset based on an experiment.
- Choose the model structure and the criterion to fit.

- Calculate the model using a suitable algorithm, e.g. least squares method.
- Validate the obtained model.
- If the model is satisfactory, use it, otherwise, revise modeling/identification strategy and repeat the above steps.

The objective of time-domain identification is to obtain a fractional model of the form

$$G(s) = \frac{Y(s)}{U(s)} = \frac{b_m s^{\beta_m} + b_{m-1}s^{\beta_{m-1}} + \cdots + b_0 s^{\beta_0}}{a_n s^{\alpha_n} + a_{n-1}s^{\alpha_{n-1}} + \cdots + a_0 s^{\alpha_0}} \quad (8)$$

from registered system input and output. The transfer function is obtained by optimizing the parameters α_n, β_m, a_n and b_m.

An output error method can be used with a least-square approach. The optimization criterion will be the output error norm $\| e(t) \|^2$ given by

$$e(t)=y(t)-\hat{y}(t) \quad (9)$$

where y(t) is the experimental output signal and $\hat{y}(t)$ is the one obtained by simulation of the identified model under the experimental input signal u(t).

The time domain data of step response carried out at a liquid level of 900mm was used along with the fractional order transfer function identification tool (fotfid) to obtain the fractional order transfer function of the spherical tank. [3]

From previous experience it is known that in case of an integer-order model this system can be approximated by a second order model. Thus, it is possible to obtain the initial guess model by generating a fractional pole polynomial of the form

$$G(s)=\frac{1}{s^2+s+1} \quad (10)$$

With this initial model, the system is identified in several iterations, and after a simple transformation using the normalize() command the system transfer function becomes

$$G(s) = \frac{5.001}{47.942s^{1.617} + 12.017s^{1.288} + 1} \quad (11)$$

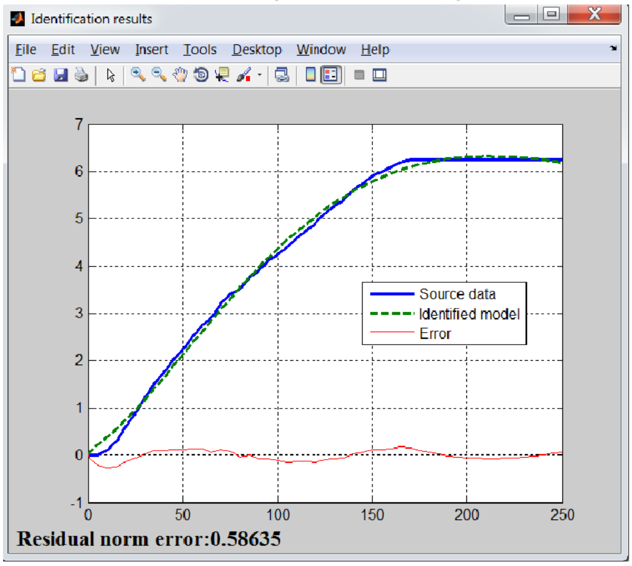


Figure 3 : Source data and identified fractional order model data over-plotted

The plot of the source data and the identified fractional order model is shown in Figure 3. It shows a residual normalised error of 0.58635.

V. TUNING OF PID CONTROLLERS

The iopid_tune graphical tool is used to first approximate the fractional order model by a conventional FOPDT(First Order Plus delay Time) model, and then classical tuning formulae is applied to get the PID controller parameters. The identified integer order model has K= 5.10833, L=6.13942, T=6.85373. The FOPDT model is

$$G(s)=\frac{5.10833}{1+6.85373s}e^{-6.13942s} \quad (12)$$

The Ziegler-Nichols tuning formula is used to obtain the integer order PID controller parameters:

$K_p = 0.262242$, $K_i = 0.0213572$, $K_d = 0.805006$

VI. FRACTIONAL ORDER PID CONTROLLER OPTIMIZATION

The constraint based optimisation method is the most popular tuning method. Design specifications can be derived from the frequency domain evaluation of the open loop transfer function G(jω)P(jω), where G(jω) is the controller and P(jω) is the plant transfer functions.[5]

- Phase margin constraint

$$\mathrm{Arg}[G_c(j\omega)] = \mathrm{Arg}[G(j\omega)] + \mathrm{Arg}[P(j\omega)] = -\Pi + \phi m \quad (13)$$

- Gain crossover frequency constraint

$$| G_c (j\omega_c) |= |G(j\omega_c)P(j\omega_c)| = 1 \quad (14)$$

where ω_c is the gain cross over frequency

- Robustness to loop gain variation constraint

$$\left(\frac{d}{d\omega}\mathrm{Arg}[P(j\omega)G(j\omega)]\right)_{\omega=\omega c} =0 \quad (15)$$

- Rejection of high-frequency noise

$$\left|\frac{P(\omega h)G((\omega h)}{1+P(\omega h)G((\omega h)}\right|< H\ \mathrm{dB} \quad (16)$$

H is the desired noise attenuation for frequencies $\omega \geq \omega_h$

- Sensitivity constraint

$$\left|\frac{1}{1+P(\omega l)G((\omega l)}\right|< N\ \mathrm{dB} \quad (17)$$

N is the desired value of sensitivity function for frequencies $\omega \leq \omega l$

The system of five non-linear equations and five unknown parameters Kp, Ki, Kd, λ and μ can be solved by the optimization toolbox of MATLAB to give the best solution with minimum error.

Here a gain margin =10dB, phase margin=60̊, noise rejection =-20dB and sensitivity =-20dB were considered as design criteria and minimization of Integral of Absolute Error(IAE) was considered as the optimization criteria.

The design is carried out using the fpid_optim tool. The controller obtained after IAE minimisation is

$$C(s) = 0.21699 + \frac{0.6145}{s^{0.98085}} + 33.133s^{0.51581} \quad (18)$$

VII. SIMULATION RESULTS

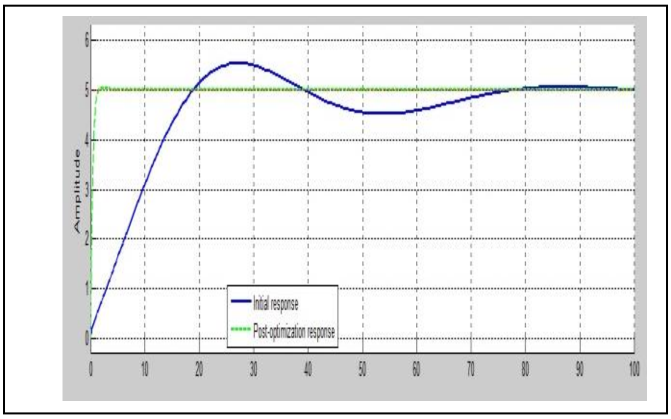
The different measures which can be used to compare the quality of controlled responses are Integral Absolute Error (IAE), Integral Squared Error (ISE) ,Integral Time-Weighted Squared Error (ITSE) and Integral Time-weighted Absolute Error (ITAE).

IAE integrates the absolute error over time. It doesn't add weight to any of the errors in a systems response. It tends to produce slower response than ISE optimal systems, but usually with less sustained oscillation. ISE integrates the square of the error over time. ISE will penalise large errors more than smaller ones (since the square of a large error will be much bigger). Control systems specified to minimise ISE will tend to eliminate large errors quickly, but will tolerate small errors persisting for a long period of time. Often this leads to fast responses, but with considerable, low amplitude, oscillation.

ITAE integrates the absolute error multiplied by the time over time. It weights errors which exist after a long time much more heavily than those at the start of the response. ITAE tuning produces systems which settle much more quickly than the other two tuning methods. The downside of this is that ITAE tuning also produces systems with sluggish initial response .

A comparison between the errors (IAE, ISE, ITSE and ITAE) when using fractional order controller and integer order controller are given in the table below. It clearly shows that the fractional controller gives better error performances compared to the integer order controller for the same fractional order plant.

Figure 4: Comparison between the initial response and post optimisation response for IAE minimisation



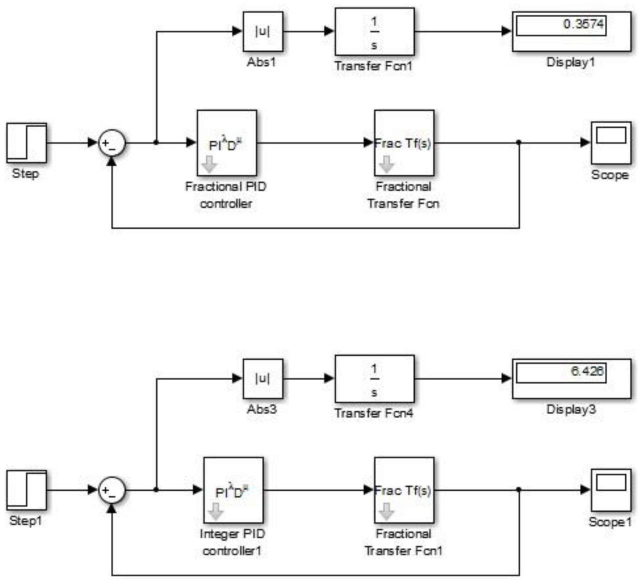


Figure 5:Comparison of the Integral Absolute error with Fractional order controller and Integer order controller

Table 1: Comparison of Errors

Controller	IAE	ISE	ITSE	ITAE
Fractional PID	0.367	0.7505	0.8031	0.2993
Integer PID	6.426	4.792	123.2	31.83

VIII. CONCLUSION

The time domain system identification of the spherical tank was carried out and a piecewise linear fractional order transfer function was obtained. The controller parameters for the integer order controller was obtained using Zeigler-Nichols method. The Fractional order proportional integral derivative (FOPID) controller was designed for liquid level control in a spherical tank which is modeled as fractional order system. The designed FOPID controller provides better results as compared with the traditional IOPID controller in simulation.

The FOPID controller was designed following a set of imposed tuning constraints, which can guarantee the desired control performance and the robustness of the designed controllers to the loop gain variations. From the simulation results, it is observed that the overshoot and rise time of the designed fractional order PID controller is less when compared to the integer order controller and thus more efficient.

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