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Fractional Order PID Controller for Liquid Level System

Ramkumar V P
PG Scholar,
Electrical &Electronics Dept.
Mar Baselios college of Engineering
Thiruvananthapuram, Kerala, India

Elizabeth Varghese
Associate Professor,
Electrical &Electronics Dept.
Mar Baselios college of Engineering
Thiruvananthapuram, Kerala, India

***Abstract*— Control of Liquid Level System has been a proven area by control engineers implementing the conventional controllers such as PID controller. But with development of Fractional calculus the control technique are also being improved. The thesis work deals with design of Fractional order PID [FOPID] controller for a Liquid Level System (LLS).**

The Liquid Level system is modeled mathematically to obtain the transfer function, as first order system plus delay. Then the FOPID controller is designed by using Zeigler-Nichols and Astrom-Hagglund method based on certain design specifications. The frequency response of the FOPID controller is compared with the frequency response of normal PID controller.

Index Terms*— *Fractional order PID controller, PID controller, Zeigler-Nichols, Astrom-Hagglund.

I. INTRODUCTION

Liquid Level System has become an inevitable part in many industries due to the wide use of steam generators and other liquid based production techniques. Therefore the control of liquid level has gained its priority in these industries, so is the controller used.

The idea of fractional order PID is proposed by Podlubny I. [1]. In 1980, Irving et al. introduced a linear parameter varying model in order to describe the steam generator dynamics over the entire operating power range and proposed a model reference adaptive proportional integral derivative (PID) level controller [2]. The Irving model and its modifications have probably been the most widely accepted steam generator models for the design of water-level controllers. On the basis of classical MPC theory for linear time varying system, Kothare and *et al.* established a framework to design water level controller for Steam Generator. In 1999, Bendotti set water-level control problem for Steam Generator as a benchmark for robust control techniques, and the evaluation of water-level control performance using six different linear control algorithms such as PID, etc., were also obtained [3]. The performance of these linear robust controllers is higher than that of the classical PID-like controllers. With the development of neural networks, fuzzy set theory and evolutionary computing, some intelligent water level controllers have also been designed which result in better transient response with comparison to those PID controllers.

With the development of Fractional calculus, the control engineers are extending the conventional control technique to the fractional level so that the performance of controller is improved. The Fractional order PID controller is actually an improved form of normal PID controller with more number of control parameters thereby improving the performance of the controller. The main advantages of fractional order PID controller over integer-order PID controllers is that, it has five adjustable parameters (the proportional gain (K_P , the integrating gain (K_i, the derivative gain (K_d, the integrating order (λ) and the derivative order (μ)), thus, expands the scope of parameter tuning, increase design freedom and can achieve better control qualities; it can effectively suppress noise; it has better robustness for the model uncertainty.
Zeigler- Nichols and Astrom-Hagglund methods are used for obtaining the PID control parameters Kp, Ki ,and Kd. In order to obtain the λ and μ parameters two nonlinear equations as explained in [4] are solved which is described in the coming sections. The frequency response of FOPID controller and the conventional PID controllers are compared.

II. MATHEMATICAL MODELLING OF LIQUID LEVEL SYSTEM

III.

The diagram of Liquid Level system (plant) under consideration is shown in the fig 1. The LLS mainly consists of process tank, reservoir tank, level transmitter, pump, control valve governed by pneumatic signal and data acquisition card.

Fig 1: The Liquid Level System under consideration

The functional diagram is shown in the figure 2. The RF capacitance level transmitter is used to measure the liquid level in the process tank. In level control action, the pump sucks water from reservoir tank and gives it to control valve. The error signal is generated by the PC and according to this signal. the control signal is generated and given to the Electro-Pneumatic converter. It controls the flow of the fluid in pipeline by varying stem position of the control valve. For maintaining the level of the process tank, flow is manipulated level signal is given to the Data acquisition card. By pass line is provided to avoid the pump overloading.

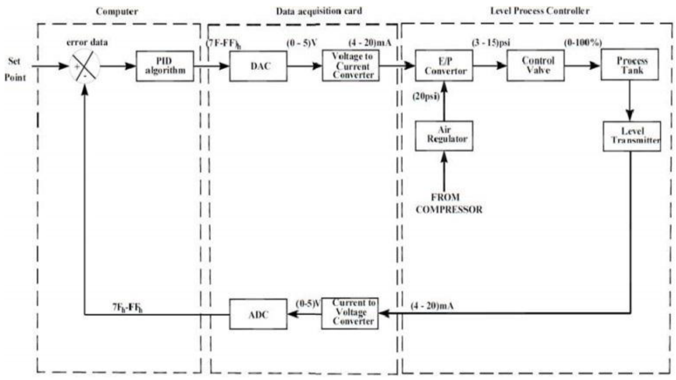

Fig 2: Function diagram of Liquid Level system

The components specification of the plant is given in table 1. For designing the control parameters, the first step is to obtain the mathematical model of the plant, LLS. The linear modeling is explained in this section in a simple method [5] by considering the figure 3.

Table 1: Specification of Liquid Level System

Pump		Process tank		Reservoir tank	
Model	Tullu 80	Material	Acrylic	Material	Mild Steel
Speed	6500RPM	Capacity	2 liters	Capacity	7 liters

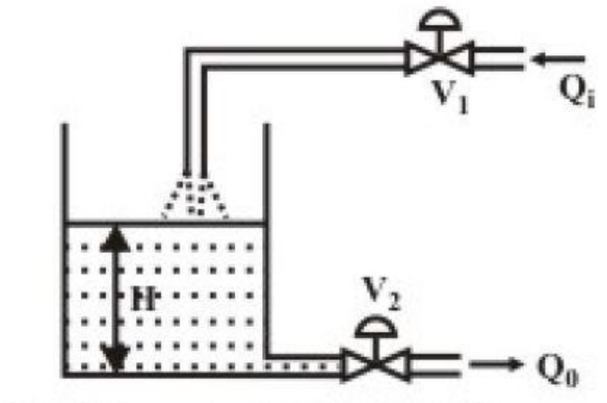

Fig 3: Modeling diagram of Process tank

Let Q_i and Q_o are the inflow rate and outflow rate (in m3/sec) of the tank, and H is the height of the liquid level at any time instant. We assume that the cross sectional area of the tank be A. In a steady state, both Q_i and Q_o are same, and the height H of the tank will be constant. But when they are unequal, we can write,

$$Q_i - Q_o = A\frac{dH}{dt} \quad (1)$$

But the outflow rate Q_o is dependent on the height of the tank. Considering the Valve V_2 as an orifice, considering that the opening of the orifice (valve V_2 position) remains same throughout the operation, therefore,

$$Q_o = C\sqrt{H} \quad (2)$$

where, C is a constant. So from equation (1) we can write that,

$$Q_i - C\sqrt{H} = A\frac{dH}{dt} \quad (3)$$

The nonlinear nature of the process dynamics is evident from equation 3, due to the presence of the term H. In order to linearize the model and obtain a transfer function between the input and output, let us assume that initially $Q_i = Q_o = Q_s$; and the liquid level has attained a steady state value H_s.

Now expanding   in Taylor's series, we can have

$$Q_o = Q_o(H_s) + \dot{Q}_o(H_s)(H - H_s) + \cdots \quad (4)$$

Taking first order approximation, we obtain linear model as

$$q = A\frac{dh}{dt} + \frac{1}{R} \quad (5)$$

where q= Q_i - Q_o
h=H – H_s
H_s is the steady state height

From (5) the transfer function of the plant is obtained as:

$$\frac{h(s)}{q(s)} = \frac{R}{\tau s + 1} \quad (6)$$

where,

$$R = \frac{2\sqrt{H_s}}{C}$$

and

$$\tau = R * A$$

Equation 6 is the transfer function of the plant.

IV. FRACTIONAL ORDER PID CONTROLLER

In the last two decades, fractional calculus has been rediscovered and applied in many number of fields, mainly in the area of control theory [6], [7], [8], [9]. Fractional order proportional-integral-derivative (FOPID) controllers have received a considerable attention in the last years and they provide more flexibility in the controller design, with respect to the standard PID controllers, because they have five parameters to tune. Other than K_p, K_i, and K_d control parameters of normal PID controller, λ and μ, fractional powers (λ and μ) of the integral and derivative parts, respectively add to the better flexibility of Fractional PID controller.

The differential equation of Fractional order PID controller is given as:

$$U(t) = K_p e(t) + K_d J_t^{\lambda} e(t) + K_i D_t^{\mu} e(t) \quad (7)$$

The continuous transfer function of FOPID is also obtained through Laplace transform as:

$$C(s) = K_p + \frac{K_i}{s^\lambda} + K_d s^\mu \qquad (8)$$

V. FRACTIONAL ORDER PID: TUNING

Fractional PID controller is actually an extended and much more advanced form of PID controller with more number of control parameters which increase the design freedom and also makes the controller more flexible. The tuning is done to obtain the parameters of PID controller K_p, K_i and K_d by Ziegler-Nichols and Astrom-Hagglund method. The initial values of K_p and K_i are obtained by using Zeigler-Nichols tuning method. The initial value of K_d is obtained by using Asttrom-Hagglund method. The integral and differential order λ and μ are then obtained by solving the non-linear equations which are obtained by considering the phase margin is equal to the desired phase margin and the criteria

$$\left|C(j_{\omega cp})G(j_{\omega cp})\right| = 1$$

the equation below must be satisfied:

$$C(j_{\omega cp}) = \frac{1}{|G(j_{\omega cp})|} e^{j\emptyset pm} = K_c \cos\emptyset_{pm} + jK_c \sin\emptyset_{pm} \qquad (9)$$

LHS of the equation can be written as below:

$$C(j_{\omega cp}) = K_p + K_i \omega_{cp}{}^{-\lambda} \cos\left(\frac{\pi}{2}\lambda\right) + K_d \omega_{cp}{}^{\mu} \cos\left(\frac{\pi}{2}\mu\right) + j[-K_i \omega_{cp}{}^{-\lambda} \sin\left(\frac{\pi}{2}\lambda\right) + K_d \omega_{cp}{}^{\mu} \sin\left(\frac{\pi}{2}\mu\right)] \qquad (10)$$

Thus the non-linear equations are obtained as:

$$f_1(\lambda,\mu) = k_p + k_i \omega_{cp}{}^{-\lambda} \cos(\frac{\pi}{2}\lambda) + k_d \omega_{cp}{}^{\mu} \cos(\frac{\pi}{2}\mu) - k_c(\cos\emptyset_{pm}) \qquad (11)$$

$$f_2(\lambda,\mu) = -k_i \omega_{cp}{}^{-\lambda} \cos(\frac{\pi}{2}\lambda) + k_d \omega_{cp}{}^{\mu} \sin(\frac{\pi}{2}\mu) - k_c(\sin\emptyset_{pm}) \qquad (12)$$

Hence all the control parameters of Fractional order PID controller is obtained. The values can be optimized to obtain better values of control parameters.

A. APPLICATION OF TUNING METHOD

The transfer function of LLS shown in figure 3 is obtained after taking the measurements radius of inlet valve, outlet valve, process tank and undergoing certain calculations. The transfer function of LLS is therefore obtained as:

$$G(s) = \frac{1.23}{0.924s+1} e^{-1s} \qquad (13)$$

which is of first order plus a delay system.
The tuning of PID controller by Zeigler-Nichols method is done, and the result is shown in the table 2.

Table 2: Control parameters from Zeigler-Nichols method

CONTROL SCHEME	K_p	K_i	K_d
ZEIGLER-NICHOLS	1.0440	1.0158	0.257

After obtaining K_p, K_i, and K_d values, same parameters are tuned using Asrtom-Hagglund method with the desired phase margin as 30°, whose result is shown in table 3**.** The K_d value is then fine-tuned to obtain the better response. The corresponding result is shown in table 4.

Table 3: Control parameters from Astrom-Hagglund method

CONTROL SCHEME	K_p	K_i	K_d
ASTROM-HAGGLUND	1.7399	1.77179	0.42714

After obtaining these results, the K_p, K_i, and K_d values from Zeigler-Nichols and K_d from Astrom-Hagglund method is selected to find the integral and differential order of Fractional order PID controller which is obtained by solving equations 10 and 11, the results are tabulated as in table 5**.**

Table 4: Control parameters after fine tuning

CONTROL SCHEME	K_p	K_i	K_d
ASTROM-HAGGLUND	1.7399	1.77179	0.63485

Table 5: Fractional order PID control parameter

CONTROL SCHEME	K_p	K_i	K_d	λ	μ
FOPID	1.0440	1.0158	0.63485	0.4433	1.0467

The conventional PID designed for phase margin 30°is shown in table 6.

Table 6: Fractional PID control parameter

CONTROL SCHEME	K_p	K_i	K_d
PID CONTRO-LLER	0.3420	8.13	12.8908

VI. SIMULATION RESULTS

The simulation results of control designs are discussed here. The figure 4 shows the step response of Zeigler-Nichols tuning.

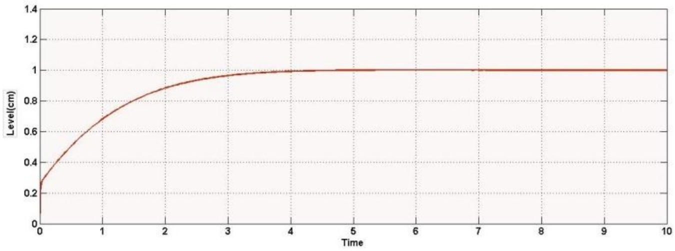

Fig 4: Step response of Zeigler-Nichols tuning method

The figure 5 shows the simulation result of Astrom-Hagglund method.

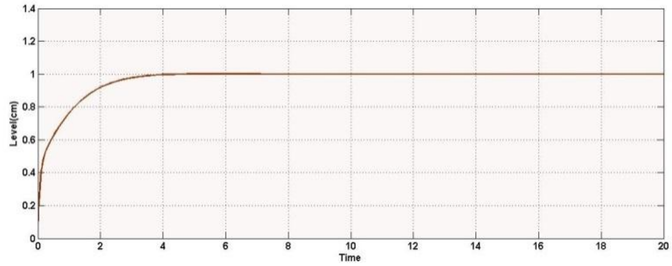

Fig 5: Step response of Astrom-Hagglund tuning method

The Fractional order PID controller is designed using the results of Zeigler-Nichols and Astrom-Hagglund method, and the frequency response is obtained as in figure 6.

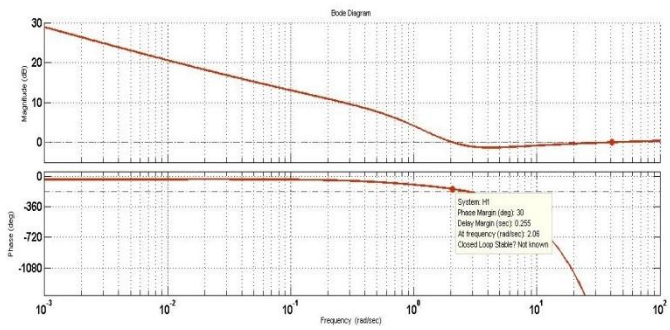

Fig 6: Frequency Response of proposed controller

The desired phase margin is obtained from the Fractional PID controller designed as shown in figure 6. For comparing the Frequency response of FOPID controller with conventional technique, a normal PID controller is designed for desired phase margin of 30° and the Frequency response is compared as in figure 7.

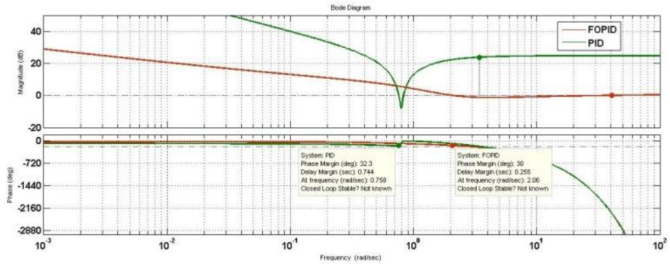

Fig 7: Frequency Response of PID controller

VI. CONCLUSION

The Liquid Level System has been modeled and Transfer function is obtained as in equation 13. The Fractional Order PID controller has been designed for the LLS and the frequency response is taken and has been validated that the required phase margin is satisfied by the controller. The conventional PID is also degined for same phase margin and its observed that Fractional PID is better than PID as it satisfies the Phase Margin more accurately as in figure 7.

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