International Journal of Engineering Research & Technology (IJERT)
ISSN: 2278-0181
Vol. 1 Issue 6, August - 2012

Free Convection Flow of Non-Newtonian Fluids in an Anisotropic Porous Medium

G.Soudjada[1] Dr.Subbulakshmi[2]

[1]*Assistant Professor, Department of Mathematics, AAGA&S College, Karaikal.*

[2] *Assistant Professor, Department of Mathematics, D.G.G.A College(W), Mayiladuthurai*

"Abstract"

Convection in porous media plays a vital role in recent advancements. The applications of porous media are found in different areas like geophysics, petroleum processes, and air conditioning porosity. In this study, the anisotropic effects of porous medium are investigated for suitable range of parameters. The governing partial nonlinear differential equations were transformed into a set of coupled ordinary differential equations, which was solved using the fourth-order Runge-Kutta method. Nusselt number increases almost linearly with increasing porosity.

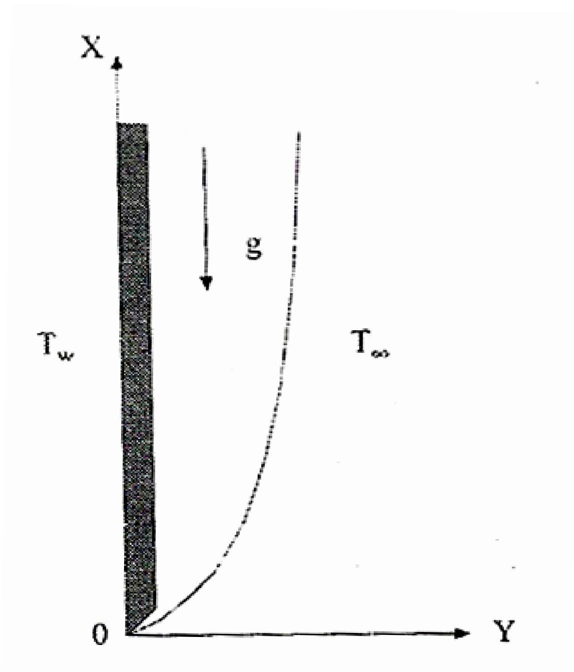
Key Words: Non-Newtonian flow, free convection, anisotropic porous medium.

"1. Introduction"

There has been an increase in interest in the effect of anisotropic porous media, because of their extensive practical application in many areas. A porous medium is a material containing pores. The skeletal portion of the material is often called the metric or frame. The pores are typically filled with a fluid. Many studies related to non-Newtonian fluids saturated in an anisotropic porous medium have been carried out. Non-Newtonian fluids are characterized by a non-linear relationship between the shear stress and shear velocity of the flow. These fluids are often encountered in nature and industrial technologies (volcanic, lava, mudflavs, oil, plastics, oil-based points and polymer solutions) Chen & Chen (1988) investigated the free convection flow along a vertical plate embedded in a porous medium. Vafai et.al (1983), carried out an experimental investigation into valuable porosity, finding that the Nusselt number depends on the Reynolds number and the free convection flow of Non-Newtonian fluids in an anisotropic porous medium is investigated numerically. Sekar. R *et. al*. (1996) have investigated ferro convection in an anisotropic porous medium. Sengupta.T.K (2004) have investigated foundation of computational fluid dynamics. G. Degan *et. al(2007).* have studied transient natural convection of non-Newtonian fluids about a vertical surface embedded in an anisotropic porous medium. Han-Taw Chen *et. al.* (1988) have investigated the free convection flow of non-Newtonian fluids along a vertical plate embedded in a porous medium. Rajendra *et. al.* (2001) investigated the flow of non-Newtonian fluids in fixed and fluidized beds. Prakash Chandra and V. V. Satyamurty *et. al.* (2011) have studied the non-Darcian and Anisotropic Effects on Free Convection in a Porous Enclosure. Gorla *et. al.* (2001) have studied the free convection in non-Newtonian fluids along a horizontal plate in a porous medium. E. Kim (1997) have investigated the natural convection along a wavy vertical plate to non-Newtonian fluids A similarity solution is sought for the governing equations. Then the effect of variable porosity on the temperature distribution and Nusselt number in both cases is stated.

"2. Mathematical formulation"

The following figure represents a Non-Newtonian power law fluid flow along a constant temperature vertical plate embedded in an anisotropic porous medium.



The Governing equations are:

The continuity equation is

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

The power law fluid is

$$u^n = \frac{-k_2(n)}{K}\left[\frac{\partial p}{\partial x} + \rho g\right] \tag{2}$$

The temperature equation is

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\left[\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right] \tag{3}$$

The density equation is

$$\rho = \rho_\infty[1 - \beta(T - T_\infty)] \tag{4}$$

The boundary conditions for these equations are

$$v = 0\,, T = T_w \quad \text{at} \quad y = 0 \tag{5}$$

$$u = 0, T = T_\infty \quad \text{as} \quad y \to \infty \tag{6}$$

where K, n and $k_2(n)$ are the power law constant, power law index and the permeability of the porous medium in the vertical direction respectively.

$k_2(n) = \varepsilon\, k_1(n)$

where ε is the anisotropic parameter and k_1 is the permeability along the horizontal direction.

$$K = \frac{5n_1}{2+3n_1}\left(\frac{150}{32}\right)^{n_2} K_1 \tag{7}$$

$$n = n_1 + 0.3(1 - n_1) \tag{8}$$

$$n_2 = \left(\frac{3n_1}{2+n_1}\right) \tag{9}$$

$$k_2(n) = \frac{2}{e}\left[\frac{De^2}{8(1-e)}\right]^{n-1} \tag{10}$$

$$k_1(n) = \frac{2}{\varepsilon e}\left[\frac{De^2}{8(1-e)}\right]^{n-1}$$

The dimensionless terms used are

$$x^* = {}^{x}/_{l}\ ,\ y^* = {}^{y}/_{l} \tag{11}$$

$$v^* = \frac{v}{\left[\rho_\infty \beta g \varepsilon k_1(n)\frac{(T_w - T_\infty)}{K}\right]^{1/n}} \tag{12}$$

$$u^* = \frac{u}{\left[\rho_\infty \beta g \varepsilon k_1(n)\frac{(T_w - T_\infty)}{K}\right]^{1/n}} \tag{13}$$

$$\mathrm{Ra} = \rho_\infty \beta g \varepsilon k_1(n)\frac{(T_w - T_\infty)}{K} l^n/(K\alpha^n) \tag{14}$$

$$\mathrm{R}a^* = \left[\rho_\infty \beta g \varepsilon k_1(n)\frac{(T_w - T_\infty)}{K} l^n \Big/ (K\alpha^n)\right]^{1/n} \tag{15}$$

$$\theta = \frac{(T - T_\infty)}{(T_w - T_\infty)} \tag{16}$$

The non-dimensionless form of governing equations are

$$\frac{\partial u^*}{\partial x^*} + \frac{\partial v^*}{\partial y^*} = 0 \tag{17}$$

$$(u^*)^n = \theta \tag{18}$$

$$u^* \frac{\partial \theta}{\partial x^*} + v^* \frac{\partial \theta}{\partial y^*} = {}^{1}/_{Ra^*} \frac{\partial^2 \theta}{\partial y^{*2}} \tag{19}$$

The above partial differential equations are transformed into ordinary differential equations using the following dimensionless variables defined by,

$$\xi(x^*) = \left(\frac{x^*}{Ra^*}\right)^{1/2} \tag{20}$$

$$\eta = \left(\frac{y^*}{\xi(x^*)}\right) = y^*\left(\frac{Ra^*}{x^*}\right)^{1/2} \tag{21}$$

$$\psi = \cup\,(x^*)\xi(x^*)f(\eta) = \left(\frac{x^*}{Ra^*}\right)^{1/2} f(\eta) \tag{22}$$

Consequently, the velocity components become,

$$u^* = \frac{\partial}{\partial y^*} = f'(\eta) \tag{23}$$

$$v^* = -\frac{\partial \psi}{\partial x^*} = {}^1\!/_2\,(f^1\eta - f)/(Ra^* x^*)^{1/2} \tag{24}$$

The boundary conditions are

$$\varepsilon\theta = f'^{(n)} \tag{25}$$

$$\theta'' + {}^1\!/_2\,\theta' f = 0 \tag{26}$$

$$\theta = 1, f = 0\ at\ \eta = 0 \tag{27}$$

$$\theta = 0, f' = 0\ at\ \eta \to \infty \tag{28}$$

The local heat flux at the wall is

$$q_w = -k_m\left(\frac{\partial T}{\partial y}\right)_{y=0}$$

$$= -k_m(T_w - T_\alpha)\varepsilon\theta'(0)\,{}^1\!/_l\left(\frac{Ra^*}{x^*}\right)^{1/2} \tag{29}$$

Simplifying, using Runge-Kutta method, we get the local Nusselt number as

$$Nu_x = \frac{h_x}{k_m} = \frac{q_w x}{k_m(T_w - T_\alpha)} \tag{30}$$

Substituting eq. 30 into 29 gives

$$\frac{Nu_x}{(x^*)^{1/2}(Ra^*)^{1/2}} = -\,\varepsilon\theta'(0) \tag{31}$$

Table -1

Variation of η with $f' = (\varepsilon\theta)^{\frac{1}{n}}$ for a value of

$\varepsilon = 1, 10, 30$ *and* $n = 1$.

ε	η	$f' = (\varepsilon\theta)^{1/n}$
1	0.0000 1.0000 4.0000 7.0000	1.0000 0.5871 0.6631 0.00609
10	0.0000 1.0000 4.0000 7.0000	10.0000 5.8719 0.6631 0.0609
30	0.0000 1.0000 4.0000 7.0000	30.0000 17.6157 1.9893 0.1827

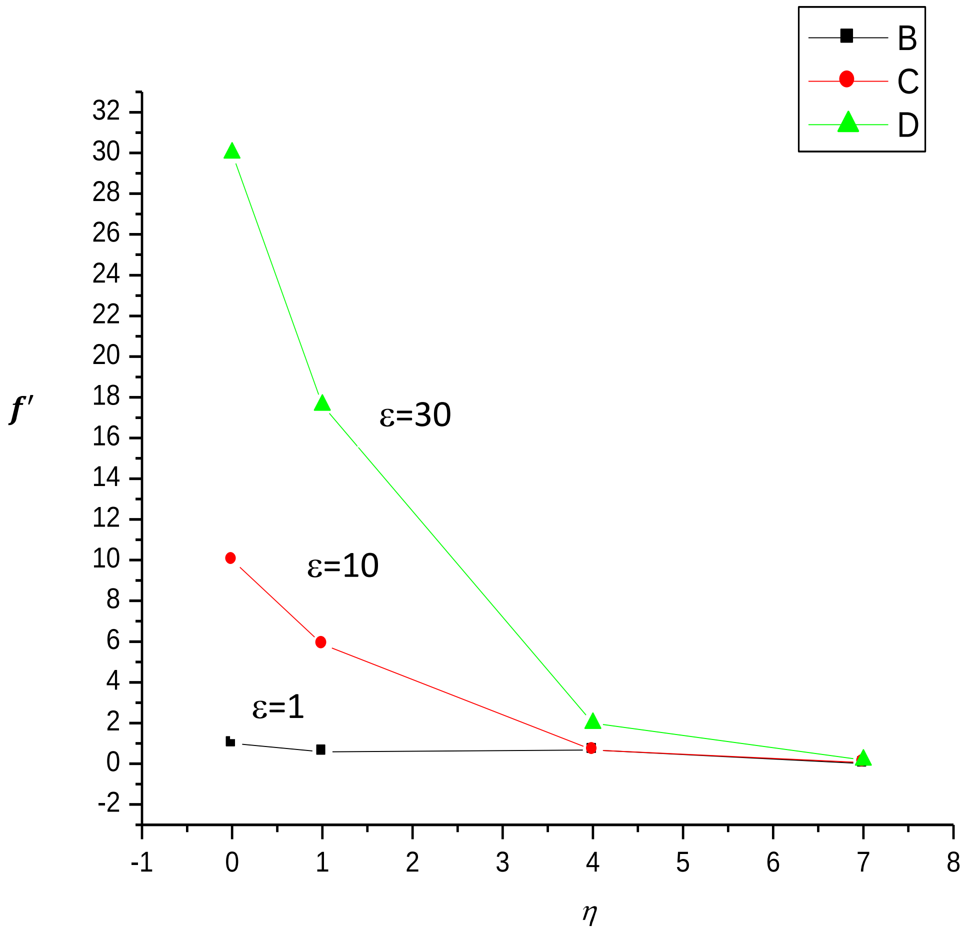


Fig : 1, Variation of η with $f' = (\varepsilon\,\theta)^{1/n}$ for a value of $\varepsilon = 1, 10, 30$ and $n = 1$.

Table - 2

Variation of η with $f' = (\varepsilon\,\theta)^{1/n}$ for a value of $\varepsilon = 1, 10, 30$ and $n = 2$.

ε	η	$f' = (\varepsilon\,\theta)^{1/n}$
1	0.0000	1.0000
	1.0000	0.7663
	4.0000	0.2575
	7.0000	0.0789
10	0.0000	3.1623
	1.0000	2.4232
	4.0000	0.8143
	7.0000	0.2468
30	0.0000	5.4772
	1.0000	4.1971
	4.0000	1.4104
	7.0000	0.4274

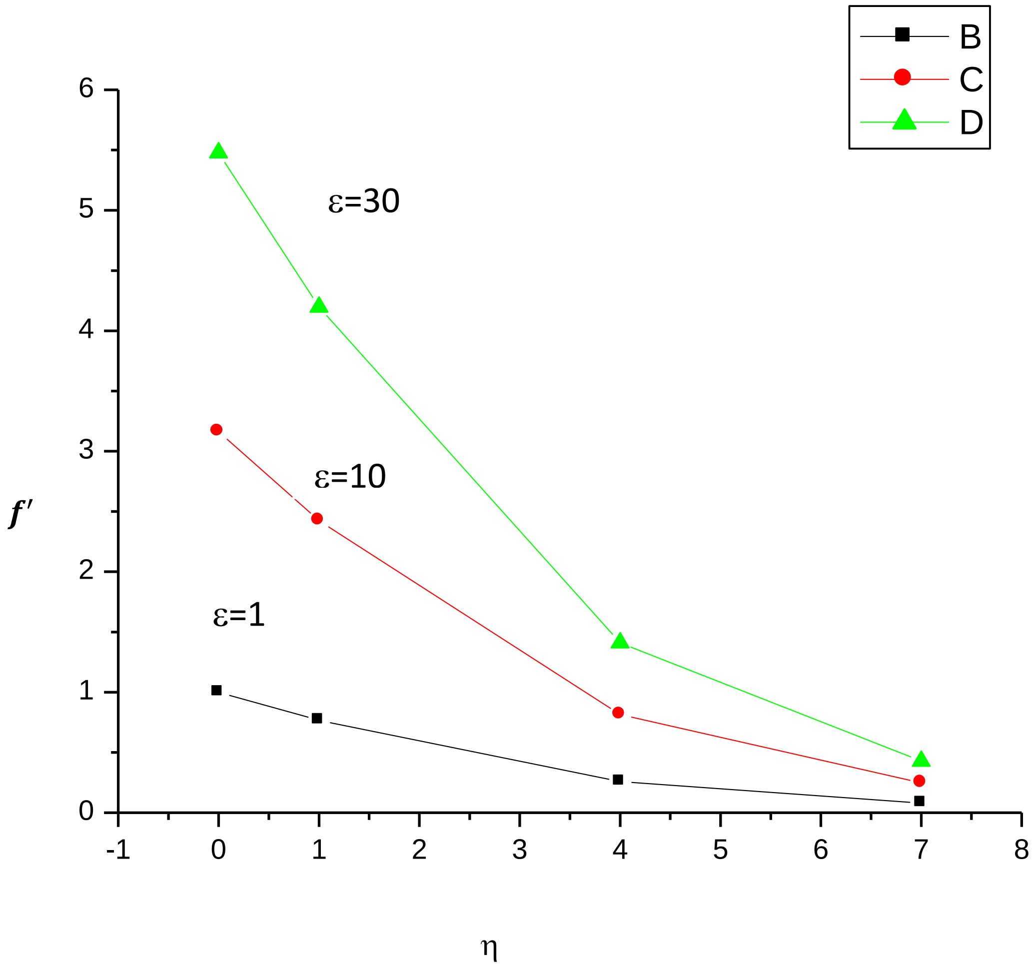


Fig.- 2 Variation of η with $f' = (\varepsilon\,\theta)^{1/n}$ for a value of $\varepsilon = 1, 10, 30$ *and* $n = 2$.

Table - 3

Variation of η with $f' = (\varepsilon\,\theta)^{1/n}$ for a value of $\varepsilon = 1, 10, 30$ and $n = 3$.

ε	η	$f' = (\varepsilon\,\theta)^{1/n}$
1	0.0000	1.0000
	1.0000	0.8374
	4.0000	0.4047
	7.0000	0.1826
10	0.0000	2.1544
	1.0000	1.8041
	4.0000	0.8720
	7.0000	0.3934
30	0.0000	3.1072
	1.0000	2.6019
	4.0000	1.2576
	7.0000	0.5674

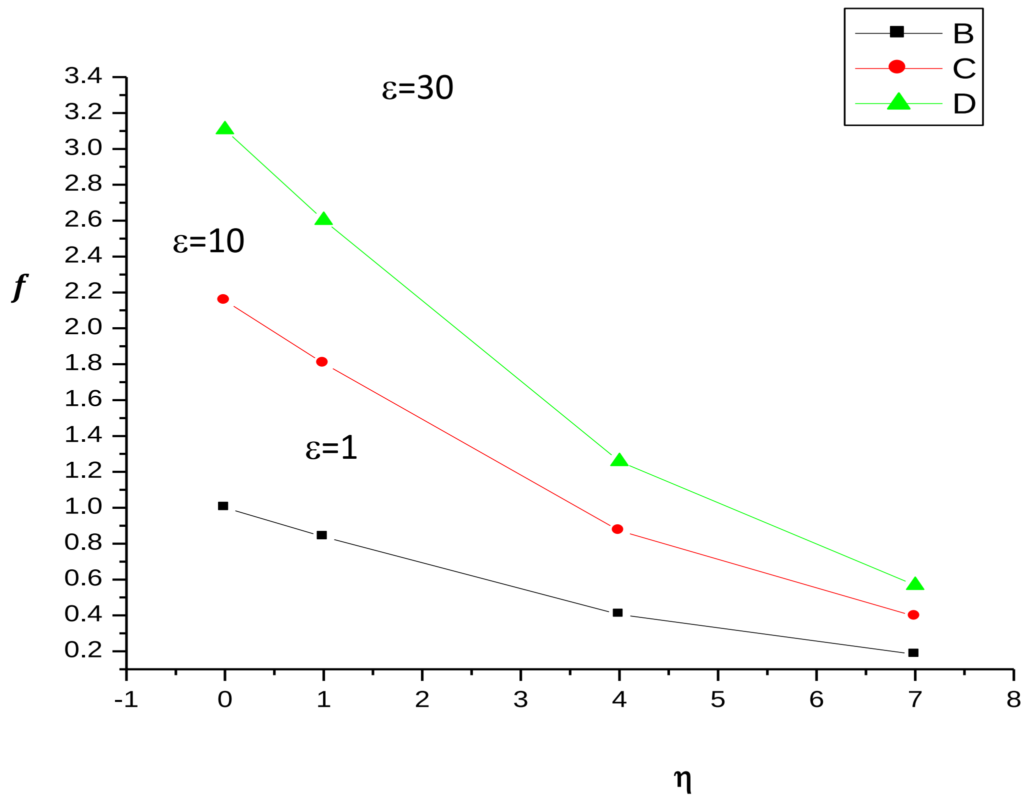


Fig - 3, Variation of η with $f' = (\varepsilon\, \theta)^{1/n}$ for a value of

$$\varepsilon = 1, 10, 30 \;\; and \;\; n = 3.$$

"4. Result and discussion"

The results of equation (25) are shown in the above three tables. The above three figures shows the dimensionless temperature f' versus the dimensionless similarity variable η for different n.

"5. Conclusion"

In this study the effects of porosity on free convection flow of Non-Newtonian fluids in an anisotropic porous medium was investigated. When n=1, the results is in agreement with a Newtonian fluid.

When n>1,

i. As porosity increases Nusselt number increases.

ii. As temperature variation becomes steeper, heat transfer rate increases and Nusselt number increases with increasing porosity.

"6. References"

1. Chen.H and Chen C, "Free convection flew of non Newtonian Embedded in a porous medium" Journal of Heat Transfer. Vol 110 PP. 257 – 260, 1988
2. Sekar.R. Vaidanathan.G Ramanathan.A. Ferro Convection in an anisotropic porous medium. Int. J. Engng.Sci 1996, Vol 34(4) PP. 399-405
3. Sengupta.T.K. 2004 Foundation of computational fluid dynamics (Hyderabad University press)
4. Vafai K. Et.al "An Experimental investigation of heat transfer in Variable porosity medium", ADME.J. Heat transfer, Vol 107, PP. 642-647, 1985
5. G.Degan, C.Akowanou, N.C.Awanou, " Transient natural convection of Non-Newtonian fluids about a vertical surface embedded in an anisotropic porous medium", I.J of Heat and Mass Transfer, 50 (2007),4629-4639.
6. Prakash Chandra and V.VSatyamurthy", Non-Darcian and Anisotropic Effects on Free Convection in a porous Enclosure", Transp porous Med, 90 (2011),301-320.
7. Rajendra.P, Chhabra, Jacques Comiti,Ivan Machad, " Flow of non-Newtonian fluids in fixed and fludised beds",Chemical Engineering Science, 56 (2001),1-27.
8. Han-Taw Chen, Cha'o-Kuang, Chen,"Free convection flow of non-Newtonian fluids along a vertical plate embedded in a porous medium, Journal of Heat Transfer, 110-1 (1988),257-260.
9. Gorla, Rama Subba Reddy and Kumari, Mahesh, "Free convection in non-Newtonian fluids along a horizontal plate in a porous medium", In: Heat and Mass Transfer, 39 (2) (2001), pp.101-106.
10. E.Kim, "Natural convection along a wavy vertical plate to non-Newtonian fluids" ,In: Heat and Mass Transfer, 40(13) (1997), pp.3069-3078.