International Journal of Engineering Research & Technology (IJERT)
ISSN: 2278-0181
Vol. 1 Issue 10, December- 2012

Fundamental Natural Frequencies Of Double-Walled Carbon With Different Boundary Conditions

S Sanyasi Naidu[1] Ch Varun[2] G Satyanarayana[3]

Abstract

In the present work fundamental natural frequencies of double walled carbon nanotubes (DWCNTs) are studied using Bubnov-Galerkin method. The main objective of this method is for quick and effective evaluation of fundamental frequencies. The inner and outer carbon nanotubes are modeled as two individual Euler-Bernoulli's elastic beams interacting each other by Vander Waals force. The fundamental natural frequencies of double walled carbon nanotubes (DWCNTs) are studied for three cases, i.e. (i) inner and outer carbon nanotubes (CNTs) with same boundary conditions, such as simply supported- simply supported and clamped-clamped (ii) inner and outer carbon nanotubes (CNTs) with different boundary conditions, such as simply supported- clamped, cantilever-clamped, and cantilever-simply supported and (iii) left and right ends of carbon nanotubes (CNTs) with different boundary conditions, such as simply supported- clamped, simply supported- free and clamped-free.

The fundamental natural frequencies are validating with those available in literature and observed a good agreement between them. The Effect of aspect ratio is studied for different boundary conditions. Out of all these boundary conditions clamped –clamped boundary condition has the highest natural frequencies. These observations may be useful for the designer to estimate the fundamental natural frequencies in each two series.

1 Introduction:

Afterthediscoveryofmulti-walledcarbonnanotubes **(**MWCNTs**)** in1991 by Iijima, hasstimulatedever**-**broaderresearchactivities in science and engineering devoted entirely to carbon nanostructures and their applications. This is due in large part to the combination of their expected structural perfection, small size, low density, high stiffness, high strength (the strength of the outer most shell of MWCNT is approximately 100 times greater than that of aluminum), and excellent electronic properties. As a result, carbon nanotube (CNT) may find use in a wide range of application in material reinforcement, field emission pane display, chemical senthesing, drug delivery, Nano electronicsetc

Carbon nanotubes (CNTs) have the most promising materials for nanoelectronics, nanodevices, and nanocomposites because of their unusual electronic properties and superior mechanical strength [1]. Many proposed applications and designs of CNTs are involved with aspect ratio about 10. Such examples include suspended crossing CNTs with spans about 20 nm, CNTs as single -electron transistors of length down to 20, MWNTs of aspect ratio around 20 as electrometers or building blocks in nanoelectronics, CNT-nanomechanical switches of aspect ratio around 10 and CNTs of aspect ratio about 10-25 as atomic force microscope (AFM).Owing to the hollow structure of CNTs, short CNTs are preferred in many cases to prevent undesirable kinking and buckling. Therefore, the vibrational behavior of short CNTs say, of aspect ratio between 10 and 30, is of practical significance.

Most CNTs to date have been synthesized with closed ends[1]. For applications of MWCNTs, both its ends can be restricted only on the outer tube. For example, in a nanoelectricalmechanicalsystem (NEMS), the small size and unique properties of CNTs suggest that they can be used in sensor devices with unprecedented sensitivity[2]. Other relevant issues to be clarified are the effects of differential boundary supports between the inner and outer tubes on the vibration of MWCNTs and boundary effects on transverse vibration devices composed of rods in microelectromechanicalsystems (MEMSs)[3]. It is expected that the differences of boundary condition, which are ignored in existing beam model, would play an important role in the vibration of a DWCNT when the vibrational modes at a resonant frequency between the two tubes are considered. Especially for short DWCNTs, some changes of boundary conditions may effect the vibrational modes more sensitively. For this reason, the relevance of the existing model, in which both tubes have the same boundary conditions for the vibration of DWCNTs, is questionable. To clarify this issue, free vibrations of DWCNTs with differential boundary supports between inner and outer tubes are studied in this work.

2 Analysis:

The governing differential equations for free vibration of the DWCNTs are

$$c_1(w_2 - w_1) = EI_1 \frac{\partial^4 w_1}{\partial x^4} + \rho A_1 \frac{\partial^2 w_1}{\partial t^2}$$

$$-c_1(w_2 - w_1) = EI_2 \frac{\partial^4 w_2}{\partial x^4} + \rho A_2 \frac{\partial^2 w_2}{\partial t^2} \qquad (1)$$

Where x is the axial coordinate, t is the time, $w_j(x,t)$ the transverse displacement, I_j the moment of inertia and A_j the cross-sectional area of the j^{th} nanotube; the indexes $j = 1,2$ denote the inner and outer nanotube, respectively.

The exact solution for various boundary conditions were considered by Xu et at.[4,5,6]. Their derivation necessitates numerical evaluation of 8x8 determinant and attendant cumbersome numerical analysis. Therefore the expressions for natural frequencies are obtained in this work by approximate method areexplained in the following sections.

3 Case: 1 both inner and outer nanotubes are with same boundary condition

3.1 Simply Supported DWCNTs: Polynomial Approximate Solution

Here transverse displacement is consider as $w = D\varphi(\xi)\sin(\omega t)$ (2)

Where, $\varphi(\xi)$ is a coordinate function and $\xi = x/L$ is a non-dimensional axial coordinate.

The coordinate function depends upon the boundary conditions of the carbon nanotubes (CNTs). For simply supported: at left end $\xi = -1$, deflection and bending moment are zero (i.e. $w = 0, \frac{\partial^2 w}{\partial \xi^2} = 0$) and at right end $\xi = 1$, deflection and bending moment are zero (i.e. $w = 0, \frac{\partial^2 w}{\partial \xi^2} = 0$). We have to select the degree of coordinate function equal to number of independent boundary conditions plus one. So the coordinate function for this boundary condition is $\varphi = \xi^5 + a\xi^4 + b\xi^3 + c\xi^2 + d\xi$. The boundary conditions are applied to the coordinate function and found that $a = 0$, $b = \frac{-10}{3}$, $c = 0$, $d = \frac{7}{3}$. Then the coordinate function becomes $\varphi = 3\xi^5 - 10\xi^3 + 7\xi$ (3)

Now the displacements consider as follows:

$w_1 = D_1 \varphi \sin(\omega t)$, $w_2 = D_2 \varphi \sin(\omega t)$ (4)

Substitute the expressions (4) into governing differential eq. (1), and multiplying the result of the substitution by φ and integrating over the length of the beam. The following two equations are obtained in D_1 and D_2 as

$$(-L^4 \rho A_1 \omega^2 + L^4 c_1 + 99EI_1)D_1 + (-L^4 c_1)D_2 = 0$$

$$(-L^4 c_1)D_1 + (-L^4 \rho A_2 \omega^2 + L^4 c_1 + 99EI_2)D_2 = 0 \qquad (5)$$

We demand the determinant

$$\begin{vmatrix} (-L^4 \rho A_1 \omega^2 + L^4 c_1 + 99EI_1 & -L^4 c_1 \\ -L^4 c_1 & -L^4 \rho A_2 \omega^2 + L^4 c_1 + 99EI_2 \end{vmatrix} \qquad (6)$$

to vanish . This leads to the frequency equation as given below

$$L^8 \rho^2 A_1 A_2 \omega^4 + (99L^4 A_1 EI_2 - L^8 A_1 c_1 - 99EI_1 L^4 A_2 - L^8 c_1 A_2)\rho\omega^2 + 99L^4 c_1 EI_2 + 99EI_1 L^4 c_1$$

$$+ 9801E^2 I_1 I_2 = 0 \; (7)$$

With roots $\omega_{1,1}^2 = [\ L^4 A_1 c_1 + L^4 c_1 A_2 + 99A_1 EI_2 + 99EI_1 A_2 - (L^8 A_1^2 c_1^2 + 2L^8 A_1 c_1^2 A_2 + 198L^4 A_1^2 c_1 EI_2$

$- 198L^4 A_1 c_1 EI_1 A_2 + L^8 c_1^2 A_2^2 - 198L^4 c_1 A_2 A_1 EI_2 + 198L^4 c_1 A_2^2 EI_1 + 9801A_1^2 E^2 I_2^2$

$- 19602A_1 E^2 I_2 I_1 A_2 + 9801E^2 A_2^2 I_1^2)^{1/2}]/2L^4 \rho A_1 A_2$

$\omega_{2,1}^2 = [\ L^4 A_1 c_1 + L^4 c_1 A_2 + 99A_1 EI_2 + 99EI_1 A_2 + (L^8 A_1^2 c_1^2 + 2L^8 A_1 c_1^2 A_2 + 198L^4 A_1^2 c_1 EI_2$

$- 198L^4 A_1 c_1 EI_1 A_2 + L^8 c_1^2 A_2^2 - 198L^4 c_1 A_2 A_1 EI_2 + 198L^4 c_1 A_2^2 EI_1 + 9801A_1^2 E^2 I_2^2$

$- 19602A_1 E^2 I_2 I_1 A_2 + 9801E^2 A_2^2 I_1^2)^{1/2}]/2L^4 \rho A_1 A_2$ (8)

3.2 Clamped DWCNTs:

The above procedure is repeated with $\varphi = 1 - \cos(2\pi\xi)$, which is satisfying the clamped boundary condition. The frequency equation as given below

$$9L^8 \rho^2 A_1 A_2 \omega^4 + (\frac{-23378}{5} L^4 A_1 EI_2 - 9L^8 A_1 c_1 - \frac{23378}{5} EI_1 L^4 A_2 - 9L^8 c_1 A_2)\rho\omega^2$$

$$+ \frac{23278}{5} L^4 c_1 EI_2 + \frac{23278}{5} EI_1 L^4 c_1 + 2429100E^2 I_1 I_2 = 0$$

With roots

$$\omega_{1,1}^2 = [\ \frac{1}{2}L^4 A_1 c_1 + \frac{1}{2}L^4 c_1 A_2 + \frac{8572}{33} A_1 EI_2 + \frac{8572}{33} EI_1 A_2 - (\frac{1}{4}L^8 A_1^2 c_1^2 + \frac{1}{2}L^8 A_1 c_1^2 A_2$$

$$\frac{8572}{33}L^4 A_1^2 c_1 EI_2 - \frac{85733}{33}L^4 A_1 c_1 EI_1 A_2 + \frac{1}{4}L^8 c_1^2 A_2^2 - \frac{8572}{33}L^4 c_1 A_2 A_1 EI_2$$
$$+\frac{8572}{33}L^4 c_1 A_2^2 EI_1 + 67474 A_1^2 E^2 I_2^2 - 134950 A_1 E^2 I_2 I_1 A_2 + 67474 E^2 A_2^2 I_1^2)^{1/2}]/L^4 \rho A_1 A_2$$

$$\omega_{2,1}^2 = [\ \frac{1}{2}L^4 A_1 c_1 + \frac{1}{2}L^4 c_1 A_2 + \frac{8572}{33} A_1 EI_2 + \frac{8572}{33} EI_1 A_2 + (\frac{1}{4}L^8 A_1^2 c_1^2 + \frac{1}{2}L^8 A_1 c_1^2 A_2$$

$$\frac{8572}{33}L^4 A_1^2 c_1 EI_2 - \frac{85733}{33}L^4 A_1 c_1 EI_1 A_2 + \frac{1}{4}L^8 c_1^2 A_2^2 - \frac{8572}{33}L^4 c_1 A_2 A_1 EI_2$$
$$+\frac{8572}{33}L^4 c_1 A_2^2 EI_1 + 67474 A_1^2 E^2 I_2^2 - 134950 A_1 E^2 I_2 I_1 A_2 + 67474 E^2 A_2^2 I_1^2)^{1/2}]/L^4 \rho A_1 A_2$$

4 Case 2: Both inner and outer nanotubes with different boundary conditions

4.1 Simply supported-clamped DWCNT: Approximate Solution

Here transverse displacement as $w = D\varphi(\xi)\sin(\omega t)$ (9)

Where, $\varphi(\xi)$ is a coordinate function.

The coordinate function is depends upon the boundary conditions of the carbon nanotubes (CNTs). For simply supported-clamped, at left end $\xi = -1$ transverse displacement, slope and bending moment are zero (i.e. $w = 0$, $\frac{\partial w}{\partial \xi} = 0$, $\frac{\partial^2 w}{\partial \xi^2} = 0$) and at right end $\xi = 1$,transverse displacement, slope and bending moment are zero (i.e. $w = 0$, $\frac{\partial w}{\partial \xi} = 0$, $\frac{\partial^2 w}{\partial \xi^2} = 0$).We have to select the degree of coordinate function as equal to number of independent boundary conditions plus one. So the coordinate functionfor this boundary condition $\varphi = \xi^7 + a\xi^6 + b\xi^5 + c\xi^4 + d\xi^3 + e\xi^2 + f\xi$. The boundary conditions are applied to the coordinate function and find that $a = 0$, $b = -3$, $c = 0$, $d = 3$, $e = 0$, $f = -1$.Then the coordinate function is $\varphi = \xi^7 - 3\xi^5 + 3\xi^3 - 1$ (10)

We now the displacements as follows:

$w_1 = D_1 \varphi \sin(\omega t)$, $w_2 = D_2 \varphi \sin(\omega t)$ (11)

We substitute the expressions (11) into governing differential eq. (1), and multiplying the result of the substitution by φ and integrating over the length of the beam, the following two equations are obtain in D_1 and D_2: $(-L^4 \rho A_1 \omega^2 + L^4 c_1 + 241 EI_1) D_1 + (-L^4 c_1) D_2 = 0$

$$(-L^4 c_1) D_1 + (-L^4 \rho A_2 \omega^2 + L^4 c_1 + 241 EI_2) D_2 = 0 \quad (12)$$

We demand the determinant

$$\begin{vmatrix} (-L^4\rho A_1\omega^2 + L^4 c_1 + 241EI_1 & -L^4 c_1 \\ -L^4 c_1 & -L^4\rho A_2\omega^2 + L^4 c_1 + 241EI_2 \end{vmatrix} \quad (13)$$

to vanish . This leads to the frequency equation

$$L^8\rho^2 A_1 A_2\omega^4 + (241L^4 A_1 EI_2 - L^8 A_1 c_1 - 241EI_1 L^4 A_2 - L^8 c_1 A_2)\rho\omega^2 + 241L^4 c_1 EI_2$$

$$+ 241EI_1 L^4 c_1 + 58081E^2 I_1 I_2 = 0 \quad (14)$$

With roots

$$\omega_{1,1}^2 = [\ L^4 A_1 c_1 + L^4 c_1 A_2 + 241A_1 EI_2 + 241EI_1 A_2 - (L^8 A_1^2 c_1^2 + 2L^8 A_1 c_1^2 A_2 + 482L^4 A_1^2 c_1 EI_2$$

$$- 482L^4 A_1 c_1 EI_1 A_2 + L^8 c_1^2 A_2^2 - 482L^4 c_1 A_2 A_1 EI_2 + 482L^4 c_1 A_2^2 EI_1 + 58081A_1^2 E^2 I_2^2$$

$$-116162A_1 E^2 I_2 I_1 A_2 + 58081E^2 A_2^2 I_1^2)^{1/2}]/2L^4\rho A_1 A_2$$

$$\omega_{2,1}^2 = [\ L^4 A_1 c_1 + L^4 c_1 A_2 + 241A_1 EI_2 + 241EI_1 A_2 - (L^8 A_1^2 c_1^2 + 2L^8 A_1 c_1^2 A_2 + 482L^4 A_1^2 c_1 EI_2$$

$$- 482L^4 A_1 c_1 EI_1 A_2 + L^8 c_1^2 A_2^2 - 482L^4 c_1 A_2 A_1 EI_2 + 482L^4 c_1 A_2^2 EI_1 + 58081A_1^2 E^2 I_2^2$$

$$-116162A_1 E^2 I_2 I_1 A_2 + 58081E^2 A_2^2 I_1^2)^{1/2}]/2L^4\rho A_1 A_2 \quad (15)$$

4.2 Cantilever-Clamped DWCNTs

The above procedure is repeated with $\varphi = \xi^7 - 2\xi^6 - \xi^5 + 4\xi^4 - \xi^3 - 2\xi^2 + \xi$, which is satisfies the clamped boundary condition. The frequency equation as given below

$$L^8\rho^2 A_1 A_2\omega^4 + (448L^4 A_1 EI_2 - L^8 A_1 c_1 - 448EI_1 L^4 A_2 - L^8 c_1 A_2)\rho\omega^2 + 448L^4 c_1 EI_2$$

$$448EI_1 L^4 c_1 + 200704E^2 I_1 I_2 = 0 \quad (16)$$

With roots

$$\omega_{1,1}^2 = [\ L^4 A_1 c_1 + L^4 c_1 A_2 + 448A_1 EI_2 + 448EI_1 A_2 - (L^8 A_1^2 c_1^2 + 2L^8 A_1 c_1^2 A_2 + 896L^4 A_1^2 c_1 EI_2$$

$$- 896L^4 A_1 c_1 EI_1 A_2 + L^8 c_1^2 A_2^2 - 896L^4 c_1 A_2 A_1 EI_2 + 896L^4 c_1 A_2^2 EI_1 + 200704A_1^2 E^2 I_2^2$$

$$- 401408A_1 E^2 I_2 I_1 A_2 + 200704E^2 A_2^2 I_1^2)^{1/2}]/2L^4\rho A_1 A_2$$

$$\omega_{2,1}^2 = [\ L^4 A_1 c_1 + L^4 c_1 A_2 + 448A_1 EI_2 + 448EI_1 A_2 + (L^8 A_1^2 c_1^2 + 2L^8 A_1 c_1^2 A_2 + 896L^4 A_1^2 c_1 EI_2$$

$$- 896L^4 A_1 c_1 EI_1 A_2 + L^8 c_1^2 A_2^2 - 896L^4 c_1 A_2 A_1 EI_2 + 896L^4 c_1 A_2^2 EI_1 + 200704A_1^2 E^2 I_2^2$$

$$-401408A_1E^2I_2I_1A_2+200704E^2A_2^2I_1^2)^{1/2}]/2L^4\rho A_1A_2 \qquad (17)$$

4.3 Cantilever-Simply Supported DWCNTs

The above procedure is repeated with $\varphi=8\xi^7-5\xi^6-27\xi^5+14\xi^4+34\xi^3-9\xi^2+15\xi$, which is satisfies the clamped boundary condition. The frequency equation as given below

$$L^8\rho^2A_1A_2\omega^4+(146L^4A_1EI_2-L^8A_1c_1-146EI_1L^4A_2-L^8c_1A_2)\rho\omega^2+146L^4c_1EI_2$$

$$146EI_1L^4c_1+21316E^2I_1I_2=0 \qquad (18)$$

With roots

$$\omega_{1,1}^2=[\;L^4A_1c_1+L^4c_1A_2+146A_1EI_2+146EI_1A_2-(L^8A_1^2c_1^2+2L^8A_1c_1^2A_2+292L^4A_1^2c_1EI_2$$

$$-292L^4A_1c_1EI_1A_2+L^8c_1^2A_2^2-292L^4c_1A_2A_1EI_2+292L^4c_1A_2^2EI_1+21316A_1^2E^2I_2^2$$

$$-42632A_1E^2I_2I_1A_2+21316E^2A_2^2I_1^2)^{1/2}]/2L^4\rho A_1A_2$$

$$\omega_{2,1}^2=[\;L^4A_1c_1+L^4c_1A_2+146A_1EI_2+146EI_1A_2+(L^8A_1^2c_1^2+2L^8A_1c_1^2A_2+292L^4A_1^2c_1EI_2$$

$$-292L^4A_1c_1EI_1A_2+L^8c_1^2A_2^2-292L^4c_1A_2A_1EI_2+292L^4c_1A_2^2EI_1+21316A_1^2E^2I_2^2$$

$$-42632A_1E^2I_2I_1A_2+21316E^2A_2^2I_1^2)^{1/2}]/2L^4\rho A_1A_2 \qquad (19)$$

5 Case:3 Both left and right ends of nanotubes with different boundary condition

5.*1 Simply supported-Clamped DWCNT:* Approximate Solution

Here consider the transverse displacement is considered as $w=D\varphi(\xi)\sin(\omega t)$ (20)

Where, $\varphi(\xi)$ is a coordinate function.

The coordinate function depending upon the boundary conditions of the carbon nanotubes (CNTs). For simply supported-clamped boundary condition at left end $\xi=-1$ transverse displacement and bending moment are zero (i.e. $w=0,\frac{\partial^2 w}{\partial\xi^2}=0$) and at right end $\xi=1$ transverse displacement and deflection are zero (i.e. $w=0,\frac{\partial w}{\partial\xi}=0$).We have to select the degree of coordinate function is equal to number of independent boundary conditions plus one. So the coordinate function for this boundary condition is $\varphi=\xi^5+a\xi^4+b\xi^3+c\xi^2+d\xi$. The boundary conditions are applied to the coordinate function and found that $a=\frac{1}{2},b=\frac{-5}{2},c=\frac{-1}{2},d=\frac{3}{2}$. Then the coordinate function becomes

$$\varphi=2\xi^5+\xi^4-5\xi^3-\xi^2+3\xi \qquad (21)$$

Now the displacements considered are as follows $w_1 = D_1\varphi\sin(\omega t)$,, $w_2 = D_2\varphi\sin(\omega t)$ (22)

We substitute the expressions (18) into governing differential Eq. (1), and multiplying the result of the substitution by φ and integrating over the length of the beam, the following two equations for D_1 and D_2: $(-L^4\rho A_1\omega^2 + L^4c_1 + 145EI_1)D_1 + (-L^4c_1)D_2 = 0$

$(-L^4c_1)D_1 + (-L^4\rho A_2\,\omega^2 + L^4c_1 + 145EI_2)D_2 = 0$ (23)

We demand the determinant

$$\begin{vmatrix} (-L^4\rho A_1\omega^2 + L^4c_1 + 145EI_1 & -L^4c_1 \\ -L^4c_1 & -L^4\rho A_2\omega^2 + L^4c_1 + 145EI_2 \end{vmatrix} \quad (24)$$

to vanish . This leads to the frequency equation

$$L^8\rho^2A_1A_2\omega^4 + (145L^4A_1EI_2 - L^8A_1c_1 - 145EI_1L^4A_2 - L^8c_1A_2)\rho\omega^2 + 145L^4c_1EI_2$$

$$145EI_1L^4c_1 + 21025E^2I_1I_2 = 0 \quad (25)$$

With roots

$$\omega_{1,1}^2 = [\ L^4A_1c_1 + L^4c_1A_2 + 145A_1EI_2 + 145EI_1A_2 - (L^8A_1^2c_1^2 + 2L^8A_1c_1^2A_2 + 290L^4A_1^2c_1EI_2$$

$$-290L^4A_1c_1EI_1A_2 + L^8c_1^2A_2^2 - 290L^4c_1A_2A_1EI_2 + 290L^4c_1A_2^2EI_1 + 21025A_1^2E^2I_2^2$$

$$-42050A_1E^2I_2I_1A_2 + 21025E^2A_2^2I_1^2)^{1/2}]/2L^4\rho A_1A_2$$

$$\omega_{2,1}^2 = [\ L^4A_1c_1 + L^4c_1A_2 + 145A_1EI_2 + 145EI_1A_2 + (L^8A_1^2c_1^2 + 2L^8A_1c_1^2A_2 + 290L^4A_1^2c_1EI_2$$

$$-290L^4A_1c_1EI_1A_2 + L^8c_1^2A_2^2 - 290L^4c_1A_2A_1EI_2 + 290L^4c_1A_2^2EI_1 + 21025A_1^2E^2I_2^2$$

$$-42050A_1E^2I_2I_1A_2 + 21025E^2A_2^2I_1^2)^{1/2}]/2L^4\rho A_1A_2 \quad (26)$$

5.2 Clamped-Free DWCNTs:

The above procedure is repeated with $\varphi = 17\xi^5 - 36\xi^4 - 26\xi^3 + 124\xi^2 + 97\xi$, which is satisfies the Clamped-Free boundary condition. The frequency equation as given below

$$L^8\rho^2A_1A_2\omega^4 + (7L^4A_1EI_2 - L^8A_1c_1 - 7EI_1L^4A_2 - L^8c_1A_2)\rho\omega^2 + 7L^4c_1EI_2 + 7EI_1L^4c_1$$

$$+49E^2I_1I_2 = 0 \quad (27)$$

With roots

$$\omega_{1,1}^2 = [\ L^4A_1c_1 + L^4c_1A_2 + 7A_1EI_2 + 7EI_1A_2 - (L^8A_1^2c_1^2 + 2L^8A_1c_1^2A_2 + 14L^4A_1^2c_1EI_2$$

ISSN: 2278-0181
Vol. 1 Issue 10, December- 2012

$$-14L^4A_1c_1EI_1A_2 + L^8c_1^2A_2^2 - 14L^4c_1A_2A_1EI_2 + 14L^4c_1A_2^2EI_1 + 49A_1^2E^2I_2^2$$

$$-98A_1E^2I_2I_1A_2 + 49E^2A_2^2I_1^2)^{1/2}]/2L^4\rho A_1A_2$$

$$\omega_{2,1}^2 = [\ L^4A_1c_1 + L^4c_1A_2 + 7A_1EI_2 + 7EI_1A_2 + (L^8A_1^2c_1^2 + 2L^8A_1c_1^2A_2 + 14L^4A_1^2c_1EI_2$$

$$-14L^4A_1c_1EI_1A_2 + L^8c_1^2A_2^2 - 14L^4c_1A_2A_1EI_2 + 14L^4c_1A_2^2EI_1 + 49A_1^2E^2I_2^2$$

$$-98A_1E^2I_2I_1A_2 + 49E^2A_2^2I_1^2)^{1/2}]/2L^4\rho A_1A_2 \qquad (28)$$

5.3 Simply Supported-Free DWCNTs:

The above procedure is repeated with $\varphi = 3\xi^5 - 5\xi^4 - 10\xi^3 + 30\xi^2 + 32\xi$, which is satisfies the Simply Supported-Free boundary condition. The frequency equation as given below

$$L^8\rho^2A_1A_2\omega^4 + (5L^4A_1EI_2 - L^8A_1c_1 - 5EI_1L^4A_2 - L^8c_1A_2)\rho\omega^2 + 5L^4c_1EI_2$$

$$5EI_1L^4c_1 + 25E^2I_1I_2 = 0 \qquad (29)$$

With roots

$$\omega_{1,1}^2 = [\ L^4A_1c_1 + L^4c_1A_2 + 5A_1EI_2 + 5EI_1A_2 - (L^8A_1^2c_1^2 + 2L^8A_1c_1^2A_2 + 10L^4A_1^2c_1EI_2$$

$$-10L^4A_1c_1EI_1A_2 + L^8c_1^2A_2^2 - 10L^4c_1A_2A_1EI_2 + 10L^4c_1A_2^2EI_1 + 25A_1^2E^2I_2^2$$

$$-50A_1E^2I_2I_1A_2 + 25E^2A_2^2I_1^2)^{1/2}]/2L^4\rho A_1A_2$$

$$\omega_{2,1}^2 = [\ L^4A_1c_1 + L^4c_1A_2 + 5A_1EI_2 + 5EI_1A_2 + (L^8A_1^2c_1^2 + 2L^8A_1c_1^2A_2 + 10L^4A_1^2c_1EI_2$$

$$-10L^4A_1c_1EI_1A_2 + L^8c_1^2A_2^2 - 10L^4c_1A_2A_1EI_2 + 10L^4c_1A_2^2EI_1 + 25A_1^2E^2I_2^2$$

$$-50A_1E^2I_2I_1A_2 + 25E^2A_2^2I_1^2)^{1/2}]/2L^4\rho A_1A_2 \qquad (30)$$

6 For numerical analysis the following data is taken for DWCNTs

Young's modulus (E) = 1 TPa

Mass density (ρ) = 2.3 g/ cm^3

Vander Waals interlayer interaction coefficient (c_1) = 71.11GPa

Inner radius (R_1) = 0.35 nm

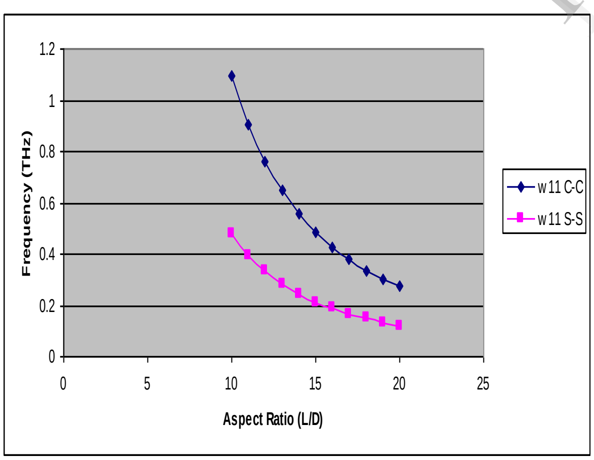
Outer radius (R_2) = 0.70 nm

Wall thickness each nanotube = 0.34 nm

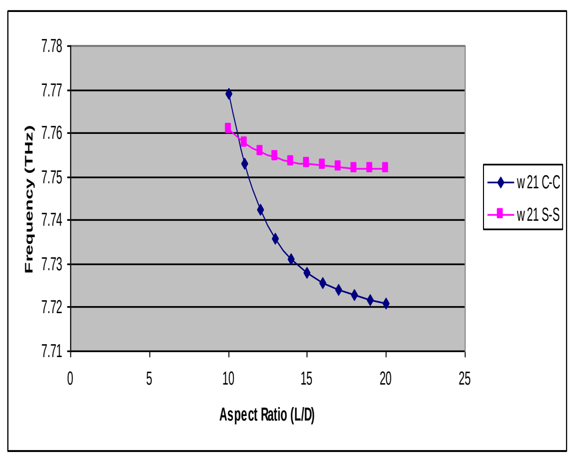
7 Results and Discussions:

Table:1 First natural frequenciesof DWCNTs with same boundary conditions:

S. No.	Aspect ratio	S-S		C-C	
		$\omega_{1,1}$ (THz)	$\omega_{2,1}$ (THz)	$\omega_{1,1}$ (THz)	$\omega_{2,1}$ (THz)
1	10	0.4794	7.7609	1.0919	7.7692
2	11	0.3963	7.7578	0.9029	7.7529
3	12	0.3330	7.7558	0.7590	7.7426
4	13	0.2838	7.7545	0.6469	7.7358
5	14	0.2447	7.7536	0.5579	7.7312
6	15	0.2131	7.7530	0.4860	7.7280
7	16	0.1873	7.7526	0.4272	7.7257
8	17	0.1659	7.7522	0.3785	7.7240
9	18	0.1480	7.7520	0.3376	7.7228
10	19	0.1329	7.7518	0.3030	7.7218
11	20	0.1199	7.7517	0.2735	7.7211



(a)

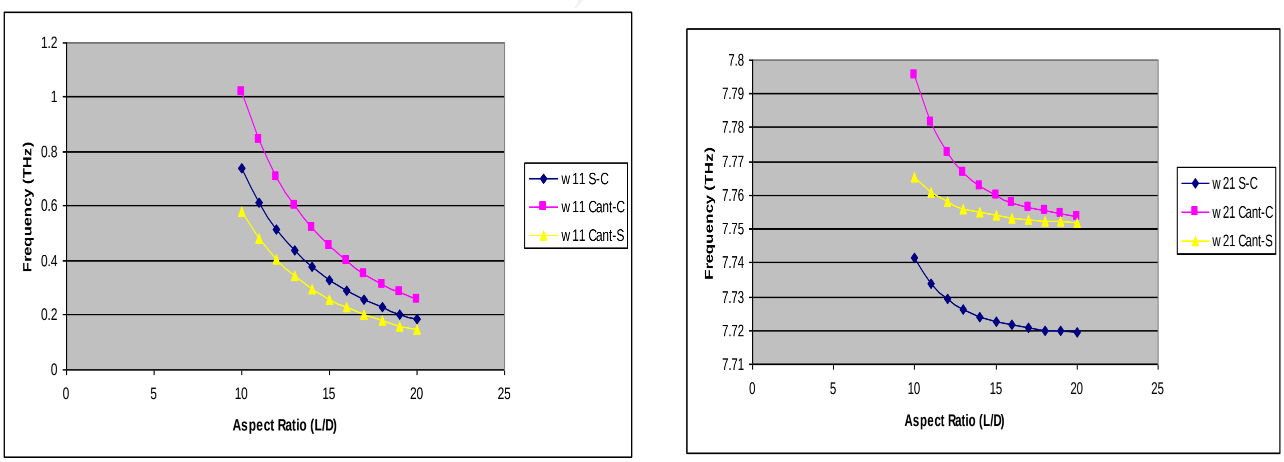


(b)

Fig.1 Variation of Co-axial and Non Co-axial Frequencies with aspect ratio with same boundary conditions

Table: 2 First natural frequencies of DWCNTs with different boundary conditions:

S. No.	Aspect ratio	S-C		Cant.-C		Cant.-S	
		$\omega_{1,1}$ (THz)	$\omega_{2,1}$ (THz)	$\omega_{1,1}$ (THz)	$\omega_{2,1}$ (THz)	$\omega_{1,1}$ (THz)	$\omega_{2,1}$ (THz)
1	10	0.7424	7.7413	1.0192	7.7956	0.5821	7.7655
2	11	0.6133	7.7339	0.8427	7.7815	0.4812	7.7609
3	12	0.5153	7.7292	0.7084	7.7725	0.4044	7.7580
4	13	0.4390	7.7261	0.6037	7.7666	0.3446	7.7561
5	14	0.3785	7.7240	0.5206	7.7626	0.2971	7.7548
6	15	0.3297	7.7225	0.4536	7.7598	0.2588	7.7539
7	16	0.2897	7.7215	0.3987	7.7578	0.2275	7.7533
8	17	0.2566	7.7207	0.3532	7.7564	0.2015	7.7528
9	18	0.2289	7.7201	0.3150	7.7553	0.1798	7.7524
10	19	0.2054	7.7197	0.2827	7.7545	0.1613	7.7522
11	20	0.1854	7.7193	0.2552	7.7538	0.1456	7.7520

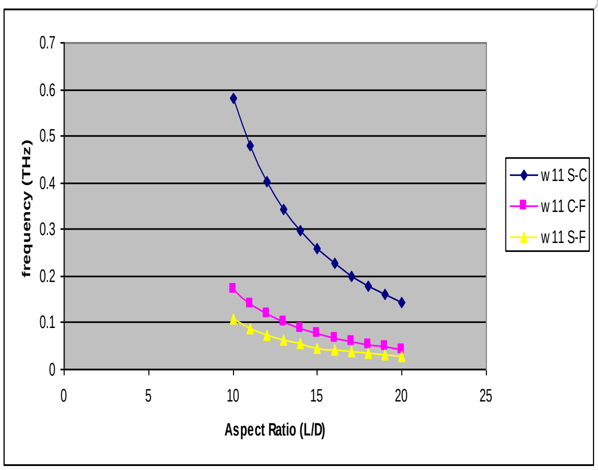


(a) ***(b)***

Fig.2 Variation of Co-axial and Non Co-axial Frequencies with aspect ratio with different boundary conditions

Table:3First natural frequencies of DWCNTs with different boundary conditions at left and right ends:

S. No.	Aspect ratio	S-C		S-F		C-F	
		$\omega_{1,1}$ (THz)	$\omega_{2,1}$ (THz)	$\omega_{1,1}$ (THz)	$\omega_{2,1}$ (THz)	$\omega_{1,1}$ (THz)	$\omega_{2,1}$ (THz)
1	10	0.5801	7.7654	0.1078	7.7516	0.1708	7.7518
2	11	0.4795	7.7609	0.0891	7.7514	0.1412	7.7515
3	12	0.4030	7.7580	0.0748	7.7513	0.1187	7.7514
4	13	0.3434	7.7561	0.0638	7.7512	0.1012	7.7513
5	14	0.2961	7.7548	0.0550	7.7512	0.0872	7.7512
6	15	0.2579	7.7539	0.0439	7.7512	0.0759	7.7512
7	16	0.2267	7.7533	0.0421	7.7511	0.0667	7.7512
8	17	0.2008	7.7528	0.0373	7.7511	0.0591	7.7511
9	18	0.1791	7.7524	0.0333	7.7511	0.0527	7.7511
10	19	0.1608	7.7522	0.0299	7.7511	0.0473	7.7511
11	20	0.1451	7.7520	0.0269	7.7511	0.0427	7.7511



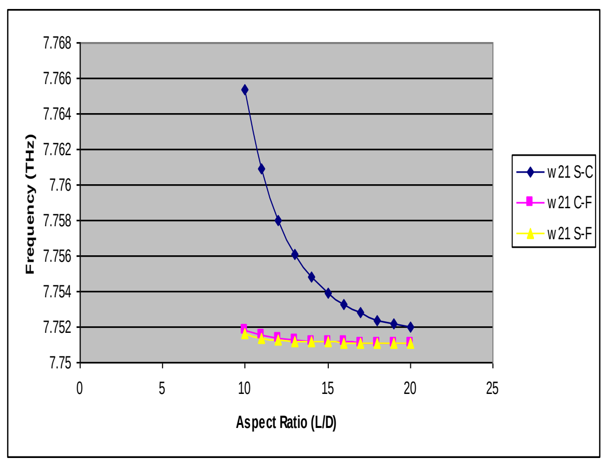


Fig.3 Variation of Co-axial and Non Co-axial Frequencies with aspect ratio with left and right ends of CNT's with different boundary conditions

7 Comparison with K.Y. xuet.al[5]:

K.Y. xu et.al[5] studied vibration of a double-walled carbon nanotube aroused by nonlinear interlayer van der Waals _vdW_forces with different boundary conditions for aspect ratio 10 and 20. The natural frequencies obtained by present method is compare with the with the natural frequencies K.Y. xu et.al[5]. The frequencies are tabulated below. It is observed that a good agreement between them. This shows the accuracy of the present method.

S.No	Aspect ratio (L/D)	Boundary condition	Present		K.Y. Xu et.al.	
			$\omega_{1,1}$ (THz)	$\omega_{2,1}$ (THz)	$\omega_{1,1}$ (THz)	$\omega_{2,1}$ (THz)
1	10	S-S	0.4794	7.7609	0.4	7.71
2		C-C	1.0919	7.7692	1.06	7.75
3		C-F	0.1708	7.7518	0.17	7.7
4	20	S-S	0.1199	7.7517	0.11	7.7
5		C-C	0.2735	7.7211	0.26	7.7
6		C-F	0.0427	7.7511	0.04	7.7

8 Comparison with Natsuki et.al.[7]:

Most recently, Natusuki et al. [7] analyzed free vibration characteristics of DWCNT. He calculated the natural frequencies of DWCNT; both ends simply supported .Specifically Natusuki et al. [7] adopted the following formula for the Vander Waals interaction coefficient c_1 :

$$c_1 = \frac{\pi\varepsilon R_1 R_2 \sigma^6}{\alpha^4}\left[\frac{1001\sigma^4}{3} H^{13} - \frac{11120\sigma^6}{9} H^7\right] \tag{31}$$

Where $$H^m = (R_1 + R_2)^{-m} \int_0^{\pi/2} \frac{1}{(1 - K\cos^2\theta)^{m/2}} \, d\theta \ , \ \ (m = 7{,}13) \tag{32}$$

And $$K = \frac{4R_1 R_2}{(R_1 + R_2)^2} \tag{33}$$

For evaluation of c_1, Natsuki used the following data

$\sigma = 0.34$ nm, $\varepsilon = 2.967$ mev, $\alpha = 0.142$ nm, $d_{in} = 4.8$ nm, and $d_{out} = 5.5$ nm, Yield c_1 = $1.474825922044788 \times 10^{11}$ whereasNatuski [1] informs that his value is 1.50×10^{11}, showing an excellent comparison.

According to Natsuki [7] the first natural frequency for $L = 10$ nm equals to 4.04 THz. The numerical data which is used by Natsuki is applied to our exact and approximate methods for simply supported boundary conditions of DWCNT, which yields 4.0339 THz and 4.0453 THz respectively, which shows the close agreement with Natsuki results.

S. No.		$\omega_{1,1}$ (THz)	% Error
1	Natsuki et.al	4.04	_
2	Trigonometric Solution	4.0339	0.15099
3	Polynomial Solution	4.0453	0.13118

***9 Conclusions*:**

This paper studies free vibration analysis of DWNTs modeled as elastic beams for different boundary conditions between inner and outer tubes. The results obtained are compared with those available in literature and some discussions are summarizedas follows.

(i) This method estimates the natural frequencies with minimum error.

(ii) Increasing the Aspect ratio decreasing the natural frequencies.

(iii)The effect of Aspect ratio for second series is very less when compare to the first series.

(iv) The natural frequencies of DWCNTs in Clamped boundary conditions showshighest than other**.**

Nomenclature:

A_1 =Cross-sectional area of inner nanotube (nm ^{2})

A_2 =Cross-sectional area of outer nanotube (nm ^{2})

c_1 =Vander Waals interlayer interaction coefficient (GPa)

D =Diameter of outer nanotube (nm)

E =Young's modulus (TPa)

I_1 =Moment of inertia of inner nanotube (nm ^{4})

I_2 =Moment of inertia of outer nanotube (nm ^{4})

L =Length of the nanotube (nm)

m = Mode number (m = 1, 2, 3 …..)

t = Time

x = Axial coordinate

w_1 = Transverse displacement of inner nanotube

w_2 = Transverse displacement of outer nanotube

ρ = Mass density (g/cm^3)

ξ = Non-dimensional axial coordinate

$\omega_{1,1}$ = Fundamental natural frequency of first series (THz)

$\omega_{2,1}$ = Fundamental natural frequency of second series (THz)

σ = Vander Waals radius (nm)

ε = The well depth of the Lennard-Jones potential (mev)

α = The carbon-carbon bond length (nm)

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