International Journal of Engineering Research & Technology (IJERT)
ISSN: 2278-0181
Vol. 3 Issue 9, September- 2014

Fuzzy Generalization for Optimal Operation of Processes with Large Operating Condition Changes

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***Abstract*— The proposed method in this paper deals with optimal operation. The objective is to handle optimality in case of variation in local approximation of optimization problem. If a disturbance moves the process far from the nominal point (large disturbance), the local model approximation used for the calculation of optimality by linearization of the nominal operating point and the local assumption of the quadratic cost function may be poor. In this paper the optimal operation is generalized to cover large changes in operating conditions by modelling with Takage-Sugeno (T-S) type fuzzy inference engine. For parameter tuning of this model, a hybrid GA (genetic algorithm)-LS (least square) algorithm is used. Also, the performance of the proposed method is studied in simulation of a nonquadratic objective function with large disturbance. The proposed fuzzy based method provided near optimal operation and showed very low loss for large magnitude of disturbances in compare with conventional method.**

Keywords*— *T-S fuzzy inference system; optimal operation; nonquadratic objective function; operating condition changes

I. Introduction

Fuzzy modelling formulates the system knowledge with rules in a transparent way to interpret and analysis so as to gain insights into the system being modeled. As stated by Zadeh [1] “The closer one looks at a real-world problem, the fuzzier becomes its solution”. According to his incompatibility principle [1], as the complexity of a system increases, human’s ability to make precise and significant statements about its behaviors decreases, until a threshold is reached beyond which precision and significance become impossible. Under this principle a modeling method with fuzzy numbers rather than crisp numbers was proposed. The main idea is that the key elements are classes of objects or concepts in which the membership of each element to the class is gradual (fuzzy) rather than sharp (crisp), while the classical theory of crisp sets can describe only the membership or non-membership of an item to a set. So, this precise logic of imprecision makes fuzzy inference based modeling a powerful method to generate answers based on information that are vague, ambiguous, qualitatively incomplete and imprecise in the systems present excess complexity arising out of the nonlinearities or even modeling difficulties intrinsic to the process. A great number of industrial applications via fuzzy model have been reported such as [2-7]. Among various fuzzy modelling approaches, the method based on Takagi-Sugeno (T-S) fuzzy model [8], gives a simple and effective way to solve complex nonlinear system problems.

The Takagi–Sugeno (T–S) fuzzy model can represent nonlinear system by decomposing the whole input space into several fuzzy sets and representing each output space with a linear equation. Such a model is capable of approximating a wide class of nonlinear systems. For the reason that it employs linear model in the consequent part, conventional linear system theory can be applied for the system analysis and synthesis accordingly. And hence, the T–S fuzzy models are becoming powerful engineering tools for modelling and control of complex systems.

There is a growing demand toward plant operation as close to optimality as possible because of rising energy prices, increasing competitions, and environmental demands. In many cases, steady state operation accounts for the largest part of the operating cost, and significant economical improvements can be achieved by operating the plant optimally at steady state. The null space method [9], with designing a function of some measured variable, makes a process to operate close to economically optimal steady state operation in the presence of disturbances and removes the need for solving an optimization problem online. This method is based on second order approximation of the objective function and holds its optimality for small deviation from the nominal optimum (small magnitude of the disturbance). It is globally optimal in cases where the sensitivity matrix (**F**) is not dependent on the operating point (disturbances), or, for a system with a quadratic cost objective and linear model equations. Our main concern in this paper is to extend the optimality to the process with varying optimal sensitivity matrix and large magnitude of disturbances with modeling optimal sensitivity matrix with a sugeno type fuzzy inference engine.

This paper is organized as follow. The next section describes basics of sugeno-type fuzzy modeling and then parameter tuning method. Section 4 will formulate the optimization problem. Section 5 will study the simulation of the fuzzy-based optimization method for a nonquadrartic problem with large disturbance and it is followed with result and discussion section and finally conclusion is in section 7.

II. TAKAGI–SUGENO FUZZY INFERENCE SYSTEM

Fuzzy sets are characterized by membership functions or degree of truth of ν in **A** that map R to the membership space.

$$\mathbf{A} = \{ (\nu, \mu(\nu)) \mid \nu \in R \} \tag{1}$$

The membership function is described by an arbitrary curve suitable from the point of view of simplicity, convenience, speed, and efficiency. When the membership space contains only 0 and 1, **A** is nonfuzzy and μ is a characteristic function of non-fuzzy set. The range of the membership function is a subset of the nonnegative real numbers. In this paper Gaussian membership functions is regarded as follow.

$$\mu(\nu) = \exp[-\frac{1}{2}(\frac{\nu - \alpha}{\sigma})] \tag{2}$$

Where α is the center of the membership function and σ is a constant related to spread of the membership function.

Structure of a Takagi–Sugeno fuzzy inference system is shown in Fig. 1. It is a model that maps characteristics of input data to input membership functions, input membership functions to rules, rules to output crisp functions, and output crisp functions to a single-valued output [10]. Generally, the process of formulating the mapping from a given input to an output using fuzzy logic is called the fuzzy inference.

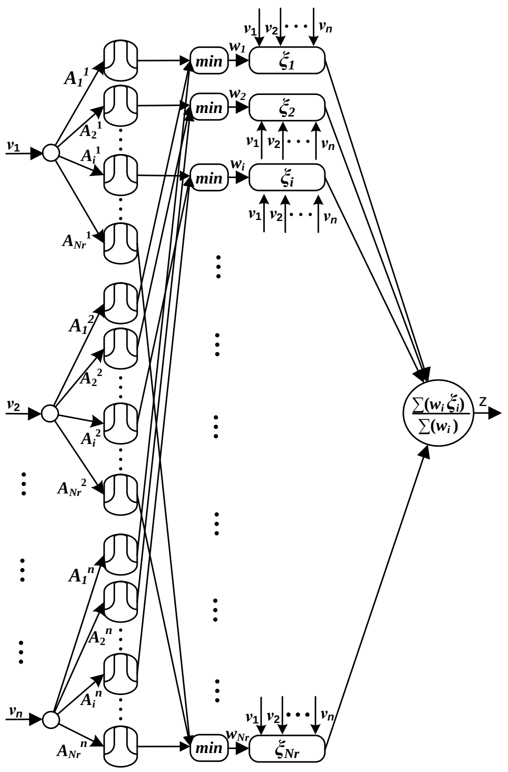

Fig. 1. Structure of T–S fuzzy model

In T-S fuzzy systems, the relationships between variables are represented by the means of fuzzy if-then rules as follow.

$$\text{Rule}_i : \text{If } \nu_1 \text{ is } \mathbf{A}_i^1 \text{ and } \nu_2 \text{ is } \mathbf{A}_i^2 \ \ldots \ \nu_n \text{ is } \mathbf{A}_i^n \text{ Then } z_i = \xi_i(\nu_1, \nu_2, \ldots, \nu_n) \tag{3}$$

Where $\boldsymbol{\nu} = [\nu_1, \nu_2, \ldots, \nu_n]$ are input variables, $\mathbf{A}^j_i$ $(1 \le j \le n)$ represent fuzzy set, z_i is the output of Rule i, and ξ_i is a crisp function. In the first-order sugeno model, a linear combination of input variables is considered as consequent crisp function as follow

$$\xi_i(\nu_1, \nu_2, \ldots, \nu_n) = b_i^0 + b_i^1 \nu_1 + b_i^2 \nu_2 + \ldots + b_i^n \nu_n \tag{4}$$

As such, each rule can be considered as a local linear model that will fuse together to produce an overall nonlinear output z. Given the input vector $\boldsymbol{\nu} = [\nu_1, \nu_2, \ldots, \nu_n]$, the model output $\tilde{y}$ is the weighted average of the individual rule outputs z_i $(1 \le i \le N_r)$ according to the following formula:

$$z = \frac{\sum_{i=1}^{N_r} w_i z_i}{\sum_{i=1}^{N_r} w_i} \tag{5}$$

Where N_r is the number of rules, and w_i, is the firing strength of *i*-th rule and calculated as follow:

$$w_i = \prod_{j=1}^{n} \mu_i^j(\nu_j) \tag{6}$$

Where ∏ denotes the fuzzy AND operator and in this paper the minimum operator is simply used and μ^j_i is the membership function corresponding to fuzzy set A^j_i.

III. PARAMETER TUNING

The parameter tuning emphasizes on tuning both the parameters of antecedent and consequent parts of fuzzy rules. These parameters are automatically tuned from numerical information (input-output data pairs from nonlinear model), using hybrid method of least square (LS) and genetic algorithm (GA). An input variable (disturbance) is changed instantly and, at the same time, the behavior of the output variables is collected. Then, the same procedure is performed for the other input variables and finally a data set for the identification of the fuzzy models is obtained by offline calculation from nonlinear model. Subsequently, the identification data set is randomly divided into training data set and test data set [10]. The training data set is used for tuning model parameters and these models are then validated through the test data.

Among the chromosomes in the population, some of them will be arbitrarily selected. This selection component in the GA guides the algorithm to the solution. One approach to guide the selection procedure which is used in this work is stochastic uniform selection function. This reproduction population will then be mated through crossover component. Crossover is the process of creating one or more offspring from the current population. In this work arithmetic crossover is used. The last component of the GA is mutation. Mutation rules apply random changes to individual to form next generation. This process is performed to avoid the algorithm from stuck at local minimum by introducing traits not in the original population and Gaussian mutation is applied in this work. The so called selection, crossover, mutation are the three main types of rules at each step to create the next generation from current population. In this work we use MATLAB software to implement genetic algorithm. For more information about genetic algorithm one can refer to MATLAB user's guide. Also, the coefficients of the crisp linear functions are constructed with least square estimation method. This GA-LS based procedure can be summarized as follow:

1. Generate random initial population.
2. Evaluate objective function for every chromosome.
   (2-i) Use LS method to define parameter of linear equations with desired membership functions parameters.
   (2-ii) Calculate mean squared error (MSE) for every chromosome.
3. Perform selection, crossover and mutation operation to produce new population.
4. Repeat steps 2 and 3 for a certain number of generations to get the best individual which will represent the best fuzzy model.

IV. Optimal Operation

The objective is to achieve optimal steady-state operation, where the degrees of freedom $\mathbf{u}$ are selected such that the scalar cost function J($\mathbf{u}$,$\mathbf{d}$) is minimized with respect to degrees of freedom by solving the following problem.

$$\min_{\mathbf{x},\mathbf{u}} J(\mathbf{x},\mathbf{u},\mathbf{d})$$

Subject to

$$f(\mathbf{x},\mathbf{u},\mathbf{d})=0$$
$$g(\mathbf{x},\mathbf{u},\mathbf{d})\le 0$$
$$\mathbf{x}\in R^{n_x},\mathbf{u}\in R^{n_u},\mathbf{d}\in R^{n_d} \tag{7}$$

Where $\mathbf{x}$, $\mathbf{u}$, and $\mathbf{d}$ are the states, inputs and disturbances, respectively; f is the set of equality constraints corresponding to the model equation; g is the set of inequality constraints that limits the operation. The null space method [9] deals with the optimal selection of the linear measurement combinations for quadratic approximation of Eq. (7) with the second-order Taylor expansion of the cost function J($\mathbf{u}$,$\mathbf{d}$)

$$\min \begin{bmatrix}\mathbf{u}\\ \mathbf{d}\end{bmatrix}^T \begin{bmatrix}\mathbf{J}_{uu} & \mathbf{J}_{ud}\\ \mathbf{J}_{du} & \mathbf{J}_{dd}\end{bmatrix}\begin{bmatrix}\mathbf{u}\\ \mathbf{d}\end{bmatrix} \tag{8}$$

Where $\mathbf{J}_{uu}=\frac{\partial^2 J}{\partial \mathbf{u}^2}$, $\mathbf{J}_{ud}=\mathbf{J}_{du}^T=\frac{\partial^2 J}{\partial \mathbf{u}\partial \mathbf{d}}$, and $\mathbf{J}_{dd}=\frac{\partial^2 J}{\partial \mathbf{d}^2}$

Consider that n_u is the number of independent unconstrained free variable $\mathbf{u}$, n_d is the number of independent disturbance $\mathbf{d}$, and n_y is the number of independent measurement $\mathbf{y}$. If $n_y \ge n_u + n_d$, it is possible to select combination matrix $\mathbf{H}$ in the left null space of $\mathbf{F}$ or

$$\mathbf{H} = \text{null}(\mathbf{F}^T) \tag{9}$$

where $\mathbf{F}$ is optimal sensitivity matrix evaluated at constant active constraints with the following definition

$$\mathbf{F}=\frac{\partial \mathbf{y}^{opt}}{\partial \mathbf{d}^T} \tag{10}$$

Also $\mathbf{F}$ could be calculated from linearized local model [11]

$$\mathbf{F}=-(\mathbf{G}^y\mathbf{J}_{uu}^{-1}\mathbf{J}_{ud}-\mathbf{G}_d^{\,y}) \tag{11}$$

However, in practice, it is usually easier to obtain $\mathbf{F}$ numerically. In the other word, for practical use it is more reliable to obtain $\mathbf{F}$ numerically from its definition Eq. (10), instead of deriving an analytical expression from Eq. (11) [12]. Moreover, providing analytical expression of $\mathbf{F}$ for entire operation space in a complex nonlinear chemical plants from explicit representation of the model equations is even a more difficult problem to be solved, but is readily to be solved numerically with a T-S fuzzy model.

V. Numerical Simulation

A nonquadratic objective function is considered. The selected cost function is

$$J=u^2-d^3u \tag{12}$$

Where the disturbance value is zero at the nominal operating point ($d^*=0$). The two available independent measurements are

$$y_1=0.9u+0.1d \tag{13}$$

And

$$y_2=0.5u-d \tag{14}$$

For one unconstrained degree of freedom (n_u =1) and with one disturbance (n_d =1), the minimum numbers of measurements is 2 ($n_y \ge n_u + n_d$). So with these two measurements, null space provides a linear combination of measurement which leads to near optimal solution locally. To find optimality, $\partial J/\partial u = 0$, is solved and optimal input is $u^{opt} = d^3/2$. So, the optimal outputs from Eq. (13) and Eq. (14) are

$$y_1^{opt}=0.45d^3+0.1d \tag{15}$$

And

$$y_2^{opt}=0.25d^3-d \tag{16}$$

Optimal sensitivity matrix is as follow

$$\mathbf{F}=\frac{\partial y^{opt}}{\partial d}=\begin{bmatrix}1.35d^2+0.1\\ 0.75d^2-1\end{bmatrix} \tag{17}$$

Therefore $\mathbf{F}$ depends on the disturbance value and is not constant with different magnitude of d. In accordance with Eq. (9), $\mathbf{H}$ is located in the left null space of optimal sensitivity matrix

$$\mathbf{HF}=[h_1 \quad h_2]\begin{bmatrix}f_1\\ f_2\end{bmatrix}=0 \tag{18}$$

Around the nominal operating point with nominal disturbance and with h_1=1, solving Eq. (18) concludes that h_2=0.1. The loss is defined as the difference between the actual cost and the optimal cost

$$L = J(\mathbf{u}, \mathbf{d}) - J(\mathbf{u}^{\mathbf{opt}}, \mathbf{d}) \quad (19)$$

Where for a given **d**, solving Eq. (7) gives $\mathbf{u}^{\mathbf{opt}}(\mathbf{d})$. The constant setpoint policy for optimal nominal point after some mathematical derivation results to the loss value of $L=d^6/4$.The loss is shown in Fig. 2 as a function of disturbance. It is clear that around the nominal operating point (zero disturbance), null space yields acceptable loss but for disturbances away from nominal point, its optimality degrades and loss increases significantly. In the rest of this section the proposed integrated null space and fuzzy is used to provide near optimal solution for disturbances away from nominal point.

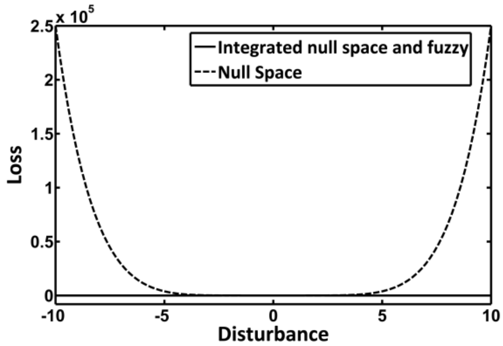

Fig. 2. Comparison of the loss values.

For developing integrated fuzzy and null space method input space is partitioned with 3 fuzzy sets. Since there is single disturbance in this problem (j=1). So, rule base consists of three rules for each element of optimal sensitivity matrix ($1 \le i \le 3$).

$$\text{Rule}_i : \text{If } d \text{ is } \mathbf{A}_i \quad \text{Then} \quad \xi_{\mathbf{i}}(k,1) = \mathbf{b}_{\mathbf{i}}^0(k,1) + \mathbf{b}_{\mathbf{i}}^1(k,1)\, d \quad (20)$$

The optimal sensitivity matrix contains 2 rows for two measurements ($1 \le k \le 2$) and one column for single disturbance. So, there are 6 rules with two constants in the linear function of the consequent part of each rule. Fig. 3 shows identification data of the fuzzy model for optimal sensitivity matrix. The parameters are tuned by hybrid GA-LS method. Tuning algorithm procedure and its specifications are in section (III). The fuzzy models are then validated through the test data and Table I demonstrates the validation results of the fuzzy model for the test data set. Small errors in Table I show that fuzzy models are close to nonlinear model. Fig. 4 shows the result membership function of input space. The parameters of membership function and rule consequents are in Table II.

Fig. 5 shows that with implementing the fuzzy based method, objective function goes near reoptimized trajectory of the problem and Fig 2 shows that the result loss is reduced to approximately 0 for entire region of the disturbance space which is considerably lower than conventional null space method.

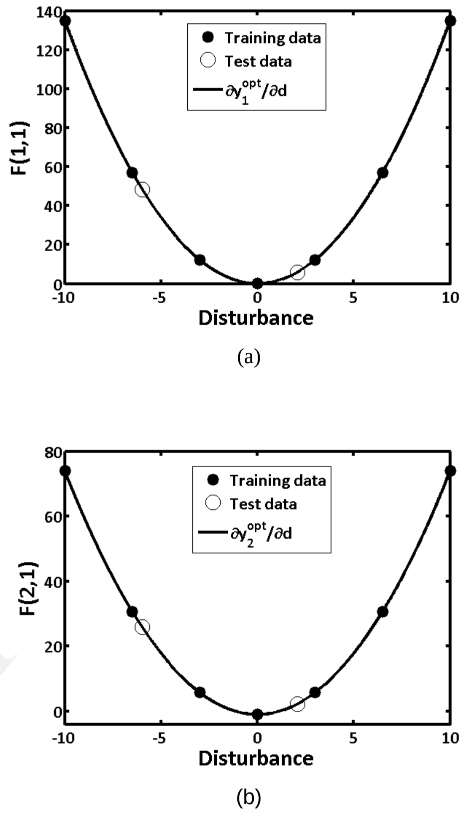

Fig. 3. Identification data of the fuzzy model for optimal sensitivity matrix. The training data is shown with ● mark and the test data is shown with ○ mark. (a) **F**(1,1) represents the element in the first row of **F** (b) **F**(2,1) represents the element in the second row of **F**

TABLE I. ERROR QUANTIFICATION FOR OUTPUT VARIABLES.

Outputs	MSE for output variables
F(1,1)	7.15×10^{-4}
F(2,1)	3.97×10^{-4}

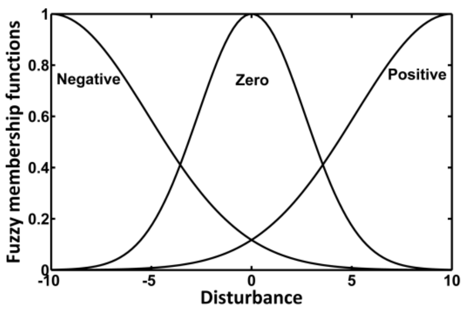

Fig. 4. The fuzzy domain partitions of the disturbance space

TABLE II. THE PARAMETERS OF FUZZY SYSTEM.

$Rule_i$	A_i			F(1,1)		F(2,1)	
i		α_i	σ_i	$b_i^0(1,1)$	$b_i^1(1,1)$	$b_i^0(2,1)$	$b_i^1(2,1)$
1	Negative	-10	4.83	-49.527	-17.596	-28.570	-9.775
2	Zero	0	2.67	17.349	0	8.5831	0
3	Positive	10	4.83	-49.527	17.596	-28.570	9.775

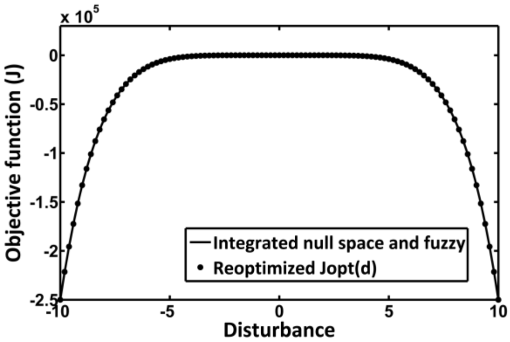

Fig. 5. Objective function value for the proposed integrated null space and fuzzy method.

VI. RESULTS AND DISCUSSIONS

Large value of loss of null space method in Fig. 2 shows that when large disturbance moves the process far from the nominal point, the local model approximation by linearization of the nominal operating point and local quadratic cost function (or approximation of the objective function by second order Taylor series) produces poor results. This is usually counteracted by re-optimization of the system with a real-time optimization algorithm. However, this can become complicated as it involves several difficult steps such as steady state detection, data estimation, reconciliation, and solving a large nonlinear optimization problem. However, it is clear from Fig. 2 that the proposed fuzzy based method has reduced the loss to approximately zero. This means that T-S fuzzy modelling of optimal sensitivity matrix makes it to meet large changes in disturbances.

An interesting characteristic of the proposed fuzzy based method is the ability to meet changes in operating condition. This is possible because the fuzzy model starts representing the process linearly through membership functions defined for the initial operating region. As the process operating conditions change, the operating point may move from the original region to the other regions governed by other linear function. By doing so, membership functions governing the process behavior, are the aggregation of the rules of operating regions, make possible the handling of the varying operating condition which leads to varying optimal sensitivity matrix issue. So, as Fig. 5 shows fuzzy based algorithm goes near reoptimized trajectory and Fig. 2 shows the loss value is reduced to 0 for entire region of the disturbance space which is considerably lower than conventional null space method.

VII. CONCLUSION

In this paper a T-S fuzzy inference engine was developed to generalize optimal operation to include operating condition away from nominal operating point where optimal sensitivity matrix will change. The advantages of proposed T-S fuzzy based method were demonstrated on a problem with nonquadratic objective function. The results showed near optimal operation in the case of successive and large disturbances in compare with conventional null space method.

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