# Generalized Semipre Regular Continuous and Irresolute Mappings in Intuitionistic Fuzzy Topological Space

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## Abstract

*In this paper, we introduce and study the notions of intuitionistic fuzzy generalized semipre regular continuous mappings and intuitionistic fuzzy generalized semipre regular irresolute mappings and study some of its properties in intuitionistic fuzzy topological spaces.*
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## 1. Introduction

The concept of fuzzy set [FS] was introduced by Zadeh [20] and later fuzzy topology was introduced by Chang [3] in 1967. By adding the degree of non membership to FS, Atanassov [1] proposed intuitionistic fuzzy set [IFS] using the notion of fuzzy sets. On the other hand Coker [4] introduced intuitionistic fuzzy topological spaces using the notion of intuitionistic fuzzy sets. In this paper we introduce intuitionistic fuzzy generalized semipre regular continuous mappings and intuitionistic fuzzy generalized semipre regular irresolute mappings and studied some of their basic properties.

## 2. Preliminaries

Throughout this paper, $(X, \tau)$ or X denotes the intuitionistic fuzzy topological spaces (briefly IFTS). For a subset A of X, the closure, the interior and the complement of A are denoted by cl(A), int(A) and $A^c$ respectively. We recall some basic definitions that are used in the sequel.

**Definition 2.1:** [1]

Let X be a non-empty fixed set. An intuitionistic fuzzy set (IFS in short) A in X is an object having the form $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle / x \in X\}$ where the functions $\mu_A : X \rightarrow [0,1]$ and $\nu_A : X \rightarrow [0,1]$ denote the degree of membership (namely $\mu_A(x)$) and the degree of nonmembership (namely $\nu_A(x)$) of each element $x \in X$ to the set A, respectively, and $0 \le \mu_A(x) + \nu_A(x) \le 1$ for each $x \in X$. Denote by IFS(X), the set of all intuitionistic fuzzy sets in X.

**Definition 2.2:** [1]

Let A and B be IFSs of the form $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle / x \in X\}$ and $B = \{\langle x, \mu_B(x), \nu_B(x)\rangle / x \in X\}$. Then

(i) $A \subseteq B$ if and only if $\mu_A(x) \le \mu_B(x)$ and $\nu_A(x) \ge \nu_B(x)$ for all $x \in X$,
(ii) $A = B$ if and only if $A \subseteq B$ and $B \subseteq A$,
(iii) $A^c = \{ \langle x, \nu_A(x), \mu_A(x)\rangle / x \in X \}$,
(iv) $A \cap B = \{ \langle x, \mu_A(x) \wedge \mu_B(x), \nu_A(x) \vee \nu_B(x)\rangle / x \in X \}$,
(v) $A \cup B = \{ \langle x, \mu_A(x) \vee \mu_B(x), \nu_A(x) \wedge \nu_B(x)\rangle / x \in X \}$.

For the sake of simplicity, we shall use the notation $A = \langle x, \mu_A, \nu_A\rangle$ instead of $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle / x \in X \}$. Also for the sake of simplicity, we shall use the notation $A = \langle x, (\mu_A, \mu_B), (\nu_A, \nu_B)\rangle$ instead of $A = \langle x, (A/\mu_A, B/\mu_B), (A/\nu_A, B/\nu_B)\rangle$. The intuitionistic fuzzy sets $0_\sim = \{ \langle x, 0, 1\rangle / x \in X \}$ and $1_\sim = \{ \langle x, 1, 0\rangle / x \in X \}$ are respectively the empty set and the whole set of X.

**Definition 2.3:** [4]

An intuitionistic fuzzy topology (IFT in short) on X is a family $\tau$ of IFSs in X satisfying the following axioms:
(i) $0_\sim, 1_\sim \in \tau$,
(ii) $G_1 \cap G_2 \in \tau$, for any $G_1, G_2 \in \tau$,
(iii) $\cup G_i \in \tau$ for any family $\{G_i / i \in J \} \subseteq \tau$.
In this case the pair $(X, \tau)$ is called an intuitionistic fuzzy topological space (IFTS in short) and any IFS in $\tau$ is known as an intuitionistic fuzzy open set(IFOS in short)in X. The complement $A^c$ of an IFOS A in an IFTS $(X, \tau)$ is called an intuitionistic fuzzy closed set (IFCS in short) in X.

**Definition 2.4:** [4]

Let $(X, \tau)$ be an IFTS and $A = \langle x, \mu_A, \nu_A \rangle$ be an IFS in X. Then

(i) $int(A) = \cup\{ G / G$ is an IFOS in X and $G \subseteq A\}$,

(ii) $cl(A) = \cap\{ K / K$ is an IFCS in X and $A \subseteq K\}$,

(iii) $cl(A^c) = (int(A))^c$,

(iv) $int(A^c) = (cl(A))^c$.

**Definition 2.5:** [11]

An IFS A of an IFTS $(X, \tau)$ is an

(i) intuitionistic fuzzy semiclosed set (IFSCS in short) if $int(cl(A)) \subseteq A$,

(ii) intuitionistic fuzzy semiopen set (IFSOS in short) if $A \subseteq cl(int(A))$.

**Definition 2.6:** [6]

An IFS A of an IFTS $(X, \tau)$ is an

(i) intuitionistic fuzzy preclosed set (IFPCS in short) if $cl(int(A)) \subseteq A$,

(ii) intuitionistic fuzzy preopen set (IFPOS in short) if $A \subseteq int(cl(A))$.

Note that every IFOS in $(X, \tau)$ is an IFPOS in X.

**Definition 2.7:** [6]

An IFS A of an IFTS $(X, \tau)$ is an

(i) intuitionistic fuzzy regular closed set (IFRCS in short) if $A = cl(int(A))$,

(ii) intuitionistic fuzzy regular open set (IFROS in short) if $A = int(cl(A))$.

**Definition 2.8:** [19]

An IFS A of an IFTS $(X, \tau)$ is an

(i) intuitionistic fuzzy semipre closed set (IFSPCS for short) if there exists an IFPCS B such that $int(B) \subseteq A \subseteq B$,

(ii) intuitionistic fuzzy semipre open set (IFSPOS for short) if there exists an IFPOS B such that $B \subseteq A \subseteq cl(B)$.

**Definition 2.9:** [9]

Let A be an IFS in an IFTS $(X, \tau)$. Then

(i) $spint(A) = \cup\{G / G$ is an IFSPOS in X and $G \subseteq A\}$.

(ii) $spcl(A) = \cap\{K / K$ is an IFSPCS in X and $A \subseteq K\}$.

Note that for any IFS A in $(X, \tau)$, we have $spcl(A^c) = (spint(A))^c$ and $spint(A^c) = (spcl(A))^c$.

**Definition 2.10:**

Let A be an IFS in an IFTS $(X, \tau)$. Then

(i) intuitionistic fuzzy regular generalized closed set (IFRGCS for short) if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is an Intuitionistic fuzzy regular open in X [12],

(ii) intuitionistic fuzzy generalized pre regular closed set (IFGPRCS for short) if $pcl(A) \subseteq U$ whenever $A \subseteq U$ and U is an Intuitionistic fuzzy regular open in X [15],

(iii) intuitionistic fuzzy generalized pre closed set (IFGPCS for short) if $pcl(A) \subseteq U$ whenever $A \subseteq U$ and U is an Intuitionistic fuzzy open in X [7].

An IFS A of an IFTS $(X, \tau)$ is called an intuitionistic fuzzy regular generalized open set, intuitionistic fuzzy generalized pre regular open set and intuitionistic fuzzy generalized pre open set (IFRGOS, IFGPROS and IFGPOS in short) if the complement $A^c$ is an IFRGCS, IFGPRCS and IFGPCS respectively.

**Definition 2.11:** [17]

An IFS A in an IFTS $(X, \tau)$ is said to be an intuitionistic fuzzy semipre generalized closed set (IFSPGCS for short) if $spcl(A) \subseteq U$ whenever $A \subseteq U$ and U is an IFSOS in $(X, \tau)$.

**Definition 2.12:** [16]

The complement $A^c$ of an IFSPGCS A in an IFTS $(X, \tau)$ is called an intuitionistic fuzzy semipre generalized open set (IFSPGOS for short) in X.

**Definition 2.13:** [9]

An IFS A is an IFTS $(X, \tau)$ is said to be an intuitionistic fuzzy generalized semipre closed set (IFGSPCS) if $spcl(A) \subseteq U$ whenever $A \subseteq U$ and U is an IFOS in $(X, \tau)$. An IFS A of an IFTS $(X, \tau)$ is called an intuitionistic fuzzy generalized semipre open set (IFGSPOS in short) if $A^c$ is an IFGSPCS in $(X, \tau)$.

**Definition 2.14:** [8]

An IFS A in an IFTS $(X, \tau)$ is said to be an intuitionistic fuzzy generalized semipre regular closed set (IFGSPRCS for short) if $spcl(A) \subseteq U$ whenever $A \subseteq U$ and U is an IFROS in $(X, \tau)$.

Every IFCS, IFRCS, IFPCS, IFSCS, IFSPCS, IFSPGCS, IFGSPCS, IFRGCS, IFGPRCS is an IFGSPRCS but the converses are not true in general.

**Definition 2.15:** [8]

The complement $A^c$ of an IFGSPRCS A in an IFTS $(X, \tau)$ is called an intuitionistic fuzzy generalized semipre regular open set (IFGSPROS for short) in X.

The family of all IFGSPROSs of an IFTS $(X, \tau)$ is denoted by IFGSPRO(X). Every IFOS, IFROS, IFPOS, IFSOS, IFSPOS, IFSPGOS, IFGSPOS, IFRGOS, IFGPROS is an IFGSPROS but the converses are not true in general.

**Definition 2.16:** [11]

Let $\alpha, \beta \in [0, 1]$ and $\alpha + \beta \leq 1$. An intuitionistic fuzzy point (IFP for short) $p_{(\alpha, \beta)}$ of X is an IFS of X defined by $p_{(\alpha, \beta)}(y) = (\alpha, \beta)$ if $y = p$ or $(0, 1)$ if $y \neq p$.

**Result 2.17:** [2]

For an IFS A in an IFTS$(X, \tau)$, we have $spcl(A) \supseteq A \cup (int(cl(int(A))))$.

**Definition 2.18:** [8]

If every IFGSPRCS in $(X, \tau)$ is an IFSPCS in $(X, \tau)$, then the space can be called as an intuitionistic fuzzy semipre regular $T_{1/2}$ (IFSPR$T_{1/2}$ for short) space.

**Definition 2.19:** [6]

Let f be a mapping from an IFTS $(X, \tau)$ into an IFTS $(Y, \sigma)$. Then f is said to be intuitionistic fuzzy continuous (IF continuous in short) if $f^{-1}(B) \in IFO(X)$ for every $B \in \sigma$.

**Definition 2.20:** [6]

Let f be a mapping from an IFTS $(X, \tau)$ into an IFTS $(Y, \sigma)$. Then f is said to be an

(i) intuitionistic fuzzy semi continuous(IFS continuous in short) if $f^{-1}(B) \in IFSO(X)$ for every $B \in \sigma$,
(ii) intuitionistic fuzzy pre continuous(IFP continuous in short) if $f^{-1}(B) \in IFPO(X)$ for every $B \in \sigma$.

**Definition 2.21:** [19]

Let f be a mapping from an IFTS $(X, \tau)$ into an IFTS $(Y, \sigma)$. Then f is said to be an intuitionistic fuzzy semipre continuous (IFSP continuous for short) mapping if $f^{-1}(B) \in IFSPO(X)$ for every $B \in \sigma$.
Every IFS continuous mapping and IFP continuous mappings are IFSP continuous mapping but the converses may not be true in general.

**Definition 2.22:** [14]

A mapping $f: (X, \tau) \to (Y, \sigma)$ is called an intuitionistic fuzzy regular generalized continuous (IFRG continuous for short) mappings if $f^{-1}(V)$ is an IFRGCS in $(X, \tau)$ for every IFCS V of $(Y, \sigma)$.

**Definition 2.23:** [13]

A mapping $f: (X, \tau) \to (Y, \sigma)$ be an intuitionistic fuzzy regular generalized irresolute (IFRG irresolute) mapping if $f^{-1}(V)$ is an IFRGCS in $(X, \tau)$ for every IFRGCS V of $(Y, \sigma)$.

**Definition 2.24:** [15]

A mapping $f: (X, \tau) \to (Y, \sigma)$ is called an intuitionistic fuzzy generalized pre regular continuous (IFGPR continuous for short) mappings if $f^{-1}(V)$ is an IFGPRCS in $(X, \tau)$ for every IFCS V of $(Y, \sigma)$.

**Definition 2.25:** [18]

A mapping $f: (X, \tau) \to (Y, \sigma)$ is called an intuitionistic fuzzy semipre generalized continuous (IFSPG continuous for short) mappings if $f^{-1}(V)$ is an IFSPGCS in $(X, \tau)$ for every IFCS V of $(Y, \sigma)$.

**Definition 2.26:** [10]

A mapping $f: (X, \tau) \to (Y, \sigma)$ is called an intuitionistic fuzzy generalized semipre continuous (IFGSP continuous for short) mapping if $f^{-1}(V)$ is an IFGSPCS in $(X, \tau)$ for every IFCS V of $(Y, \sigma)$.

**Result 2.27:** [8]

For any IFS A in $(X, \tau)$ where X is an IFSPR$T_{1/2}$ space, $A \in$ IFGSPRO(X) if and only if for every IFP $p_{(\alpha, \beta)} \in A$, there exists an IFGSPROS B in X such that $p_{(\alpha, \beta)} \in B \subseteq A$.

## 3. Intuitionistic Fuzzy Generalized Semipre Regular Continuous Mappings

Govindappa Navalagi, A. S. Chandrashekarappa and S. V. Gurushantanavar [5] have introduced GSPR-continuous mappings in topological spaces. In this section we have introduced intuitionistic fuzzy generalized semipre regular continuous mapping and investigated some of its properties in intuitionistic fuzzy topological spaces.

**Definition 3.1:**

A mapping $f: (X, \tau) \to (Y, \sigma)$ is called an intuitionistic fuzzy generalized semipre regular continuous (IFGSPR continuous for short) mappings if $f^{-1}(V)$ is an IFGSPRCS in $(X, \tau)$ for every IFCS V of $(Y, \sigma)$.
For the sake of simplicity, we shall use the notation $A = \langle x, (\mu, \mu), (\nu, \nu) \rangle$ instead of $A = \langle x, (a/\mu_a, b/\mu_b), (a/\nu_a, b/\nu_b) \rangle$ in all the examples used in this paper.

**Example 3.2:**

Let X= {a, b}, Y= {u, v} and $G_1$ = <x, (0.5, 0.4), (0.5, 0.6)>, $G_2$ = <y, (0.6, 0.7), (0.4, 0.2)>. Then $\tau = \{0_\sim, G_1, 1_\sim\}$ and $\sigma = \{0_\sim, G_2, 1_\sim\}$ are IFTs on X and Y respectively. Define a mapping f: $(X, \tau) \to (Y, \sigma)$ by f(a) = u and f(b) = v. Then f is an IFGSPR continuous mapping.

**Theorem 3.3:**

Every IF continuous mapping is an IFGSPR continuous mapping but not conversely.
**Proof:** Let f: $(X, \tau) \to (Y, \sigma)$ be an IF continuous mapping. Let V be an IFCS in Y. Then $f^{-1}(V)$ is an IFCS in X. Since every IFCS is an IFGSPRCS, $f^{-1}(V)$ is an IFGSPRCS in X. Hence f is an IFGSPR continuous mapping.

**Example 3.4:**

In Example 3.2, f: $(X, \tau) \to (Y, \sigma)$ is an IFGSPR continuous mapping but not an IF continuous mapping. Since $G_2$ = <y, (0.6, 0.7), (0.4, 0.2)> is an IFOS in Y but $f^{-1}(G_2)$ = <x, (0.6, 0.7), (0.4, 0.2)> is not an IFOS in X.

**Theorem 3.5:**

Every IFS continuous mapping is an IFGSPR continuous mapping but not conversely.
**Proof:** Let f: $(X, \tau) \to (Y, \sigma)$ be an IFS continuous mapping. Let V be an IFCS in Y. Then $f^{-1}(V)$ is an IFSCS in X. Since every IFSCS is an IFGSPRCS, $f^{-1}(V)$ is an IFGSPRCS in X. Hence f is an IFGSPR continuous mapping.

**Example 3.6:**

In Example 3.2, f: $(X, \tau) \to (Y, \sigma)$ is an IFGSPR continuous mapping but not an IFS continuous mapping. Since $G_2$ = <y, (0.6, 0.7), (0.4, 0.2)> is an IFOS in Y but $f^{-1}(G_2)$ = <x, (0.6, 0.7), (0.4, 0.2)> is not an IFSOS in X.

**Theorem 3.7:**

Every IFP continuous mapping is an IFGSPR continuous mapping but not conversely.
**Proof:** Let f: $(X, \tau) \to (Y, \sigma)$ be an IFP continuous mapping. Let V be an IFCS in Y. Then $f^{-1}(V)$ is an IFPCS in X. Since every IFPCS is an IFGSPRCS, $f^{-1}(V)$ is an IFGSPRCS in X. Hence f is an IFGSPR continuous mapping.

**Example 3.8:**

Let X = {a, b}, Y = {u, v} and $G_1$ = <x, (0.3, 0.2), (0.6, 0.6)>, $G_2$ = <y, (0.4, 0.5), (0.6, 0.5)>. Then $\tau = \{0_\sim, G_1, 1_\sim\}$ and $\sigma = \{0_\sim, G_2, 1_\sim\}$ are IFTs on X and Y respectively. Define a mapping f: $(X, \tau) \to (Y, \sigma)$ by f(a) = u and f(b) = v. Then f is an IFGSPR continuous mapping but not an IFP continuous mapping.

**Theorem 3.9:**

Every IFSP continuous mapping is an IFGSPR continuous mapping but not conversely.
**Proof:** Let f: $(X, \tau) \to (Y, \sigma)$ be an IFSP continuous mapping. Let V be an IFCS in Y. Then $f^{-1}(V)$ is an IFSPCS in X. Since every IFSPCS is an IFGSPRCS, $f^{-1}(V)$ is an IFGSPRCS in X. Hence f is an IFGSPR continuous mapping.

**Example 3.10**:

Let X = {a, b}, Y = {u, v} and $G_1$ = <x, (0.7, 0.8), (0.3, 0.2)>, $G_2$ = <x, (0.2, 0.1), (0.8, 0.9)>, $G_3$ = <x, (0.5, 0.6), (0.5, 0.4)>, $G_4$ = <x, (0.6, 0.7), (0.4, 0.3)>, $G_5$ = <y, (0.1, 0.4), (0.9, 0.6)>. Then $\tau = \{0_\sim, G_1, G_2, G_3, G_4, 1_\sim\}$ and $\sigma = \{0_\sim, G_5, 1_\sim\}$ are IFTs on X and Y respectively. Define a mapping f: $(X, \tau) \to (Y, \sigma)$ by f(a) = u and f(b) = v. Then f is an IFGSPR continuous mapping but not an IFSP continuous mapping.

**Theorem 3.11:**

Every IFRG continuous mapping is an IFGSPR continuous mapping but not conversely.
**Proof:** Let f: $(X, \tau) \to (Y, \sigma)$ be an IFRG continuous mapping. Let V be an IFCS in Y. Then $f^{-1}(V)$ is an IFRGCS in X. Since every IFRGCS is an IFGSPRCS, $f^{-1}(V)$ is an IFGSPRCS in X. Hence f is an IFGSPR continuous mapping.

**Example 3.12:**

Let X = {a, b}, Y = {u, v} and $G_1$ = <x, (0.5, 0.4), (0.5, 0.6)>, $G_2$ = <y, (0.6, 0.7), (0.4, 0.2)>. Then $\tau = \{0_\sim, G_1, 1_\sim\}$ and $\sigma = \{0_\sim, G_2, 1_\sim\}$ are IFTs on X and Y respectively. Define a mapping f: $(X, \tau) \to (Y, \sigma)$ by f(a) = u and f(b) = v. Then f is an IFGSPR continuous mapping but not an IFRG continuous mapping.

**Theorem 3.13:**

Every IFGPR continuous mapping is an IFGSPR continuous mapping but not conversely.
**Proof:** Let f: $(X, \tau) \to (Y, \sigma)$ be an IFGPR continuous mapping. Let V be an IFCS in Y. Then $f^{-1}(V)$ is an IFGPRCS in X. Since every IFGPRCS is an IFGSPRCS, $f^{-1}(V)$ is an IFGSPRCS in X. Hence f is an IFGSPR continuous mapping.

**Example 3.14:**

Let X = {a, b}, Y = {u, v} and $G_1$ = <x, (0.7, 0.8), (0.3, 0.2)>, $G_2$ = <x, (0.2, 0.1), (0.8, 0.9)>, $G_3$ = <x, (0.5, 0.6), (0.5, 0.4)>, $G_4$ = <x, (0.6, 0.7), (0.4, 0.3)> and let $G_5$ = <y, (0.3, 0.2), (0.7, 0.8)>. Then τ = {0~, $G_1$, $G_2$, $G_3$, $G_4$, 1~} and σ = {0~, $G_5$, 1~} are IFTs on X and Y respectively. Define a mapping f: (X, τ) → (Y, σ) by f(a) = u and f(b) = v. Then f is an IFGSPR continuous mapping but not an IFGPR continuous mapping.

**Theorem 3.15:**

Every IFSPG continuous mapping is an IFGSPR continuous mapping but not conversely.
**Proof:** Let f: (X, τ) → (Y, σ) be an IFSPG continuous mapping. Let V be an IFCS in Y. Then $f^{-1}$(V) is an IFSPGCS in X. Since every IFSPGCS is an IFGSPRCS, $f^{-1}$(V) is an IFGSPRCS in X. Hence f is an IFGSPR continuous mapping.

**Example 3.16:**

Let X = {a, b}, Y = {u, v} and $G_1$ = <x, (0.3, 0.6), (0.7, 0.4)>, $G_2$ = <y, (0.7, 0.4), (0.3, 0.6)>. Then τ = {0~, $G_1$, 1~} and σ = {0~, $G_2$, 1~} are IFTs on X and Y respectively. Define a mapping f: (X, τ) → (Y, σ) by f(a) = u and f(b) = v. Then f is an IFGSPR continuous mapping but not an IFSPG continuous mapping.

**Theorem 3.17:**

Every IFGSP continuous mapping is an IFGSPR continuous mapping but not conversely.
**Proof:** Let f: (X, τ) → (Y, σ) be an IFGSP continuous mapping. Let V be an IFCS in Y. Then $f^{-1}$(V) is an IFGSPCS in X. Since every IFGSPCS is an IFGSPRCS, $f^{-1}$(V) is an IFGSPRCS in X. Hence f is an IFGSPR continuous mapping.

**Example 3.18:**

In Example 3.16, f: (X, τ) → (Y, σ) is an IFGSPR continuous mapping but not an IFGSP continuous mapping. Since $G_2$ = <y, (0.7, 0.4), (0.3, 0.6)> is an IFOS in Y but $f^{-1}(G_2)$ = <x, (0.7, 0.4), (0.3, 0.6)> is not an IFGSPOS in X.

**Theorem 3.19:**

Let f: (X, τ) → (Y, σ) be a mapping where $f^{-1}$(V) is an IFRCS in X for every IFCS in Y. Then f is an IFGSPR continuous mapping but not conversely.
**Proof:** Let A be an IFCS in Y. Then $f^{-1}$(V) is an IFRCS in X. Since every IFRCS is an IFGSPRCS, $f^{-1}$(V) is an IFGSPRCS in X. Hence f is an IFGSPR continuous mapping.

**Example 3.20:**

In Example 3.2, f: (X, τ) → (Y, σ) is an IFGSPR continuous mapping but not a mapping defined in Theorem 3.19.

**Theorem 3.21:**

If f: (X, τ) → (Y, σ) is an IFGSPR continuous mapping, then for each IFP $p_{(\alpha, \beta)}$ of X and each A ∈ σ such that $f(p_{(\alpha, \beta)})$ ∈ A. Then there exists an IFGSPROS B of X such that $p_{(\alpha, \beta)}$ ∈ B and f(B) ⊆ A.
**Proof:** Let $p_{(\alpha, \beta)}$ be an IFP of X and A ∈ σ such that $f(p_{(\alpha, \beta)})$ ∈ A. Put B = $f^{-1}$(A). Then by hypothesis B is an IFGSPROS in X such that $p_{(\alpha, \beta)}$ ∈ B and f(B) = $f(f^{-1}(A))$ ⊆ A.

**Theorem 3.22:**

Let f: (X, τ) → (Y, σ) be an IFGSPR continuous mapping. Then f is an IFSP continuous mapping if X is an IFSPRT$_{1/2}$ space.
**Proof:** Let V be an IFCS in Y. Then $f^{-1}$(V) is an IFGSPRCS in X, by hypothesis. Since X is an IFSPRT$_{1/2}$ space, $f^{-1}$(V) is an IFSPCS in X. Hence f is an IFSP continuous mapping.

**Theorem 3.23:**

Let f: (X, τ) → (Y, σ) be an IFGSPR continuous mapping and g: (Y, σ) → (Z, $\eta$) is an IF continuous mapping, then g ∘ f : (X, τ) →(Z, $\eta$) is an IFGSPR continuous mapping.
**Proof:** Let V be an IFCS in Z. Then $g^{-1}$(V) is an IFCS in Y, by hypothesis. Since f is an IFGSPR continuous mapping, $f^{-1}(g^{-1}(V))$ is an IFGSPRCS in X. Hence g ∘ f is an IFGSPR continuous mapping.

**Theorem 3.24:**

Let f: (X, τ) → (Y, σ) be a mapping from an IFTS X into an IFTS Y. Then the following conditions are equivalent if X and Y are IFSPRT$_{1/2}$ space:
(i) f is an IFGSPR continuous mapping,
(ii) $f^{-1}$(B) is an IFGSPROS in X for each IFOS B in Y,
(iii) for every IFP $p_{(\alpha, \beta)}$ in X and for every IFOS B in Y such that $f(p_{(\alpha, \beta)})$ ∈ B, there exists an IFGSPROS A in X such that $p_{(\alpha, \beta)}$ ∈ A and f(A) ⊆ B.
**Proof:** (i) ⇔ (ii) is obvious, since $f^{-1}(A^c) = (f^{-1}(A))^c$.
(ii) ⇒ (iii) Let B be any IFOS in Y and let $p_{(\alpha, \beta)}$ ∈ X. Given $f(p_{(\alpha, \beta)})$ ∈ B. By hypothesis $f^{-1}$(B) is an IFGSPROS in X. Take A= $f^{-1}$(B). Now $p_{(\alpha, \beta)}$ ∈ $f^{-1}(f(p_{(\alpha, \beta)}))$. Therefore $f^{-1}(f(p_{(\alpha, \beta)}))$ ∈ $f^{-1}$(B) = A. This implies $p_{(\alpha, \beta)}$ ∈ A and f(A) = $f(f^{-1}(B))$ ⊆ B.

(iii) ⇒ (i) Let A be an IFCS in Y. Then its complement, say $B = A^c$, is an IFOS in Y. Let $p_{(\alpha, \beta)} \in C$ and $f(C) \subseteq B$. Now $C \subseteq f^{-1}(f(C)) \subseteq f^{-1}(B)$. Thus $p_{(\alpha, \beta)} \in f^{-1}(B)$. Therefore $f^{-1}(B)$ is an IFGSPROS in X by Result 2.27. That is $f^{-1}(A^c)$ is an IFGSPROS in X and hence $f^{-1}(A)$ is an IFGSPRCS in X. Thus f is an IFGSPR continuous mapping.

## 4. Intuitionistic Fuzzy Generalized Semipre Regular Irresolute Mappings

In this section we have introduced intuitionistic fuzzy generalized semipre regular irresolute mappings and studied some of their properties.

**Definition 4.1:**

A mapping f: $(X, \tau) \to (Y, \sigma)$ be an intuitionistic fuzzy generalized semipre regular irresolute (IFGSPR irresolute) mapping if $f^{-1}(V)$ is an IFGSPRCS in $(X, \tau)$ for every IFGSPRCS V of $(Y, \sigma)$.

**Theorem 4.2:**

Let f: $(X, \tau) \to (Y, \sigma)$ and g: $(Y, \sigma) \to (Z, \eta)$ be IFGSPR irresolute mapping. Then $g \circ f : (X, \tau) \to (Z, \eta)$ is an IFGSPR irresolute mapping.

**Proof:** Let V be an IFGSPRCS in Z. Then $g^{-1}(V)$ is an IFGSPRCS in Y. Since f is an IFGSPR irresolute, $f^{-1}(g^{-1}(V))$ is an IFGSPRCS in X, by hypothesis. Hence $g \circ f$ is an IFGSPR irresolute mapping.

**Theorem 4.3:**

Let f: $(X, \tau) \to (Y, \sigma)$ be an IFGSPR irresolute mapping and g: $(Y, \sigma) \to (Z, \eta)$ be IFGSPR continuous mapping, the $g \circ f : (X, \tau) \to (Z, \eta)$ is an IFGSPR continuous mapping.

**Proof:** Let V be an IFCS in Z. Then $g^{-1}(V)$ is an IFGSPRCS in Y. Since f is an IFGSPR irresolute mapping, $f^{-1}(g^{-1}(V))$ is an IFGSPRCS in X. Hence $g \circ f$ is an IFGSPR continuous mapping.

**Theorem 4.4:**

Let f: $(X, \tau) \to (Y, \sigma)$ be a mapping from an IFTS X into an IFTS Y. Then the following conditions are equivalent if X and Y are $\text{IFSPRT}_{1/2}$ space:

(i) f is an IFGSPR irresolute mapping,

(ii) $f^{-1}(B)$ is an IFGSPROS in X for each IFGSPROS B in Y,

(iii) $f^{-1}(\text{spint}(B)) \subseteq \text{spint}(f^{-1}(B))$ for each IFS B of Y,

(iv) $\text{spcl}(f^{-1}(B)) \subseteq f^{-1}(\text{spcl}(B))$ for each IFS B of Y.

**Proof:** (i) ⇔ (ii) is obvious, since $f^{-1}(A^c) = (f^{-1}(A))^c$.

(ii) ⇒ (iii) Let B be any IFS in Y and $\text{spint}(B) \subseteq B$. Also $f^{-1}(\text{spint}(B)) \subseteq f^{-1}(B)$. Since spint(B) is an IFSPOS in Y, it is an IFGSPROS in Y. Therefore $f^{-1}(\text{spint}(B))$ is an IFGSPROS in X, by hypothesis. Since X is an $\text{IFSPRT}_{1/2}$ space, $f^{-1}(\text{spint}(B))$ is an IFSPOS in X. Hence $f^{-1}(\text{spint}(B)) = \text{spint}(f^{-1}(\text{spint}(B))) \subseteq \text{spint}(f^{-1}(B))$.

(iii) ⇒ (iv) is obvious by taking complement in (iii).

(iv)⇒ (i) Let B be an IFGSPRCS in Y. Since Y is an $\text{IFSPRT}_{1/2}$ space, B is an IFSPCS in Y and spcl(B) = B. Hence $f^{-1}(B) = f^{-1}(\text{spcl}(B)) \supseteq \text{spcl}(f^{-1}(B))$, by hypothesis. But $f^{-1}(B) \subseteq \text{spcl}(f^{-1}(B))$. Therefore $\text{spcl}(f^{-1}(B)) = f^{-1}(B)$. This implies $f^{-1}(B)$ is an IFSPCS and hence it is an IFGSPRCS in X. Thus f is an IFGSPR irresolute mapping.

**Theorem 4.5:**

Let f : $(X, \tau) \to (Y, \sigma)$ be a an IFGSPR irresolute mapping from an IFTS X into an IFTS Y. Then $f^{-1}(B) \subseteq \text{spint}(f^{-1}(\text{cl}(\text{int}(\text{cl}(B)))))$ for every IFGSPROS B in Y, if X and Y are $\text{IFSPRT}_{1/2}$ spaces.

**Proof:** Let B be an IFGSPROS in Y. Then by hypothesis $f^{-1}(B)$ is an IFGSPROS in X. Since X is an $\text{IFSPRT}_{1/2}$ space, $f^{-1}(B)$ is an IFSPOS in X. Therefore $\text{spint}(f^{-1}(B)) = f^{-1}(B)$. Since Y is an $\text{IFSPRT}_{1/2}$ space, B is an IFSPOS in Y and $B \subseteq \text{cl}(\text{int}(\text{cl}(B)))$. Now $f^{-1}(B) = \text{spint}(f^{-1}(B))$ implies, $f^{-1}(B) \subseteq \text{spint}(f^{-1}(\text{cl}(\text{int}(\text{cl}(B)))))$.

**Theorem 4.6:**

Let f: $(X, \tau) \to (Y, \sigma)$ be a an IFGSPR irresolute mapping from an IFTS X into an IFTS Y. Then $f^{-1}(B) \subseteq \text{spint}(\text{cl}(\text{int}(\text{cl}(f^{-1}(B)))))$ for every IFGSPROS B in Y, if X and Y are $\text{IFSPRT}_{1/2}$ spaces.

**Proof:** Let B be an IFGSPROS in Y. Then by hypothesis $f^{-1}(B)$ is an IFGSPROS in X. Since X is an $\text{IFSPRT}_{1/2}$ space, $f^{-1}(B)$ is an IFSPOS in X. Therefore $\text{spint}(f^{-1}(B)) = f^{-1}(B) \subseteq \text{cl}(\text{int}(\text{cl}(f^{-1}(B))))$. Hence $f^{-1}(B) \subseteq \text{spint}(\text{cl}(\text{int}(\text{cl}(f^{-1}(B)))))$.

**Definition 4.7:**

A function $(X, \tau)$ is said to be a ${}_{gspr}\text{IFT}_{rg}$ space if every intuitionistic fuzzy generalized semipre regular closed set is an intuitionistic fuzzy regular generalized closed set.

**Theorem 4.8:**

Let f: $(X, \tau) \to (Y, \sigma)$ be a mapping from an IFTS X into an IFTS Y. Then the following conditions are equivalent if X and Y are ${}_{gspr}\text{IFT}_{rg}$ space:

(i) f is an IFRG irresolute mapping,

(ii) f is an IFGSPR irresolute mapping.

**Proof:** (i) ⇒ (ii): Let f: $(X, \tau) \to (Y, \sigma)$ be an IFRG irresolute. Let V be an IFGSPRCS in Y. As Y is ${}_{gspr}\text{IFT}_{rg}$

space, then V is an IFRGCS in Y. Since f is an IFRG irresolute, $f^{-1}(V)$ is an IFRGCS in X. But every IFRGCS is an IFGSPRCS in X and hence $f^{-1}(V)$ is an IFGSPRCS in X. Therefore, f is an IFGSPR irresolute mapping.

(ii) ⇒ (i): Let f: $(X, \tau) \to (Y, \sigma)$ be an IFGSPR irresolute. Let V be an IFRGCS in Y. But every IFRGCS is an IFGSPRCS and hence V is an IFGSPRCS in Y and f is an IFGSPR irresolute implies $f^{-1}(V)$ is an IFGSPRCS in X. But X is ${}_{gspr}IFT_{rg}$ space and hence $f^{-1}(V)$ is an IFRGCS in X. Thus, f is an IFRG irresolute mapping.

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